High-fidelity gate operations for quantum computing beyond dephasing time limits

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The implementation of quantum gates with fidelities that exceed the threshold for reliable quantum computing requires robust gates whose performance is not limited by the precision of the available control fields. The performance of these gates also should not be affected by the noisy environment of the quantum register. Here we use randomized benchmarking of quantum gate operations to compare the performance of different families of gates that compensate errors in the control field amplitudes and decouple the system from the environmental noise. We obtain average fidelities of up to 99.8%, which exceeds the threshold value for some quantum error correction schemes as well as the expected limit from the dephasing induced by the environment.

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Scalable quantum computing requires gate operations with fidelities above a certain threshold [1,2]. Reaching this threshold remains challenging, primarily due to errors in the control fields driving the gate operations and the effects of a noisy environment. Different techniques have been developed to overcome these obstacles, such as dynamical decoupling (DD) [3,4]. An ideal dynamical decoupling sequence completely eliminates the interactions of the system with its environment and thereby "freezes" the system, apart from the evolution under the internal system Hamiltonian. This approach can therefore implement high-fidelity quantum memories. If, however, the specific application requires that the system evolves under a suitable control Hamiltonian, such as in a quantum information processor, the protection scheme and the control operations must be applied simultaneously. It is then necessary to design gates that combine processing with decoupling in such a way that the effect of the control fields survives, while all unwanted interactions are eliminated [5].

Measuring the fidelity of gate operations in the region of the threshold for scalable quantum computation cannot be done on individual gates, since the measurement errors can be comparable to or greater than the gate errors. For this purpose, the "randomized benchmarking" procedure was developed [6]. It measures an *average* gate fidelity of a random selection of gates. This allows a much more precise determination of the average gate fidelity. It does not provide information about the fidelity of individual gates, but it provides estimates of the fidelity of sequences of gates, which is the relevant quantity for the analysis of large-scale computation.

For this paper we measured average gate fidelities for families of gate operations that combine robustness against unwanted variations in the the control fields (flip angle errors) with protection against environmental noise. For this purpose, we use Clifford gates, which allow universal quantum computation when combined with magic states [7–9]. We determine their fidelities experimentally, using a nuclear spin as qubit and a different nuclear spin system as the noisy environment. The system qubit S is coupled to a noisy environment via a dephasing interaction $\mathcal{H}_{SE} = b(t)S_z$, where the (semi-)classical field $b(t)$ has a finite correlation time τ_c. Dynamical decoupling by sequences of inversion pulses can suppress the dephasing effect of this interaction and prevent the decay of the coherence in the system [3,10]. This is well explored in the context of quantum memories, where the goal is to keep a specific quantum state unchanged. Here the effect of DD can be roughly described as a prolongation of the dephasing time T_2.

In the case of quantum computing, the goal is not the preservation of a quantum state, but the precise control of a quantum system such that it evolves along a well-defined path in Hilbert space. Dephasing then creates errors roughly $\propto (1 - e^{-\tau/T_2}) \approx \tau/T_2$, where τ is the duration of the gate operation and T_2 the dephasing time. Extending the dephasing time by DD therefore also helps for quantum computing. However, in this case, an additional complication arises: the dynamical decoupling sequences that are used for protecting quantum memories would equally decouple the system from the external fields that are applied to control the system's path. It is therefore essential to combine dynamical decoupling and gate operations in such a way that they do not interfere destructively. Two general approaches have been proposed for solving this problem: to apply gate operations and DD operations successively [11], or to interleave gate operation and DD by splitting the gate operation into as many elements as there are delays between the DD pulses and modify them in such a way that the effect of the DD pulses is to revert these modifications and the overall effect becomes that of the targeted gate operation [5,11–17].

A third option is to design gate operations that are robust against experimental imperfections and have "built-in" the effect of a dynamical decoupling sequence. Such a decoupling effect is, e.g., built into π pulses around an axis in the xy plane: they invert the system operator S_z and therefore the system environment interaction. The sequences that we consider for randomized benchmarking consist of approximately 50% of π pulses. Accordingly, they automatically reduce the effect of the system-environment interaction, although their efficiency may be lower than that of specialized DD sequences.

Scalable quantum computing remains challenging, primarily due to errors in the control fields driving the gate operations and the effects of a noisy environment. Therefore, apart from reducing the coherence time, high fidelity operations also need to be robust against errors in the control fields driving the gate operations. The BB1 composite pulse [see Fig. 1(g)] was introduced by Wimperis [18] as a possible scheme for generating composite rotation pulses that are

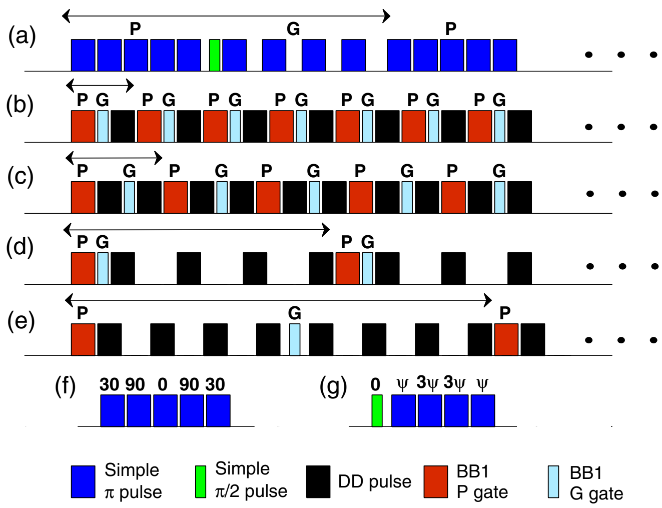

FIG. 1. (Color online) Sequences of gate operations compared in this work. The double arrows indicate the gate duration τ, defined as the time period between the beginning of one P gate to the beginning of the next P gate.

well compensated against amplitude errors. The sequence for generating a compensated rotation by an angle θ around an axis in the xy plane is

$$R_{\varphi}(\theta)R_{\beta+\varphi}(\pi)R_{3\beta+\varphi}(2\pi)R_{\beta+\varphi}(\pi), \tag{1}$$

where φ defines the orientation of the rotation axis and $\beta = \cos^{-1}(-\theta/4\pi)$. For the present purpose, we consider the cases $\theta = \pi/2$ and π, where $\beta_{90} \approx 1.7$ and $\beta_{180} \approx 1.8$. Protecting a system from the environmental noise while simultaneously driving logical gate operations can be achieved by inserting suitable free precession periods between BB1 pulses, in order to refocus the environmental perturbations. If the four π pulses of this sequence are separated in time by a delay that is twice the duration of the initial θ pulse [see Fig. 1(a)], this yields a sequence that is not only robust to amplitude errors, but also to environmental noise: it conforms to the "compute, then decouple" approach and the four π pulses generate the DD cycle.

A robust π pulse can also be generated by concatenating five π pulses with the phases $\pi/6$, 0, $\pi/2$, 0, $\pi/6$ [19] [see Fig. 1(f)]. This generates an inversion of the z component, but in addition also a $-\pi/3$ rotation around the z axis. We will refer to this pulse as KDD-5 $= R_z(-\pi/3)R_0 = R_0R_z(\pi/3)$, where R_0 is the targeted π rotation around the x axis. If it is used in a sequence like randomized benchmarking, the additional z rotation can be taken into account by adjusting the coordinate system of the qubit Bloch sphere.

To determine the average fidelity of the gate operations, we use the scheme proposed in Refs. [20,21]. It requires the application of $\pi/2$ and π rotations around the axes of the coordinate system. The individual gates are 1-qubit Clifford gates $C = PG$, where P indicates the unit operation or a π rotation around one of the coordinate axes ($\pm x, \pm y, \pm z$) and G a $\pi/2$ rotation around a coordinate axis. There are therefore 8 different P gates, 6 G gates, and 48 PG gates [20,21]. Rotations around $\pm z$ axes are implemented through a change to the rotating frame definition [20,21]. At the end of the sequence a recovery operation is applied that corresponds to the inverse of the sequence up to this point, so that the full sequence becomes a unit operation in the absence of imperfections. For optimal randomization, the recovery operation itself consists of two random P operations sandwiching the complement R, which is another Clifford gate. For a single qubit, the effect of a nonideal identity operation can be written as $\rho_{out} = (1-d)\rho_{in} + \frac{d}{2}\mathbf{1}$, where ρ_{in} is the initial state, $\mathbf{1}$ is the unit operator, and d is the depolarizing parameter. Here we use the fact that for a sequence of Clifford gates, the result can be represented as a depolarization channel [22,23]. The trace fidelity is then

$$F = \mathrm{tr}\{\rho_{out}\rho_{in}\} = 1 - \frac{d}{2} \tag{2}$$

and the error per gate is the difference EPG $= 1 - F = \frac{d}{2}$. The average fidelity for sequences with m gate operations decays as

$$F = \tfrac{1}{2}[1 + (1-d)^m] \approx \tfrac{1}{2}(1 + e^{-md}). \tag{3}$$

For the experiment we used the ^{13}C nuclear spins of adamantane, a molecular crystal. The protons in the same crystal provide the noisy environment, which leads to a dephasing time of 360 μs. A single refocusing pulse (Hahn echo) can extend the dephasing time to 740 μs, where the dephasing time here is defined as the time at which the signal decays to $1/e$ of its initial value. Strong couplings between the proton spins result in a rapidly fluctuating environment with a correlation time of the order of 100 μs [10], while the pulse times and separation between the pulses are of the order of a few microseconds. The ultimate limit on the information storage in this system is given by the energy relaxation time $T_1 = 1.52$ s. More information about the noisy environment can be found in previous works [10,24].

The system was initially in thermal equilibrium, with $\rho_{in} \propto S_z$. For each measurement we averaged over a set of 32 random sequences composed of 1 to 80 gate operations. After the restore operation, the signal was measured by applying one additional readout pulse that converted the remaining S_z component of the density operator into transverse magnetization. The free precession signal was acquired, Fourier-transformed, and the signal corresponding to the CH_2 carbon was integrated and used as a measure for the survival probability e^{-md}. Before benchmarking each gate type, the pulses used were calibrated and a reference signal acquired for normalization. We did not observe substantial difference between the calibrations performed and therefore errors due to fluctuations in the pulse generation can be neglected in our experiment. This fact can be attributed to the good stability of commercial NMR spectrometers. Since the readout pulse is much shorter than the total experimental time we can also neglect decoherence errors during the read out.

Figure 1 gives an overview over the different sequences tested in this work. First (not shown in the figure) we tested gates without DD protection, using two different implementations for P and G rotations: (i) simple rectangular pulses and (ii) BB1 pulses, which compensate errors in the amplitudes of the control fields. In the second approach [Fig. 1(a)] we generated a protected G rotation by separating the four π pulses of the BB1 composite rotation by a delay that is twice the duration of the initial $\pi/2$ pulse. The P rotation in

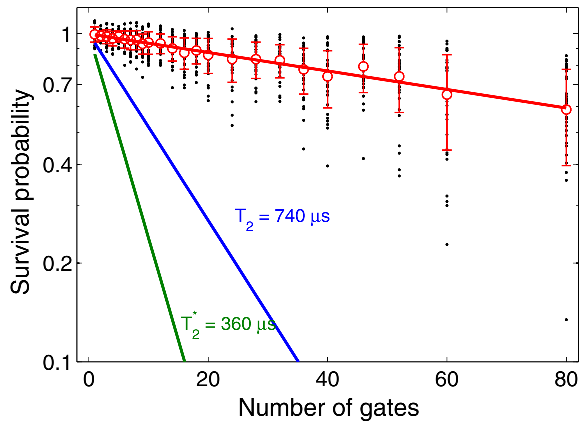

FIG. 2. (Color online) Experimental survival probabilities averaged over 32 sequences for BB1 gates as a function of the number of gates. The solid lines represent an exponential fit and theoretical predictions (see text).

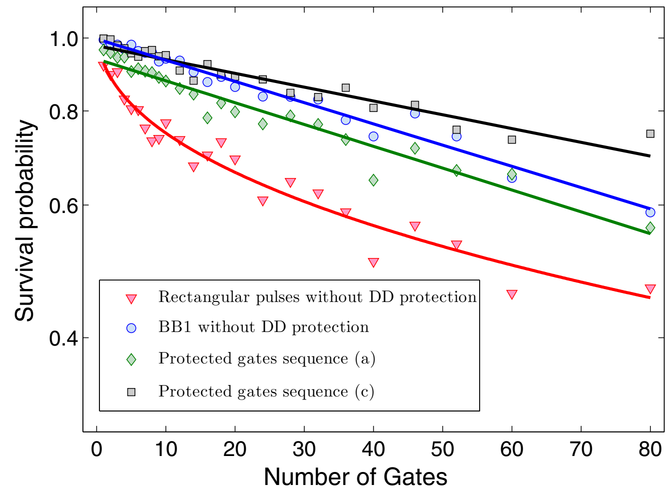

FIG. 3. (Color online) Experimental results for different types of gates.

this scheme was implemented by one KDD-5 pulse. The third set of sequences [Figs. 1(b)–1(e)] was designed to reduce experimental imperfections and decouple the system from its environment by interleaving the P and G rotations with dynamical decoupling pulses in different ways, using BB1 pulses for all PG rotations. In Figs. 1(b) and 1(c) the DD pulses were also implemented as BB1 pulses, while in Figs. 1(d) and 1(e) the DD pulses were implemented by rectangular pulses.

Figure 2 shows the experimental results obtained when composite BB1 pulses were used to implement the Clifford gates. Here the gate time was 76 μs. The blue and green lines show the theoretical prediction for pure dephasing with times of 740 and 360 μs, respectively, which would result in EPGs of 3.2% and 6.7%. The straight lines through the experimental data points are fits to exponential decays, the corresponding EPG is $0.32 \pm 0.03\%$, which is significantly better then the EPG extrapolated from the T_2 values. This is a clear indication that the environmental interaction is refocused during the benchmarking experiment, even without applying DD pulses.

In Fig. 3 we compare the BB1 pulses with the three other cases: (i) The approach defined in Fig. 1(a), (ii) interleaving BB1 pulses with the DD sequence XY-16 [4,25] [Fig. 1(c)], and (iii) rectangular pulses. The gate time of the of the first scheme is 88 μs and its performance is comparable to that of BB1, the observed EPG is $0.34 \pm 0.03\%$. In the case of interleaving BB1 pulses with DD, the gate time is 152 μs, which is approximately twice the gate time for BB1 gates without DD protection. However, the observed EPG is $0.22 \pm 0.03\%$, showing that dynamical decoupling sequences can further increase the fidelity of quantum gates. The rectangular pulses do not exhibit an exponential decay, since those pulses are not designed to compensate for field inhomogeneities. In this case we cannot assign a single value to the EPG. For a quantitative evaluation, we fitted the experimental data to the function a^{m^k}, similar to the so called stretched exponential function and often used as a phenomenological description of relaxation. The deviation of the parameter k from unity value can roughly be used to characterize how far the decay deviates from a single exponential decay [26]. The parameters obtained from this fit are $a \sim 0.88$ and $k \sim 0.44$.

If the decay is dominated by experimental imperfections, cyclic repetition of a pulse sequence is often not an optimal strategy, since the same errors are generated by each cycle and these errors accumulate in a coherent manner. This is verified by the data shown in Fig. 4. For the case of the XY-4 DD [4,25] sequence, the fidelity decays as $(1 - Q)^m$, where $Q \approx 0.05$ is the error per gate. In this case, each gate is a unity operation (NOOP gate) protected by two XY-4 cycles. If we apply a randomized sequence of gate operations instead of the NOOP gate, we obtain the data points labeled "benchmarking." Here the gate operations modify the error operation. As a result, the system does not evolve coherently away from the initial state, but its evolution corresponds more to a random walk. Accordingly, the experimental data points for the benchmarking sequence can be well described by the function $(1 - Q)^{\sqrt{m}}$, with the same error parameter Q. For

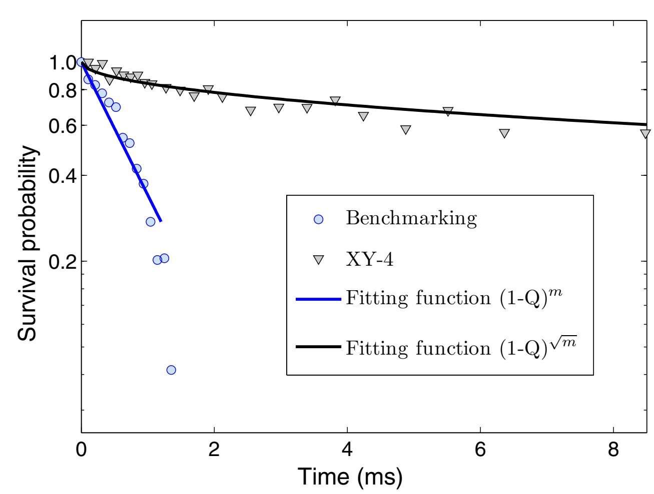

FIG. 4. (Color online) Decay of the survival probability measured by interleaving quantum gates with the dynamical decoupling XY-4 and pure dynamical decoupling.

TABLE I. Summary of the gate operations tested. For each type of gate, the gate duration τ, the experimental and limiting error per gates are given (see text). The labels (a)–(e) refers to the gates of Fig. 1.

		Gate				
	BB1	(a)	(b)	(c)	(d)	(e)
$\tau(\mu s)$	76	88	116	152	336	384
$\text{EPG}_m(10^{-4})$	5	6	8	10	22	25
$\text{EPG}_M(10^{-4})$	317	364	472	604	1191	1322
$\text{EPG}_{\text{expt}}(10^{-4})$	32 ± 3	34 ± 3	28 ± 3	22 ± 3	172 ± 6	47 ± 3

the benchmarking experiment the initial state was the thermal equilibrium $\rho_{\text{in}} \propto S_z$, while in the pure DD experiment the system was initialized in a superposition state and the separations between the DD pulses are kept the same in all cases.

These experimental data clearly show that a robust design of the gate operations can make their performance largely independent of experimental imperfections. This does not mean that the error per gate can be made arbitrarily small, since interactions that are not linear in the qubit operators $\vec{S}$ and whose correlation time is shorter than the experimentally accessible time scales are actually not suppressed. In the present system, ${}^{13}\text{C} - {}^{13}\text{C}$ couplings are bilinear and are not eliminated by the DD sequences used here, although other sequences exist for suppressing them [27,28]. Processes that contribute to the energy relaxation (T_1 processes) are characterized by correlation times comparable to or shorter than the Larmor precession period, since only fluctuations near the Larmor frequency can induce transition between the energy levels and thus cause longitudinal relaxation. The correlation time of these fluctuations is on a time scale of nanoseconds. This indicates that further improvements would require pulses that compensate also for homonuclear (bilinear) interactions.

Table I summarizes the results. For each type of gate it shows the observed EPG_{expt}. All gate operations include decoupling elements and thus result in an error per gate that is lower than EPG_M, which is estimated from the dephasing time measured by Hahn echo. This is a clear indication that gate operations alone provide some decoupling. Each sequence is also compared to the minimum $\text{EPG}_m \approx \tau/T_2$ values that is expected from the not-refocused environmental noise. These values were obtained by DD with robust pulses and very short delays between the pulses [29]. The best performance was achieved by the gate type (c), which corresponds to BB1 gates protected by the DD sequence XY-16.

The implementation of robust high-fidelity gate operations is an essential step towards reliable and scalable quantum computing. In this work we use randomized benchmarking of single qubit quantum gates to compare the performance of different families of gates that compensate errors in the control field amplitudes and decouple the system from the environmental noise. In some cases the total duration of the experiments (from ~1 to ~30 ms) exceeds the dephasing time, measured by Hahn echoes (740 μs), by almost two orders of magnitude. We could obtain average fidelities exceeding the expected limit from the dephasing induced by the environment. This is a clear indication that the effect of the noisy environment is reduced by the sequence of applied gate operations. The best average fidelity observed, 99.8%, was achieved by interleaving gate operations implemented by composite pulses with the dynamical decoupling sequence XY-16.

The improvements in gate accuracy by decoupling methods, as observed in this work, implies a reduction of the overhead cost of QEC since more noise can be tolerated by QEC codes combined with DD than by QEC alone [11]. Therefore, future work will be devoted to investigate the performance of QEC codes combined with dynamical decoupling sequences and to test the performance of DD protected gates in systems with multiple qubits.

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[1] E. Knill, R. Laflamme, and W. H. Zurek, Proc. R. Soc. London Sect. A **454**, 365 (1998).
[2] J. Preskill, Proc. R. Soc. London Sect. A **454**, 385 (1998).
[3] L. Viola, E. Knill, and S. Lloyd, Phys. Rev. Lett. **82**, 2417 (1999).
[4] A. M. Souza, G. A. Álvarez, and D. Suter, Philos. Trans. R. Soc. A **370**, 4748 (2012).
[5] J. Zhang, A. M. Souza, F. D. Brandao, and D. Suter, Phys. Rev. Lett. **112**, 050502 (2014).
[6] E. Knill, R. Laflamme, R. Martinez, and C.-H. Tseng, Nature (London) **404**, 368 (2000).
[7] S. Bravyi and A. Kitaev, Phys. Rev. A **71**, 022316 (2005).
[8] A. M. Souza, J. Zhang, C. A. Ryan, and R. Laflamme, Nat. Commun. **2**, 169 (2011).
[9] W. Zheng, Y. Yu, J. Pan, J. Zhang, J. Li, Z. Li, D. Suter, X. Zhou, X. Peng, and J. Du, Phys. Rev. A **91**, 022314 (2015).
[10] G. A. Álvarez, A. Ajoy, X. Peng, and D. Suter, Phys. Rev. A **82**, 042306 (2010).
[11] H. K. Ng, D. A. Lidar, and J. Preskill, Phys. Rev. A **84**, 012305 (2011).
[12] K. Khodjasteh and L. Viola, Phys. Rev. Lett. **102**, 080501 (2009).
[13] K. Khodjasteh, D. A. Lidar, and L. Viola, Phys. Rev. Lett. **104**, 090501 (2010).
[14] J. R. West, D. A. Lidar, B. H. Fong, and M. F. Gyure, Phys. Rev. Lett. **105**, 230503 (2010).
[15] K. Khodjasteh and L. Viola, Phys. Rev. A **80**, 032314 (2009).
[16] P. Cappellaro, L. Jiang, J. S. Hodges, and M. D. Lukin, Phys. Rev. Lett. **102**, 210502 (2009).
[17] A. M. Souza, G. A. Álvarez, and D. Suter, Phys. Rev. A **86**, 050301 (2012).
[18] S. Wimperis, J. Magn. Reson. Ser. A **109**, 221 (1994).

[19] H. M. Cho, R. Tycko, A. Pines, and J. Guckenheimer, Phys. Rev. Lett. **56**, 1905 (1986).

[20] E. Knill, D. Leibfried, R. Reichle, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland, Phys. Rev. A **77**, 012307 (2008).

[21] C. A. Ryan, M. Laforest, and R. Laflamme, New J. Phys. **11**, 013034 (2009).

[22] C. Dankert, R. Cleve, J. Emerson, and E. Livine, Phys. Rev. A **80**, 012304 (2009).

[23] J. Emerson, R. Alicki, and K. Zyczkowski, J. Opt. B: Quantum Semiclass. Opt. **7**, S347 (2005).

[24] G. A. Álvarez and D. Suter, Phys. Rev. Lett. **107**, 230501 (2011).

[25] T. Gullion, D. B. Baker, and M. S. Conradi, J. Magn. Reson. **89**, 479 (1990).

[26] D. C. Johnston, Phys. Rev. B **74**, 184430 (2006).

[27] U. Haeberlen, in *Advances in Magnetic Resonance*, edited by J. Waugh (Academic, New York, 1976).

[28] H. G. Krojanski and D. Suter, Phys. Rev. Lett. **97**, 150503 (2006).

[29] A. M. Souza, G. A. Álvarez, and D. Suter, Phys. Rev. Lett. **106**, 240501 (2011).