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Experimental protection of quantum gates against decoherence and control errors

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One of the biggest challenges for implementing quantum devices is the requirement to perform accurate quantum gates. The destructive effects of interactions with the environment present some of the most difficult obstacles that must be overcome for precise quantum control. In this work we implement a proof of principle experiment of quantum gates protected against a fluctuating environment and control pulse errors using dynamical decoupling techniques. We show that decoherence can be reduced during the application of quantum gates. High-fidelity quantum gates can be achieved even if the gate time exceeds the free evolution decoherence time by one order of magnitude and for protected operations consisting of up to 330 individual control pulses.

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Introduction. Quantum-information processing (QIP) [1] can lead to a dramatic computational speed-up over classical computers for certain problems. However, any physical QIP device is subject to errors arising from unavoidable interactions with the environment or from control imperfections. Therefore, scalable QIP needs methods for avoiding or correcting those errors. The theory of quantum error correction (QEC) states that it is possible to stabilize a quantum computation provided that the error per gate is below some threshold [2] and high-fidelity initial states are prepared [3,4]. However, QEC needs many auxiliary qubits, generating a significant overhead in additional resources. Therefore, it is highly desirable to develop methods for reducing the perturbation both between and inside the quantum gates without requiring additional qubits. In addition, reaching the error threshold required for fault tolerant quantum computation requires that the fidelity of the individual gate operations must be very high [5].

A simple way to avoid decoherence and thus reduce the error per gate consists in choosing qubit systems whose decoherence times are long compared to the duration of a gate operation. However, this is not always possible; in particular in systems that combine different types of qubits, such as electronic and nuclear spins, the decoherence of the electron spin can be faster than the possible gate durations of the nuclear spins. Furthermore, the reduction of the decoherence time of a quantum register with the number of qubits [6,7] imposes an additional difficulty for implementing quantum computation in large systems.

Dynamical decoupling (DD) [8–10] is a promising method developed to reduce decoherence by attenuating the system-environment interaction with a sequence of inversion pulses periodically applied to the qubits. Recent experiments have successfully implemented DD methods and demonstrated the resulting increase of the storage times in different systems [11–17]. In all these implementations, the goal was to preserve a given input state, i.e., to protect a quantum memory against environmental perturbations.

The necessity of protecting qubits against environmental noise occurs also in the context of quantum-information processing. In a structured environment that induces well-characterized relaxation pathways, it is possible to design protected gate operations by optimal control techniques [18]. If the relaxation mechanism is not known or it affects all modes of the system, it may still be possible to use DD techniques, provided the effect of the DD sequence is compatible with the gate operations used for information processing. In the simplest case, gate operations can be made insensitive to static environmental perturbations by refocusing them [19,20], in a manner quite similar to a Hahn echo. In the more general case of a fluctuating environment, the Hahn echo has to be replaced by multiple-pulse DD sequences. Initial experiments in this direction have been made recently on a Nitrogen-Vacancy Center [21] and on an effective qubit in a semiconductor quantum dot [22]. In the former case, two-qubit control gates were implemented by applying DD to the control qubit and adjusting the delay between the DD pulses to match the inverse of the coupling constant. Possible schemes for maintaining DD protection during the gate operation were also suggested in Refs. [23–28]. In most cases, these schemes were developed under the assumption of perfect controls, i.e., the controls operations used, e.g., for DD should not introduce any additional errors. Here, we propose a general scheme for protection against a fluctuating environment that does not rely on this assumption but is robust against experimental errors and can therefore be implemented in an experimental scheme with realistic control operations.

We show that the decoherence can be reduced during the application of quantum gates for a single qubit in a solid-state system. High-fidelity quantum gates are achieved even if the gate time exceeds the decoherence time by one order of magnitude. Since the protection scheme introduces many additional control operations, we design the protected gate operations in such a way that the effect of control imperfections on the fidelity of the system is minimized.

Protection scheme. Quantum logical gates are achieved by using time-varying control Hamiltonians $H_c(t)$ for the relevant qubit system. Their propagators can be represented by unitary operators

$$U = \mathcal{T} e^{-i\int_0^T H_c(t)dt} = e^{-iH_G T}, \tag{1}$$

where $\mathcal{T}$ is the Dyson time ordering operator and T is the gate time. The propagator can also be expressed in terms of the time-independent average Hamiltonian H_G, which would generate the same operation if it were active for the time T. For single-qubit gates, this Hamiltonian can be expressed as

$$H_G = \omega \vec{n} \cdot \mathbf{S}, \tag{2}$$

where $\mathbf{S} = (S_x, S_y, S_z)$ is the spin vector operator of the system qubit, $\vec{n}$ is a three-dimensional vector, and the strength ω is a real parameter.

In any physical implementation, the system also interacts with the environment, which introduces decoherence reducing the gate fidelity. If the environmental effects become too strong, the quantum computation cannot be stabilized by QEC codes [2]. One approach to avoid this is to use DD for reducing the effects of the environment. Consider the Hamiltonian describing a one-qubit system and its environment

$$H = H_S + H_{SE} + H_E, \tag{3}$$

where $H_S = \omega_S S_z$ is the system Hamiltonian and ω_S is the Larmor frequency of the system. H_E is the environmental Hamiltonian and H_{SE} the system-environment coupling. Here, we describe the environment as a spin bath and the coupling as a pure dephasing interaction

$$H_{SE} = \sum_k b_k S_z I_z^k, \tag{4}$$

where I_z^k is the spin operator of the kth environment spin, b_k is the coupling constant between the system and the kth spin of the environment.

The identity operation ($H_G = 0$) can be implemented by just applying DD sequences, such as the XY-4 sequence. This sequence was initially introduced in the context of nuclear magnetic resonance (NMR) [29,30]. A similar sequence is the PDD sequence [8,16,31]. Neglecting pulse errors, we can write the zeroth- and first-order terms of the average Hamiltonian of the XY-4 sequence as $\overline{H_0} = H_E$ and $\overline{H_1} = 0$. The system-environment Hamiltonian, which causes decoherence, as well as the internal Zeeman Hamiltonian of the system qubit are removed up to the first-order approximation. The only remaining term is the environmental Hamiltonian H_E, which has no effect on the system qubit.

A simple way to introduce gates with real pulses during decoupling consists in dividing the gate into two equal parts $\sqrt{U} = e^{-iH_G T/2 - iHT/2}$. Here, $e^{-iH_G T}$ is the target control operation, while the drift term $HT/2$ is an unwanted contribution that we cancel in the protected control operation. We insert the two half-gates $\sqrt{U}$ into the initial and final free precession periods of the DD sequence, as shown in Fig. 1. Adjusting $T = \tau$, the delay between the DD pulses, the propagator for the full cycle (duration $\tau_c = 4\tau$, neglecting the pulse duration) becomes, to first order in the cycle time,

$$U \approx e^{-4iH_E\tau} e^{-iH_G T}. \tag{5}$$

The free evolution terms $\propto H_S + H_{SE}$ are canceled to first order by DD. This scheme resembles the theoretical approach of Refs. [23,26], except that our scheme has higher symmetry,

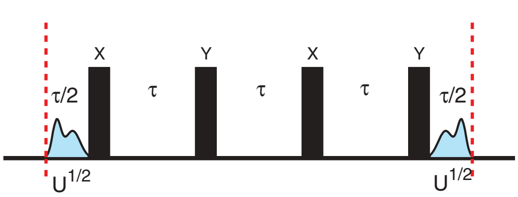

FIG. 1. (Color online) Pulse sequence for protecting quantum gates using the XY-4 sequence.

which helps to eliminate some control errors [31]. The remaining terms are H_E, which acts only on the environment, and the gate operator H_G. Since the effect of the SE coupling is absent here, decoherence has been eliminated in this first-order approximation.

Robust implementation. In real implementations, experimental imperfections must also be taken into account. In most cases, the dominant imperfection is a deviation between the actual and the ideal amplitude of the control field. The result of this amplitude error is that the rotation angle deviates from the target angle typically by a few percent. The imperfect control can affect both the implementation of H_G as well as the π pulses of the dynamical decoupling sequence, and its effect can be particularly devastating when the number of gate operations is large and the errors accumulate.

The systematic control errors of the DD pulses can be reduced by choosing robust DD sequences [10,16,32]. A well-established method for eliminating control field errors is the use of composite pulses [33]. Composite pulses are sequences of consecutive pulses designed such that the resulting total operation remains close to the ideal target operation even in the presence of some experimental imperfections. A good choice for correcting amplitude errors in a general single-qubit rotation $\mathbf{R}_\phi(\theta)$ is the BB1 composite pulse [34]:

$$\mathbf{R}_\phi(\theta) = R_\phi(\theta/2) R_{\phi+\psi}(\pi) R_{\phi+3\psi}(2\pi) R_{\phi+\psi}(\pi) R_\phi(\theta/2),$$

where ϕ describes the rotation axis, θ the rotation angle, and $\cos\psi = -\theta/4\pi$.

Figure 2 shows how a general rotation can be made robust against amplitude errors and protected against decoherence from a fluctuating environment. For this purpose, we replace each rotation $R_\phi(\theta)$ in the BB1 pulse by the protected rotation $\mathcal{R}_\phi(\theta)$ according to the scheme of Fig. 1. This scheme can obviously be extended to other DD sequences with symmetrical timing simply by replacing the XY-4 cycle with a different cycle.

Experimental performance. For the experimental tests we used natural abundance ^{13}C nuclear spins in the CH_2 groups of a polycrystalline adamantane sample as the system qubit. In this system, the carbon spins are coupled to ^{1}H nuclear spins by heteronuclear magnetic dipole interactions. The protons are coupled to each other by homonuclear dipolar interactions. Under our conditions, the couplings between the carbon nuclei can be neglected. The experiments were performed on a home-built 300-MHz solid-state NMR spectrometer.

In the context of quantum-information processing, it is important that the performance of gate operations be independent

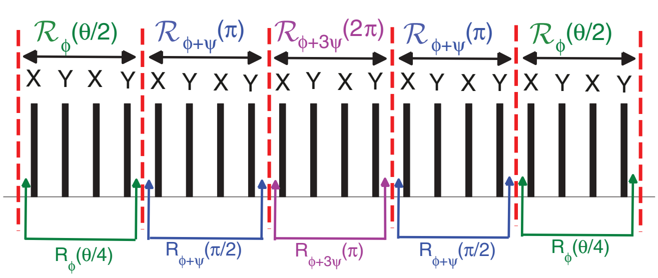

FIG. 2. (Color online) Pulse sequence for a decoherence-protected BB1 pulse using the XY-4 sequence.

TABLE I. Implemented quantum gates. The gate times and fidelities refer to the experiment in which the XY-8 cycle is used.

	Rotations	Gate time (ms)	Fidelity
$\mathcal{H}$	$R_x(\pi)R_y(\frac{\pi}{2})$	1.6	0.985
NOT	$R_x(\pi)$	0.6	0.995
$\pi/8$	$R_x(\frac{\pi}{2})R_y(\frac{\pi}{4})R_x(-\frac{\pi}{2})$	2.2	0.955

of the initial conditions. For quantifying the performance of a general quantuxm operation, the fidelity F can be used [35]:

$$F = \frac{|\mathrm{Tr}(AB^\dagger)|}{\sqrt{\mathrm{Tr}(AA^\dagger)\mathrm{Tr}(BB^\dagger)}}. \tag{6}$$

Here, A is the target propagator for the process and B the actual propagator.

The actual operations are not always unitary. We therefore write the process as

$$\rho_f = \sum_{nm} \chi_{mn} E_m \rho_i E_n^\dagger, \tag{7}$$

where ρ_i and ρ_f are the density matrices at the beginning and end of the process. The operators E_m must form a basis. For the present case, we choose them as $E_m = (I, \sigma_x, i\sigma_y, \sigma_z)$, with the Pauli matrices σ_α. The ideal and actual processes can therefore be quantified by the matrix elements χ_{mn} and experimentally determined by quantum process tomography [36].

In the experimental implementation of this concept, we tested the Hadamard ($\mathcal{H}$), NOT, and $\pi/8$ gates. For protection, we used the DD sequences XY-4 [29], XY-8 [30], and KDD [16]. The gates were decomposed into sequences of rotations around axes in the xy plane (see Table I) and each rotation was implemented as shown in Fig. 2. Figure 3 shows the results of the quantum process tomography for the case of the $\mathcal{H}$ gate protected by the XY-8 sequence. Without dynamical decoupling, the system coherence decays on a time scale $T_2^* \approx 370\,\mu$s. The decay time only due to the interaction with a fluctuating environment (measured by the Hahn echo) is $T_2 \approx 750\,\mu$s. As shown in Table I, the gate fidelities for XY-8 are > 0.95, although the durations of the gate operations are $\gtrsim T_2$. All the fidelities were calculated from the process matrices obtained directly from the raw data, without optimization methods as used in previous experiments (see, for example, Ref. [37]). The obtained fidelity values close to unity indicate that the accumulation of incoherent errors is well compensated even for a very large number of pulses.

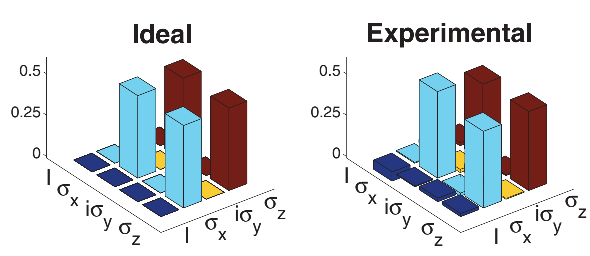

FIG. 3. (Color online) Process tomography of the Hadamard gate. The panels show the process matrix χ for an ideal gate and the experimental process matrix of the protected gate.

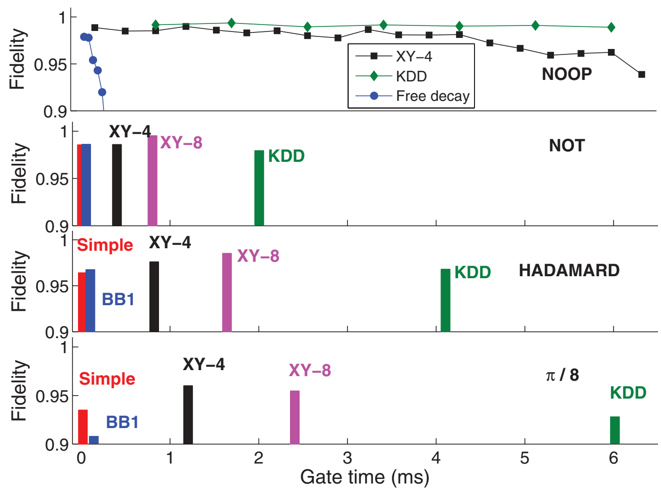

FIG. 4. (Color online) Gate fidelity as a function of gate time for different gate operations protected by different DD sequences. "Simple" indicates gates that were implemented by unprotected rotations. The delay between the pulses for the NOOP was $\approx 3\,\mu$s.

In Fig. 4, we show the achieved gate fidelities for four gate operations: identity = NOOP (no-operation), NOT, $\mathcal{H}$, and $\pi/8$ phase gate. For each type of gate, we plot the achieved fidelity against the total operation time for the gate, using the direct implementation, labeled "simple" in the figure, using only BB1 pulses and protected gates with different DD sequences. The three DD sequences differ in cycle time and robustness, with XY-4 having the shortest cycle and KDD being the most robust sequence [16]. In the top panel, we plot the fidelity of the NOOP gate for different gate durations, while the lower three panels only show the fidelity for the shortest cycle with each DD sequence. In all cases, the measured gate fidelities were >0.928. This result is very gratifying, since it shows that high gate fidelities can be obtained even if the gate duration exceeds the decoherence time T_2^* by an order of magnitude. Additionally, they exceed the T_2 time, given by the Hahn echo decay, being a clear indication that the fluctuating environment has been decoupled during the gate execution.

Discussion and conclusion. In summary, we have presented a proof of principle demonstration of decoherence suppression during quantum logical gate operations by dynamical decoupling. For this purpose, we inserted robust gate operations into different DD sequences in such a way that they do not interfere destructively with the DD. Using quantum process tomography, we have shown that high-fidelity single-qubit quantum gates can be achieved even if the gate time exceeds the decoherence time by one order of magnitude. We carefully designed the protection scheme to be robust against deviations of the control fields. As a result, even for protected operations consisting of up to 330 individual control pulses, the resulting fidelity is not significantly reduced compared to a gate implemented with a single pulse, and often it is higher. For some systems, slower gates promise higher fidelities than short gates [38,39]. In these cases, the approach that we have demonstrated here appears particularly appealing for further increasing the robustness and precision of the gate operations.

This result indicates that quantum computation can be made reliable even for systems in which the gate time is comparable to or even greater than the decoherence time of the individual qubits. In the present study, we tested the scheme with three different robust DD sequences. Our results are an additional demonstration that dynamical decoupling can be a useful tool that complements quantum error correction. We expect that the scheme is equally applicable to other types of qubit systems as well as to other types of gate operations. In particular, it will be interesting to apply this concept also to multiqubit systems. In this case we need to refocus the undesired system-environment interactions without eliminating the desired qubit-qubit interactions. This leads to more complex pulse sequences but can be achieved in principle. Efficient methods for selectively turning "on" and "off" specific Hamiltonian terms have been proposed in Refs. [40–42]. While the evolution-time overhead grows linearly in these schemes, a particular scheme [43] designed for a network of dipolar-coupled spins leads to an evolution time that is independent of the number of qubits. A proposal to combine this scheme with dynamical decoupling was made in Ref. [44].

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