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Sports statistics

# Trends and random fluctuations in athletics

Improvements in the results of athletic competitions are often considered to stem from better training and equipment, but elements of chance are always present in athletics and these also contribute. Here we distinguish between these two effects by estimating the range into which athletic records would have fallen in the absence of systematic progress and then comparing this with actual performance results. We find that only 4 out of 22 disciplines have shown a systematic improvement, and that annual best results worldwide[1] show saturation in some disciplines.

We investigated the development of sporting records by means of order statistics[2] that provide estimates of the maxima and minima of stochastic time series. We used the results to predict (retrospectively) how athletes' records would have evolved during the German championships[3,4] in the absence of systematic improvements. (These can arise from an increase in the number of competition participants, a major effect during the past century.)

We assumed that the stochastic variations are the result of a strongly stationary process in which two consecutive values are independent and all values obey the same statistical distribution, $\rho(x)$. The records considered here may be either maxima (jumping and throwing distances) or minima (running times); we rendered these two types directly comparable by converting running times into average speeds, allowing us to discuss only the case of maxima. We used the values of one time interval (here, annual best results) to determine the most probable record, $x_{\max,\mathrm{th}}$ (expectation value), and its standard deviation, $\sigma_E$, for the succeeding interval.

Assuming that $\rho(x)$ is gaussian (with mean $\mu$ and standard deviation $\sigma$), the expectation value for the record during the subsequent period of $N$ years can be estimated as

$$x_{\max,\mathrm{th}} = \mu + \sigma(a_0 + a_1 \ln\ln(N) + a_2(\ln\ln(N))^2) \quad (1)$$

where the optimal coefficients for $2 \leqslant N \leqslant 100$ are (the error of the approximation of $x_{\max,\mathrm{th}}$ for $\mu = 0$ and $\sigma = 1$ being $< 0.06\%$ over this interval): $a_0 = 0.818$, $a_1 = 0.574$ and $a_2 = 0.349$ (D.G., J.G.T. and D.S., manuscript in preparation). Analogously, we find that

$$\sigma_E = \sigma(b_0 + b_1 \ln\ln(N) + b_2(\ln\ln(N))^2) \quad (2)$$

where $b_0 = 0.8023$, $b_1 = -0.2751$ and $b_2 = 0.0020$ (approximation error $< 0.15\%$). We derived these equations from formulae for maxima estimation that hold in the limit of large $N$ (ref. 5).

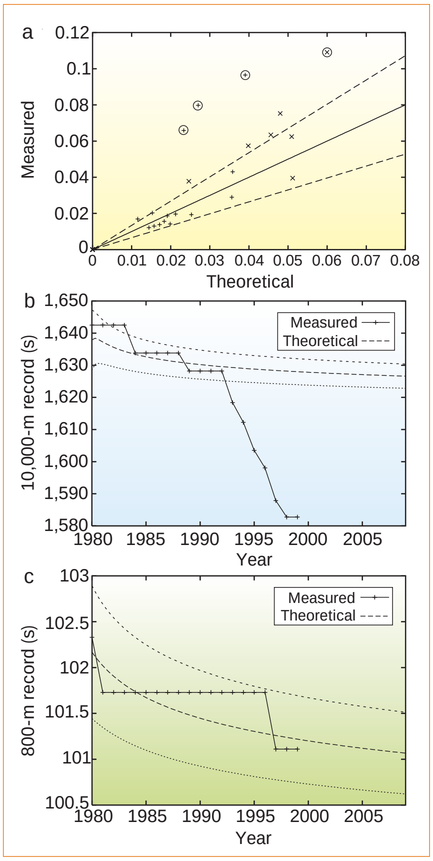


**Figure 1** Trends versus random fluctuations in athletics. **a,** Comparison of actual and theoretical results for the relative differences between the best performances of the second 12-yr interval and the mean annual best result of the first 12-yr interval for the German championships. 'Theoretical' ($x$-axis): $(x_{\max,\mathrm{th}}/\mu) - 1$; 'measured' ($y$-axis): $x_{\max,\mathrm{real}}/\mu) - 1$; $+$, running disciplines; $\times$, jumping and throwing disciplines. The solid line $y = x$ represents perfect theoretical predictions, dashed lines (not symmetrical to the $y = x$ line) indicate $\pm$s.d. intervals ($\sigma_E$). The greatest deviations correspond to the 110-m hurdles, 50-km and 20-km walking races, and pole vault. **b, c,** Representative forecasts of world records obtained using equation (1) and the annual best results from the period 1980–89. The time course in **b** (for the 10,000 m) shows a strong trend beginning in 1992; that in **c** for the 800 m seems to be purely stochastic.

We applied this method to the top performances of men in 22 disciplines during the German championships of 1973–96. We divided the data into two groups: those for the years 1973–84 served as a reference period from which we predicted the best result for the period 1985–96. A Kolmogorov–Smirnov test showed that a gaussian function is a suitable approximation of the probability distributions of our data. The distribution should have cut-offs at zero and at some physiological limit[6], but we found that these hypothetical cut-offs could be safely neglected.

Figure 1a compares the stochastic predictions with the actual data for all 22 disciplines. Most of the experimentally determined points are well described by the statistical analysis. Only four disciplines show a significant deviation — for these, the results achieved are better than the predictions, indicating systematic improvement. A previous analysis of world records[7] ignored trends and considered only record-breaking events.

We also analysed a similar data set for worldwide annual best results (for the outcome of the statistical test, see supplementary information), and found that 7 out of 19 estimates do not fall into a $3\sigma_E$ interval around $x_{\max,\mathrm{th}}$ for the time-interval pairs 1985–97 and 1990–99. Compared with the German results, stronger non-stationarities occur, indicating more systematic progress, but tend to decrease over time (D.G., J.G.T. and D.S., manuscript in preparation).

We also used the available data for 1980–89 to predict the development of world records ($N$ here being the number of years since the pre-1980 onset of the stationary process; Fig. 1).

The method we describe here should be useful for assessing the relevance of any small signal in a noisy environment[8,9], and in obtaining estimations for the quality of the best (unknown) solutions of computationally complex optimization problems.

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