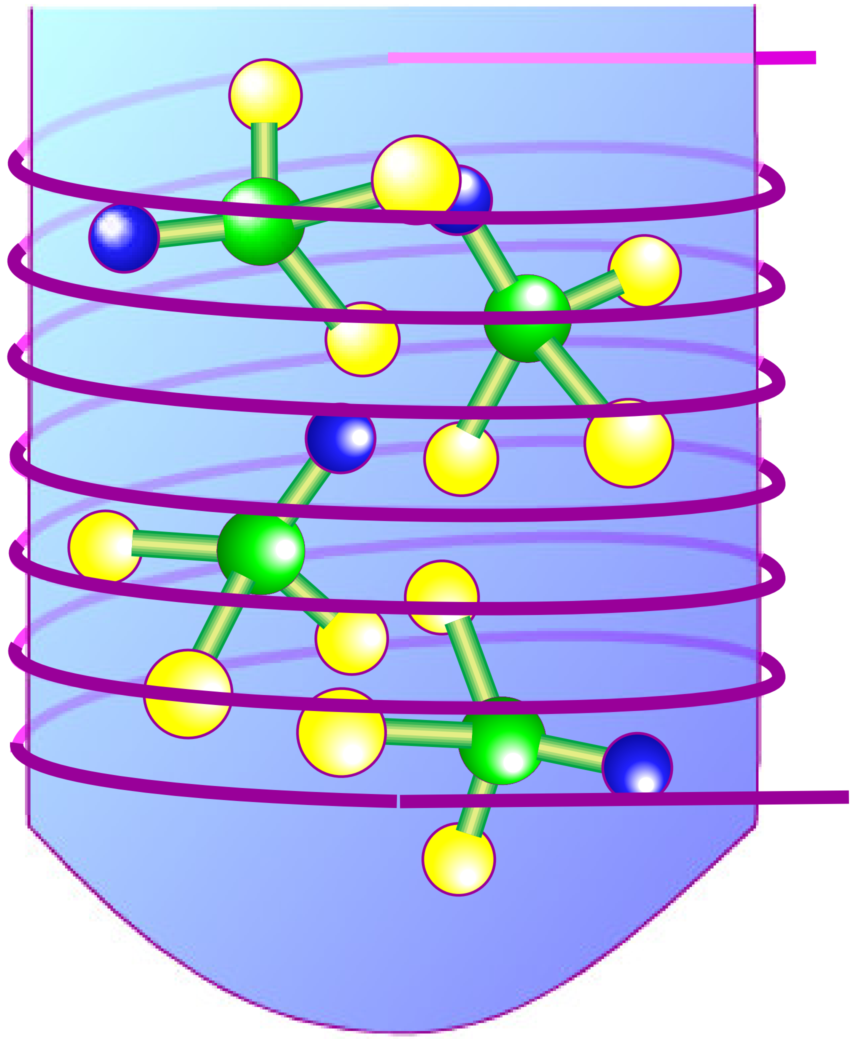
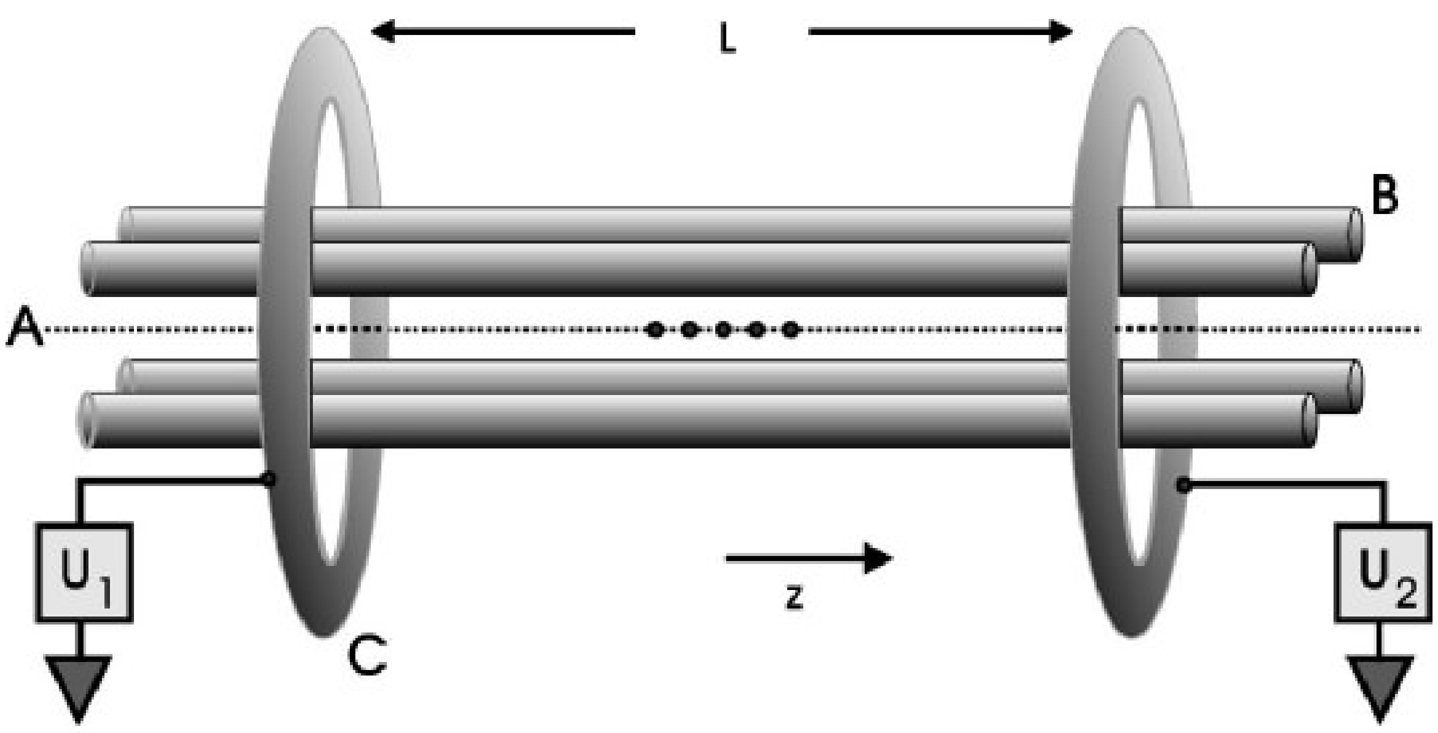
Existing Implementations

- Nuclear Magnetic Resonance



Chapter 10

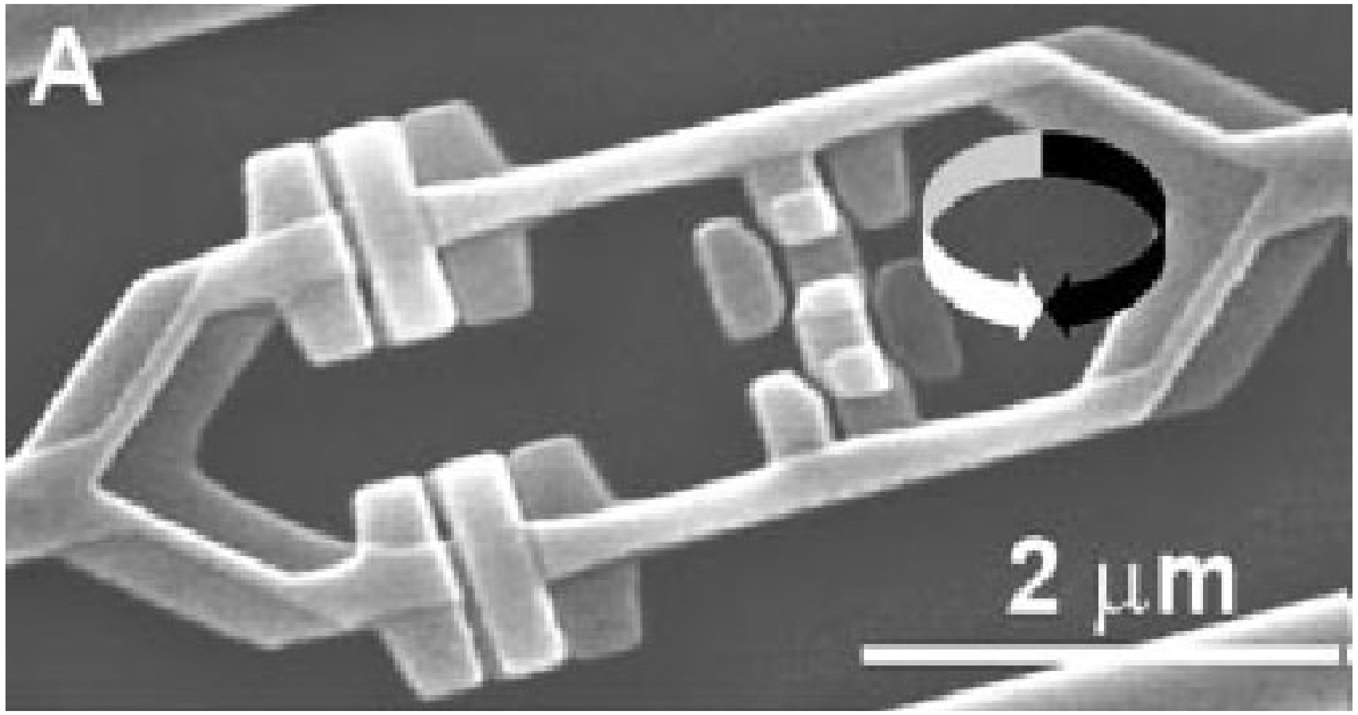
- Trapped Ions
- Neutral Atoms

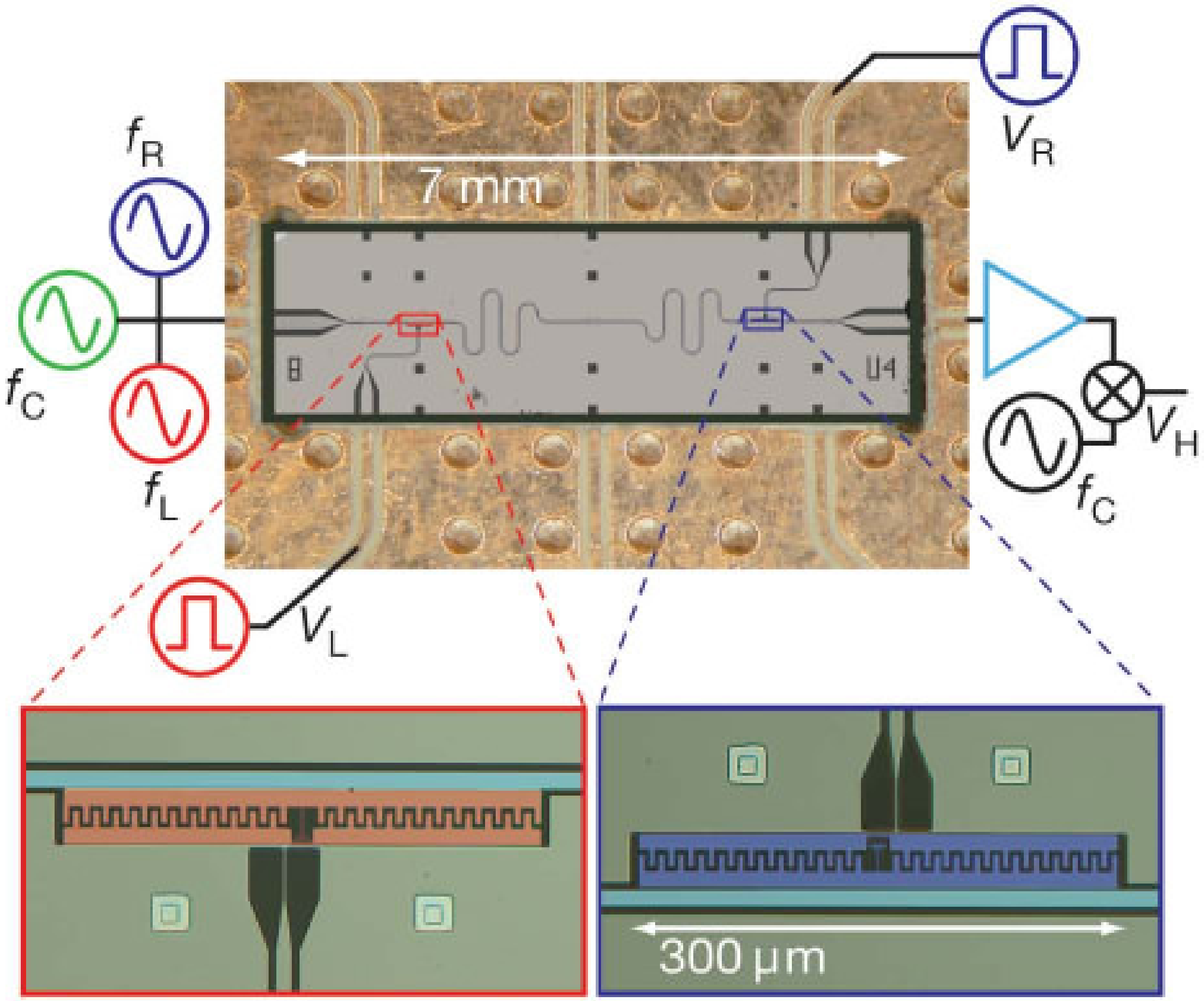


Chapter 11

Solid State Implementations

- **Quantum Dots**
- **Spins in Solids**
- **Superconductors**





Chapter 12

DiVincenzo's Criteria

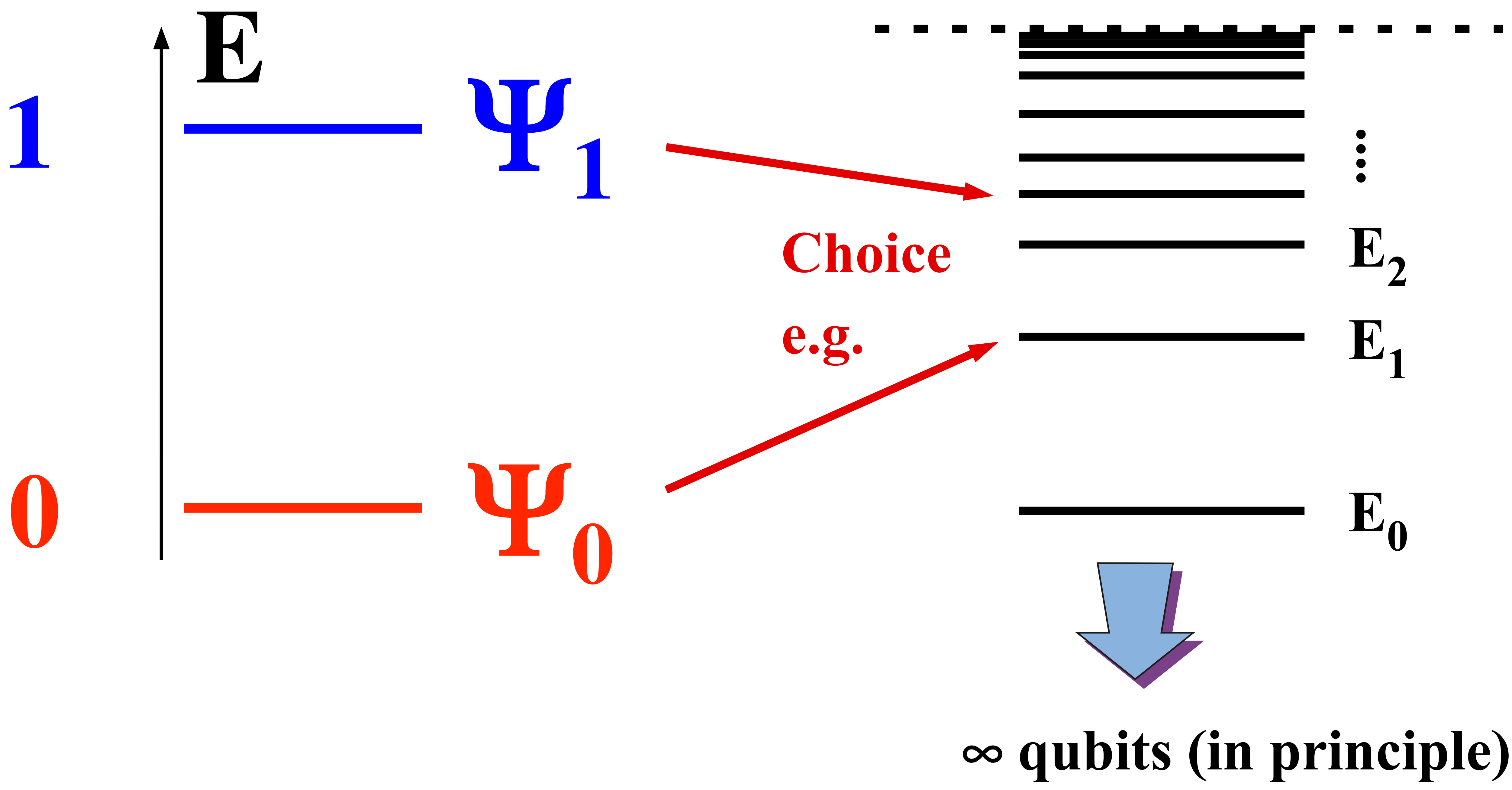
Which systems can be used to implement QIP?

DAVID P. DIVINCENZO Fortschr. Phys. **48** (2000) 9–11, 771–783

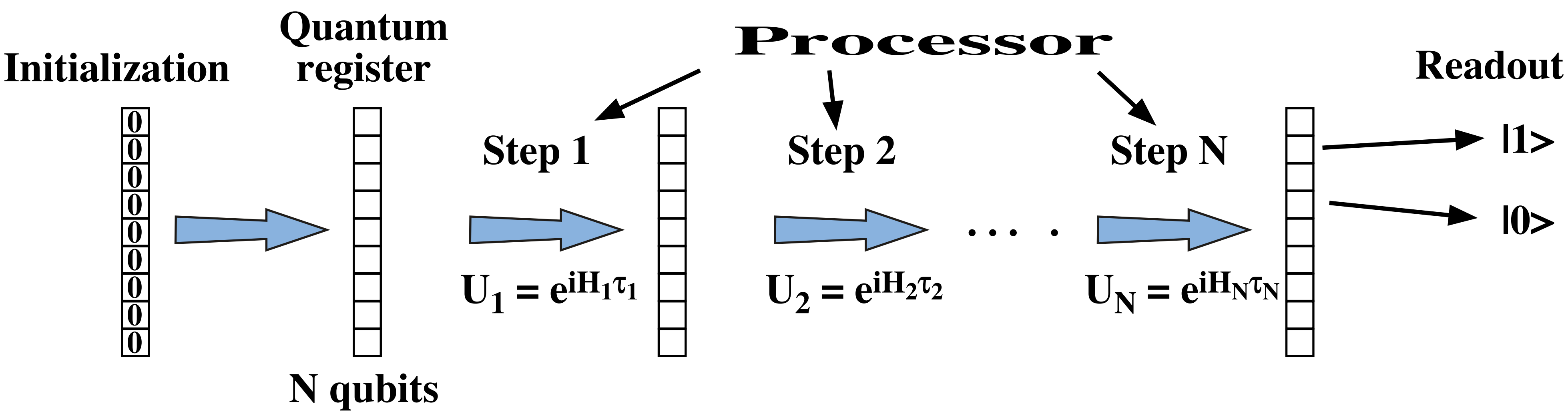
The Physical Implementation of Quantum Computation

1) **Well characterized qubits, scalable system**
2) **Initialization into a well defined state.**
3) **Long decoherence times.**
4) **Universal set of quantum gates.**
5) **Qubit-selective readout.**

Generic qubit

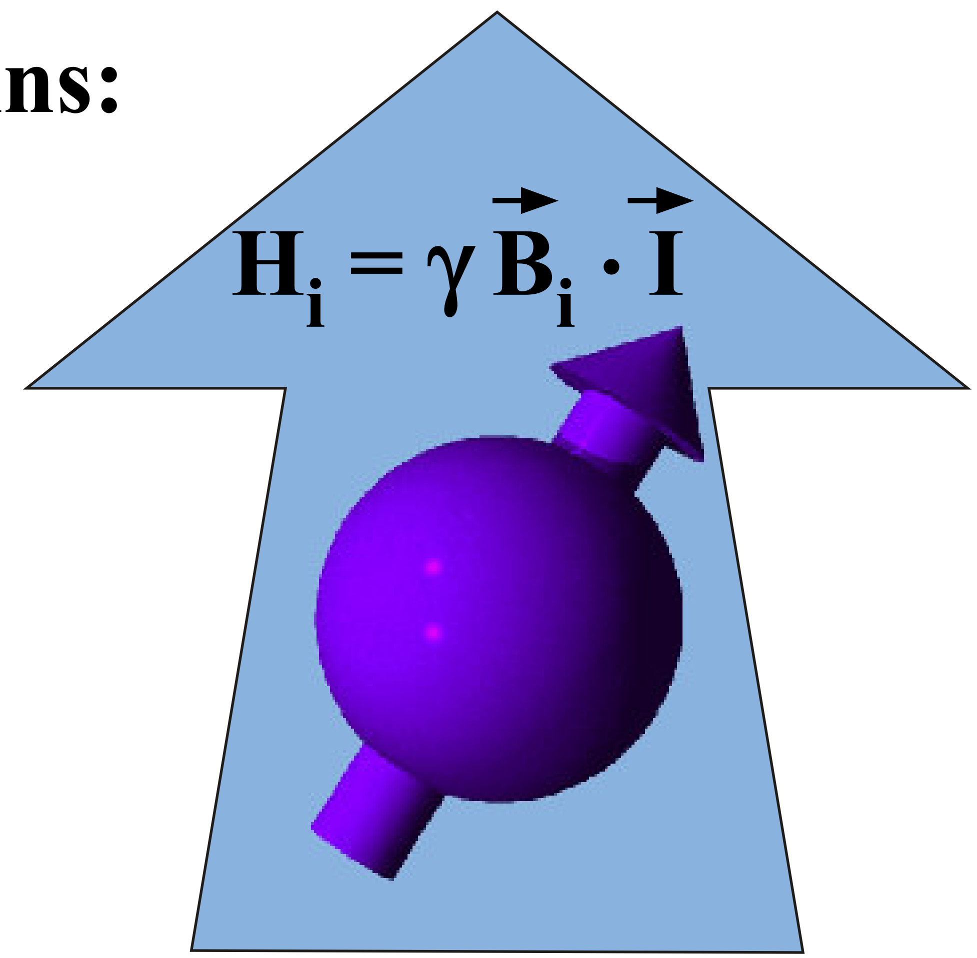


Interactions for Gates

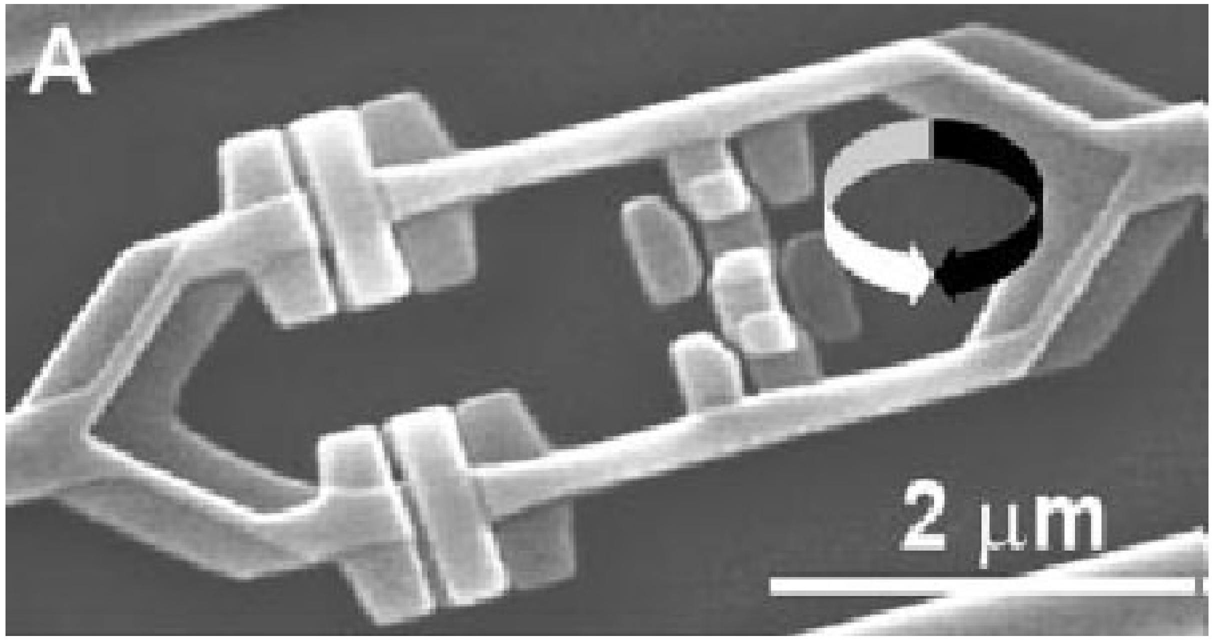


How is H_i generated? $H_i = \vec{\omega}_i \cdot \vec{I}$

Spins:



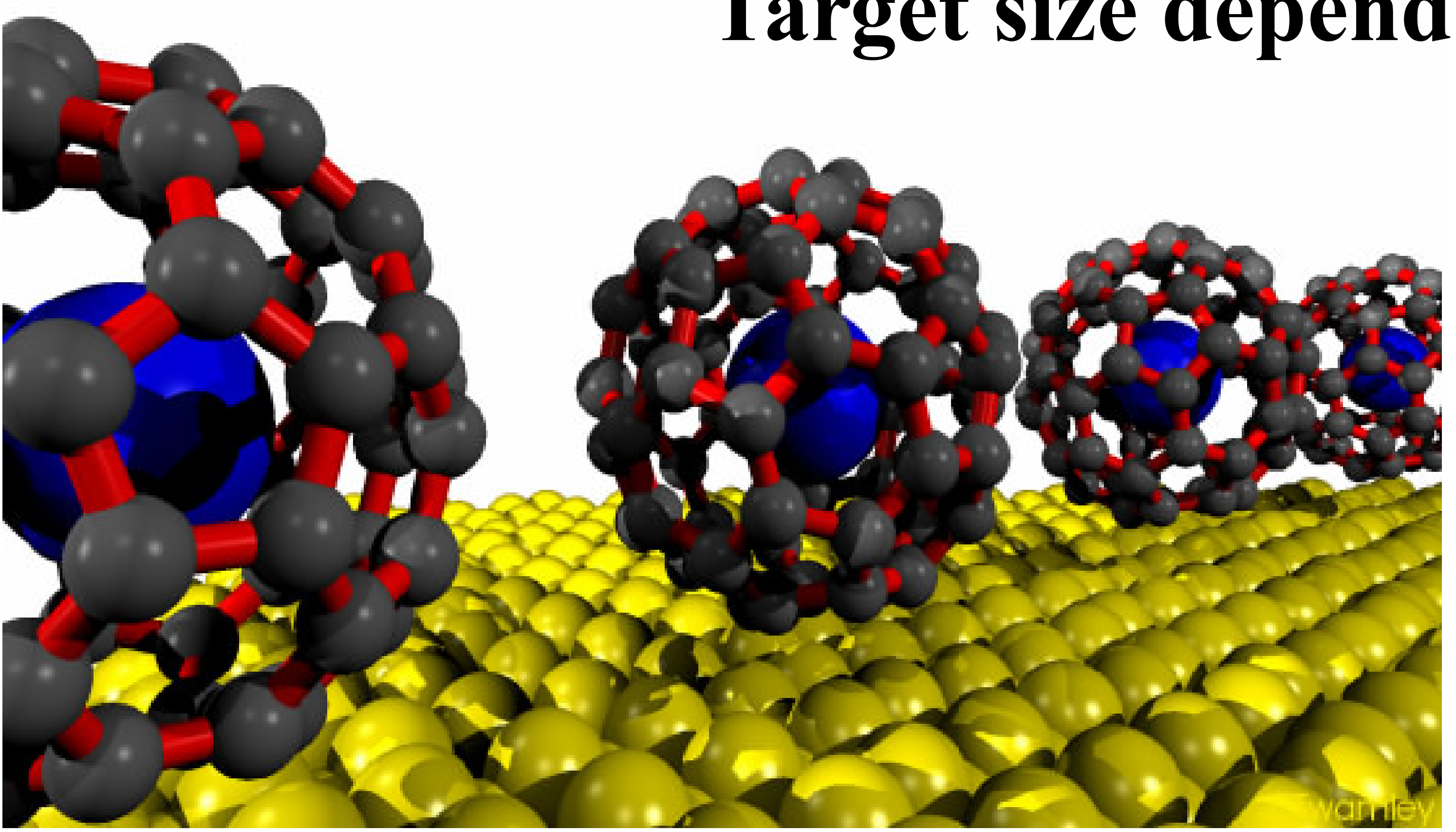
Solids: ?????

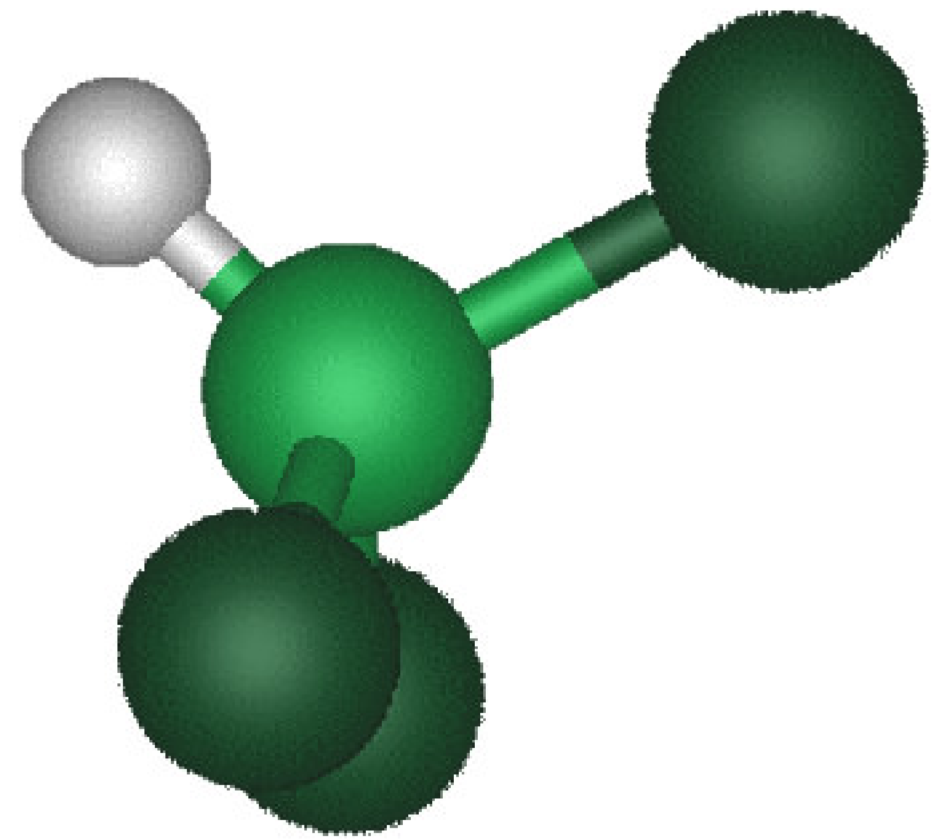


Scalability

Does the architecture allow arbitrary size of quantum register?

Target size depends on problem





Scalability includes issues of

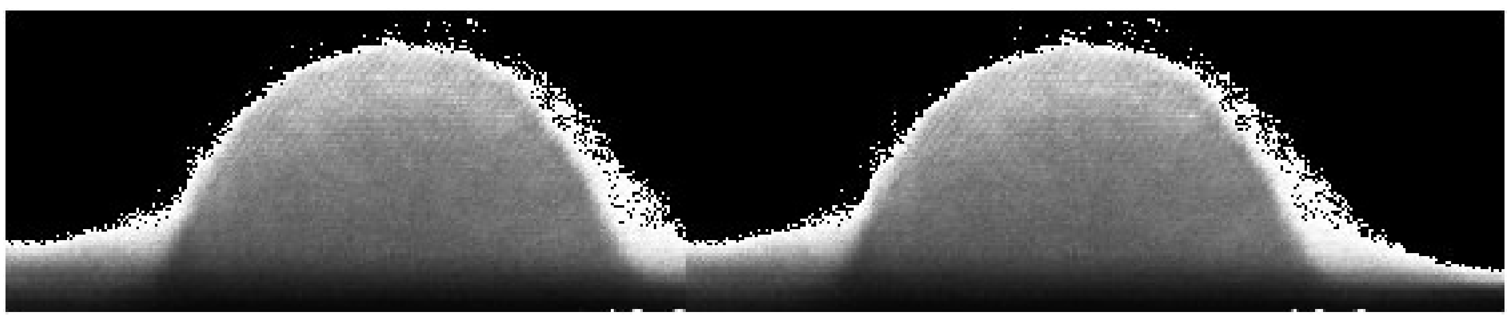
- **Addressing**
- **Errors and decoherence**

Superpositions

It must be possible to create superpositions of basis states

$$|\Psi> = c_0|0> + c_1\ |1>$$

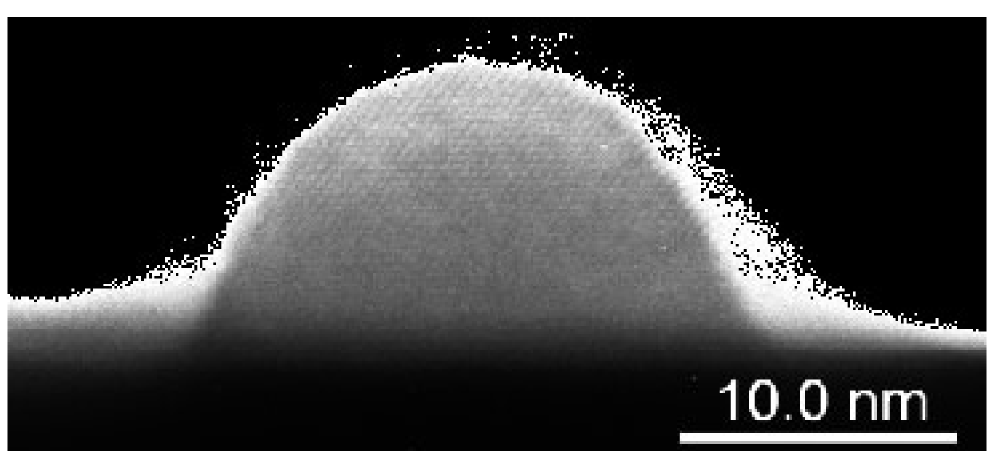
Possible:



|1> = electron right

|0> = electron left

Impossible:



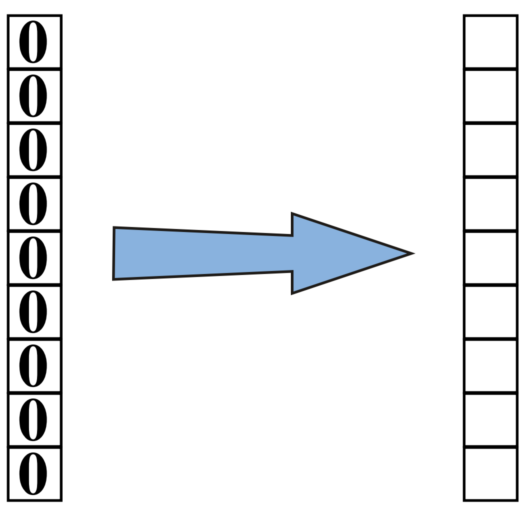
|1> = 1 electron

|0> = empty

Initialization

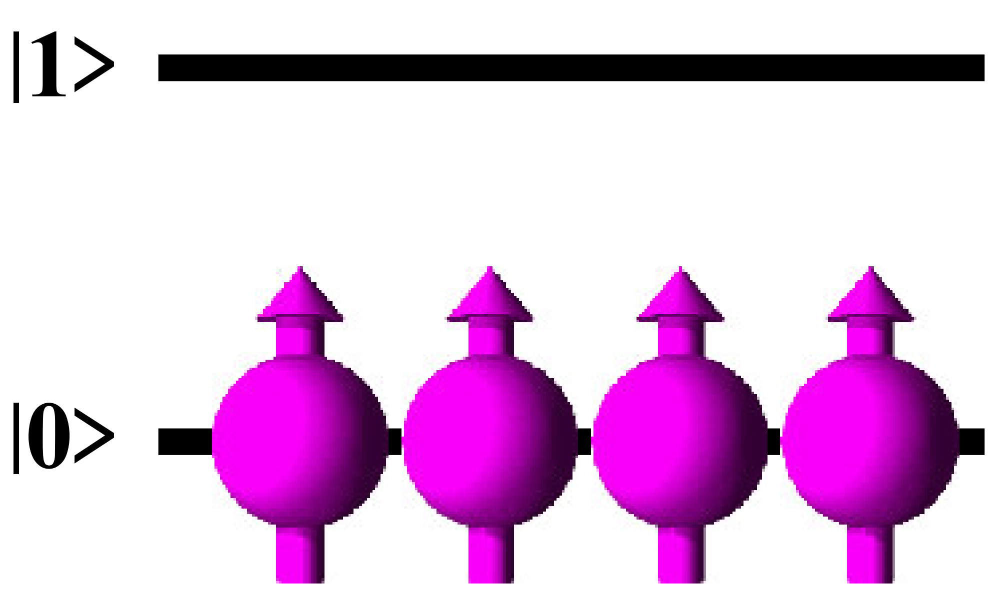
DiVincenzo's rule 2:
Initialization into a well defined state.

Quantum register



for many algorithms,

system must start in ground state



Boltzmann factor @ 100 mK, 2T:

electrons: $\frac{n\uparrow - n\downarrow}{n\uparrow + n\downarrow} \approx 1$

nuclei: $\frac{n\uparrow - n\downarrow}{n\uparrow + n\downarrow} \approx 5 \cdot 10^{-3}$

Initialization Speed

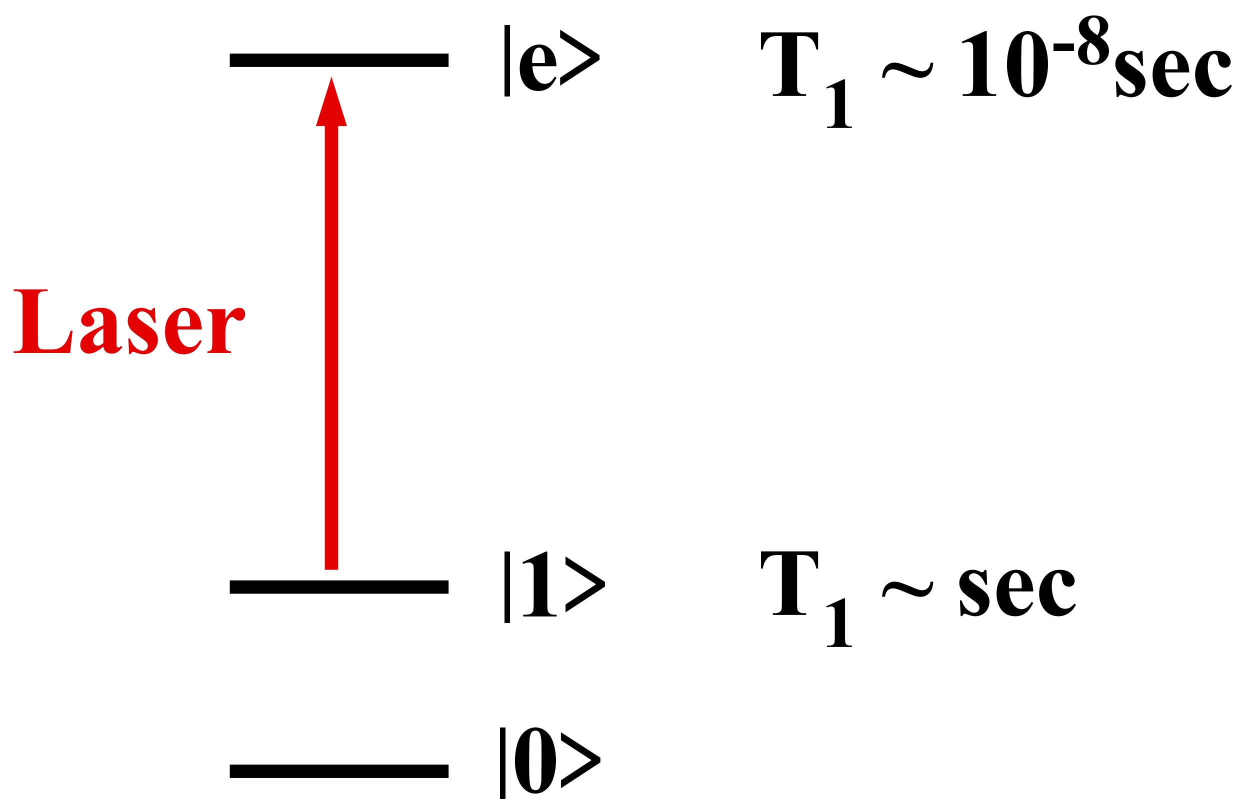
Not critical for computational qubits

Critical for ancilly qubits in error correction:

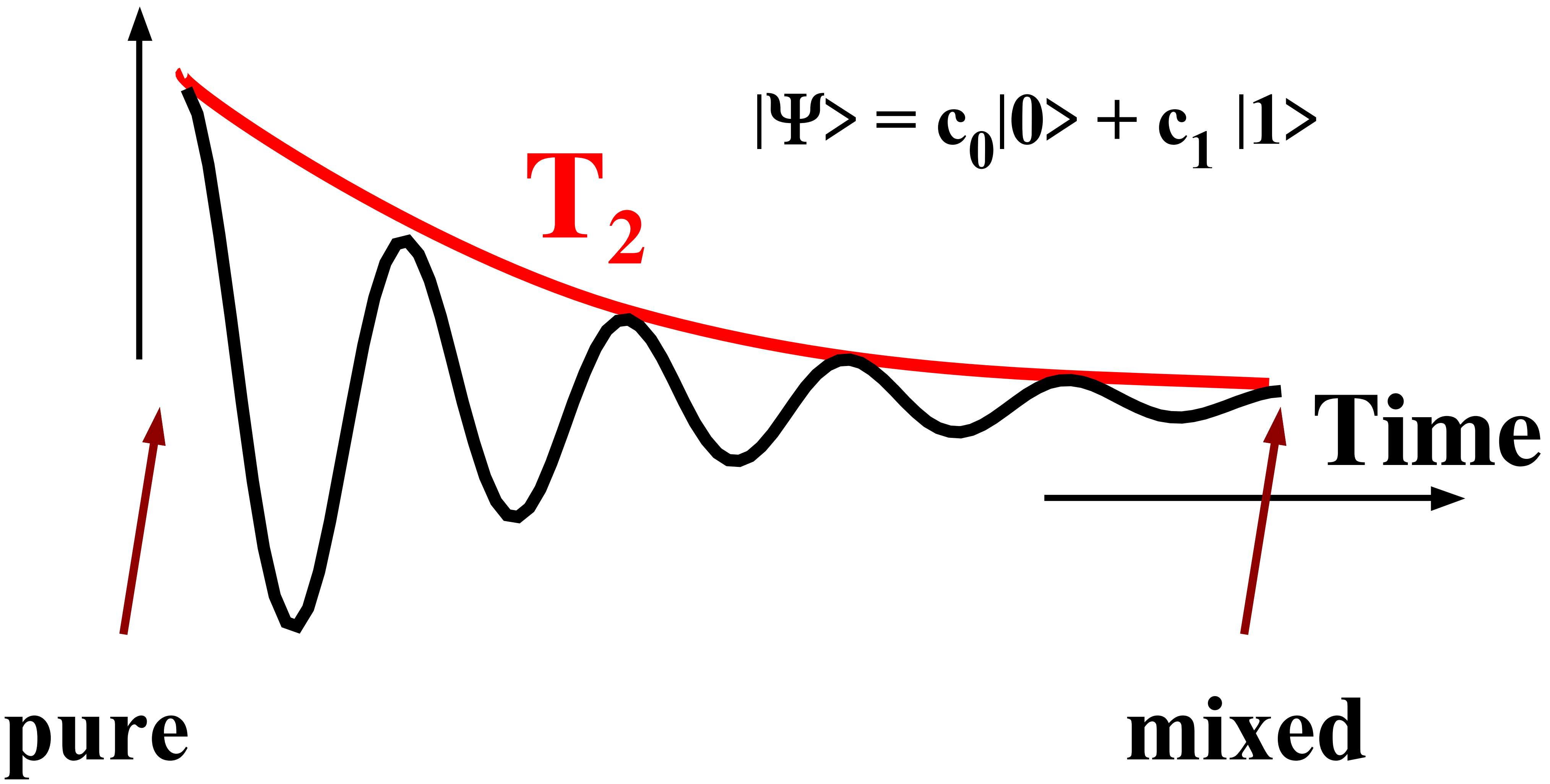
must be fast compared to dephasing

thermal relaxtion does not work !

Trapped ions:



Decoherence Time

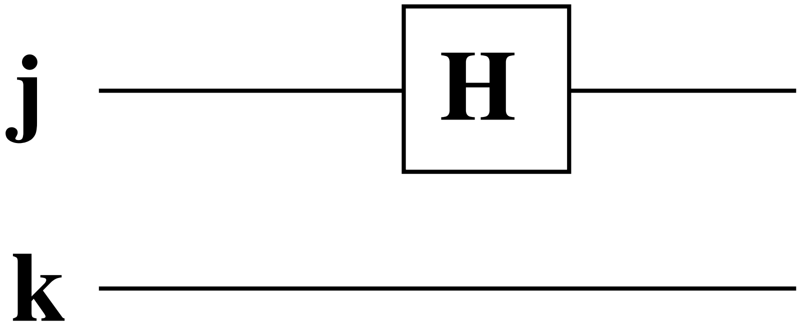


Must reach reliability threshold for quantum register

Quantum Gates

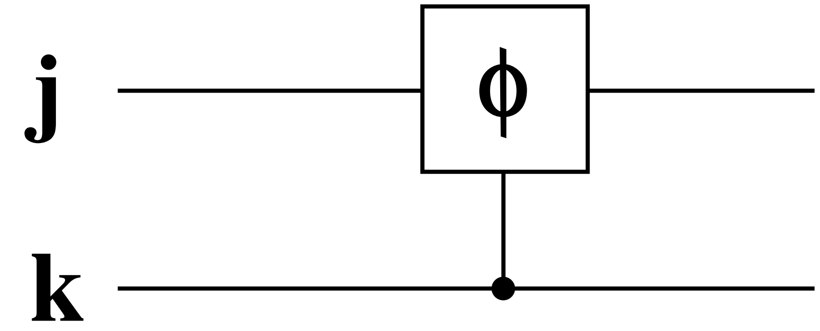
DiVincenzo 4: Universal set of quantum gates.

Single-qubit gate

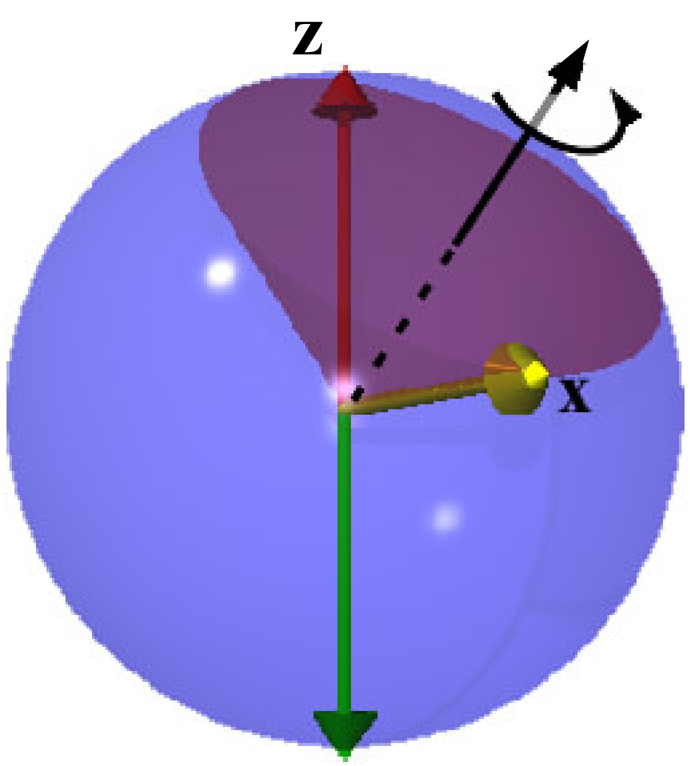


$$U_j = e^{i\frac{\pi}{2\sqrt{2}}(X_j+Z_j)}$$

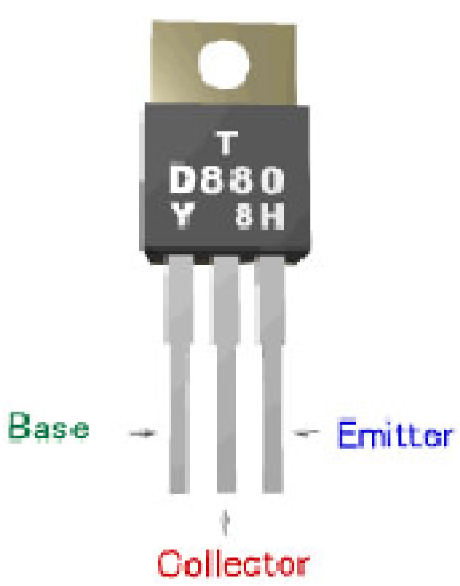
Two-qubit gate

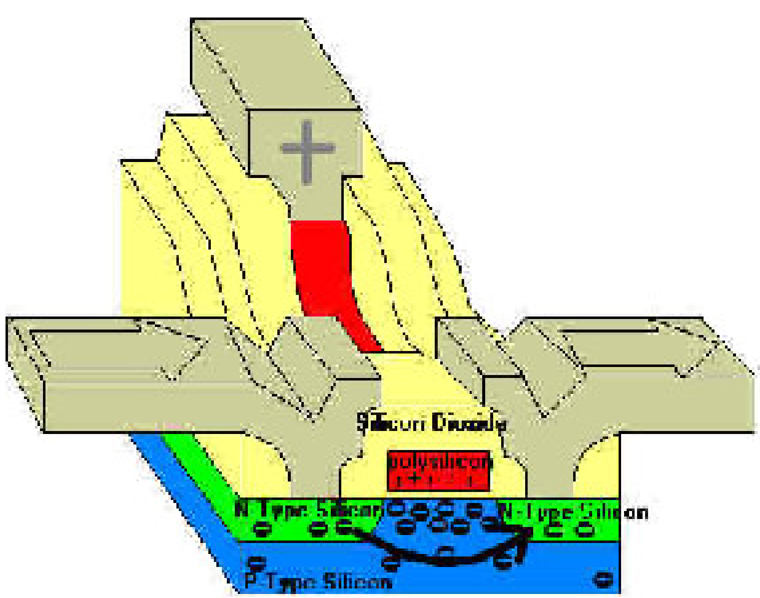


$$U_{jk} = e^{i\phi(Z_j+Z_k-Z_jZ_k)}$$



Gates must be selective!

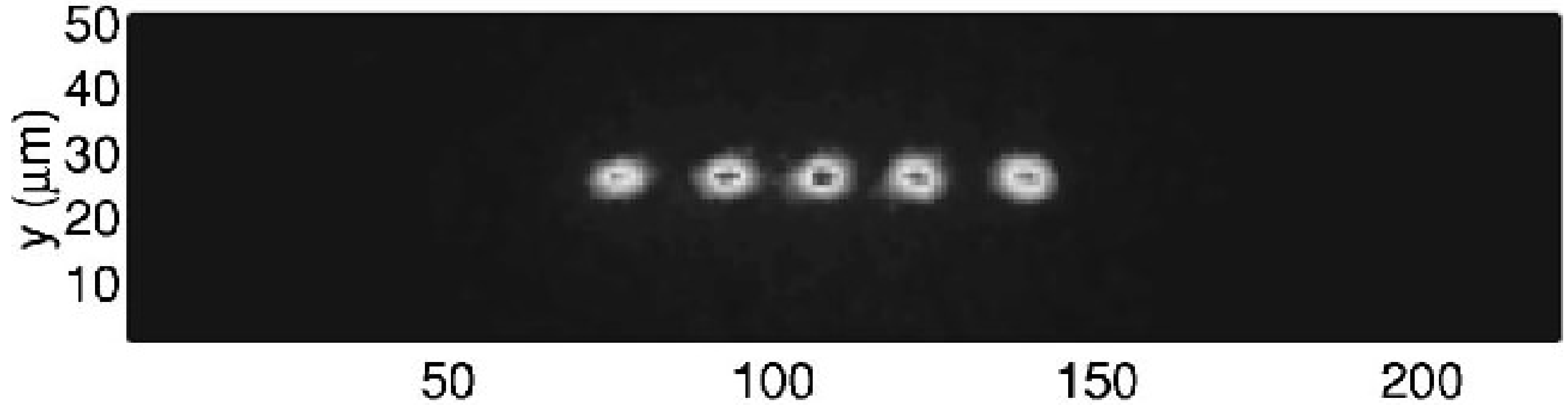




Addressing Qubits

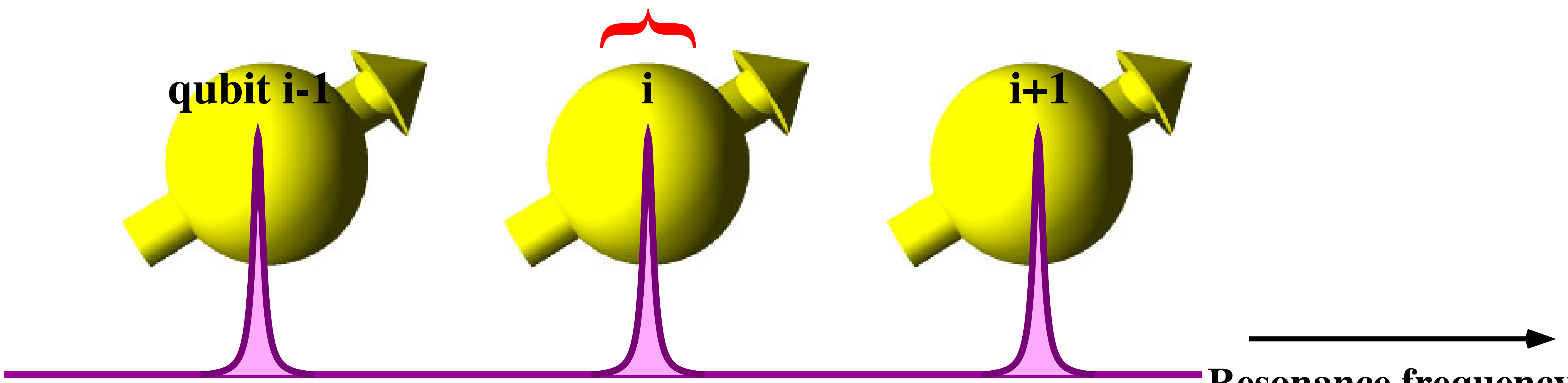
Trapped ions : optical addressing

$^{40}Ca^{+}$ ions



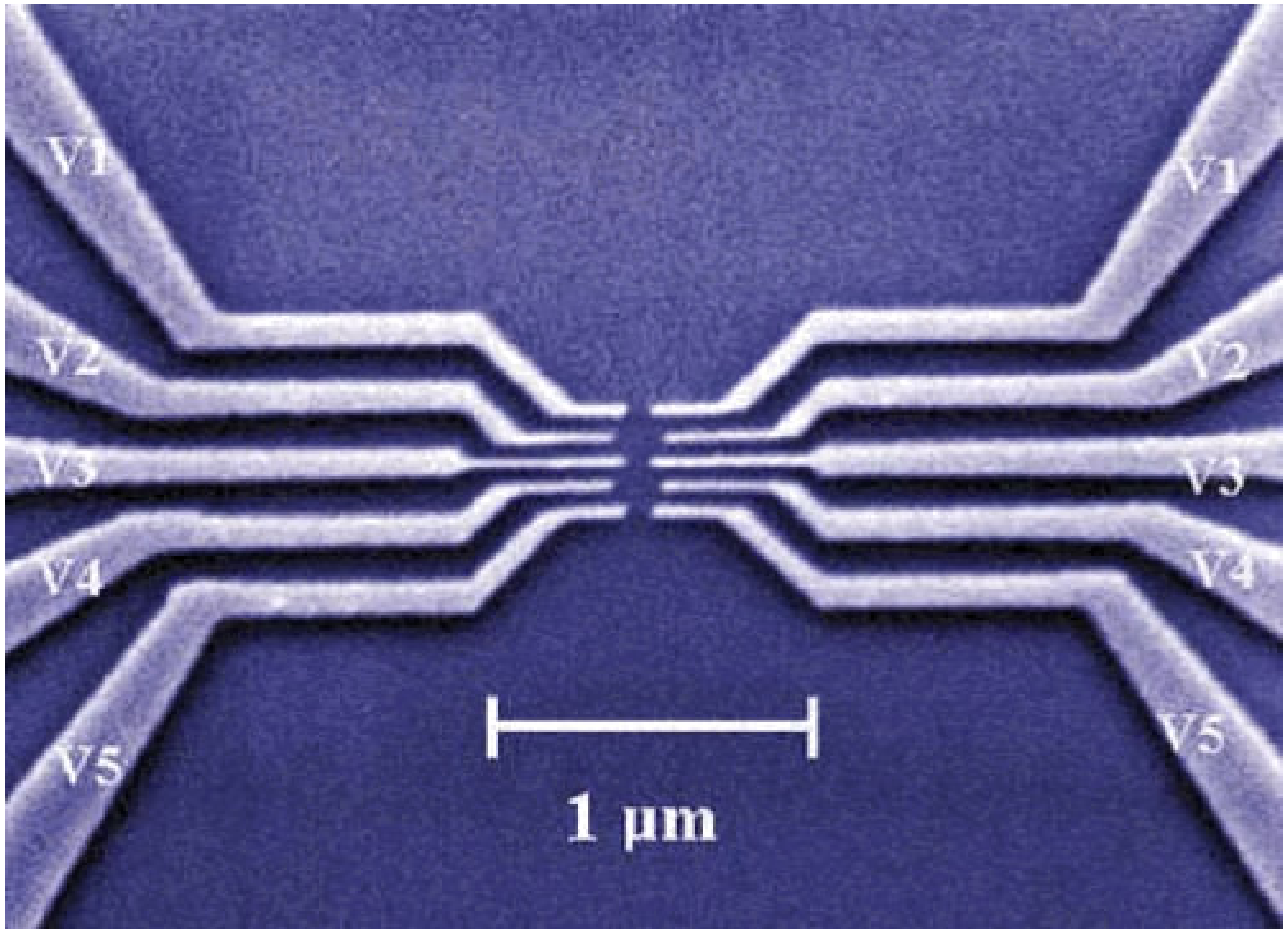
Position / µm

Resonant excitation with finite bandwidth:

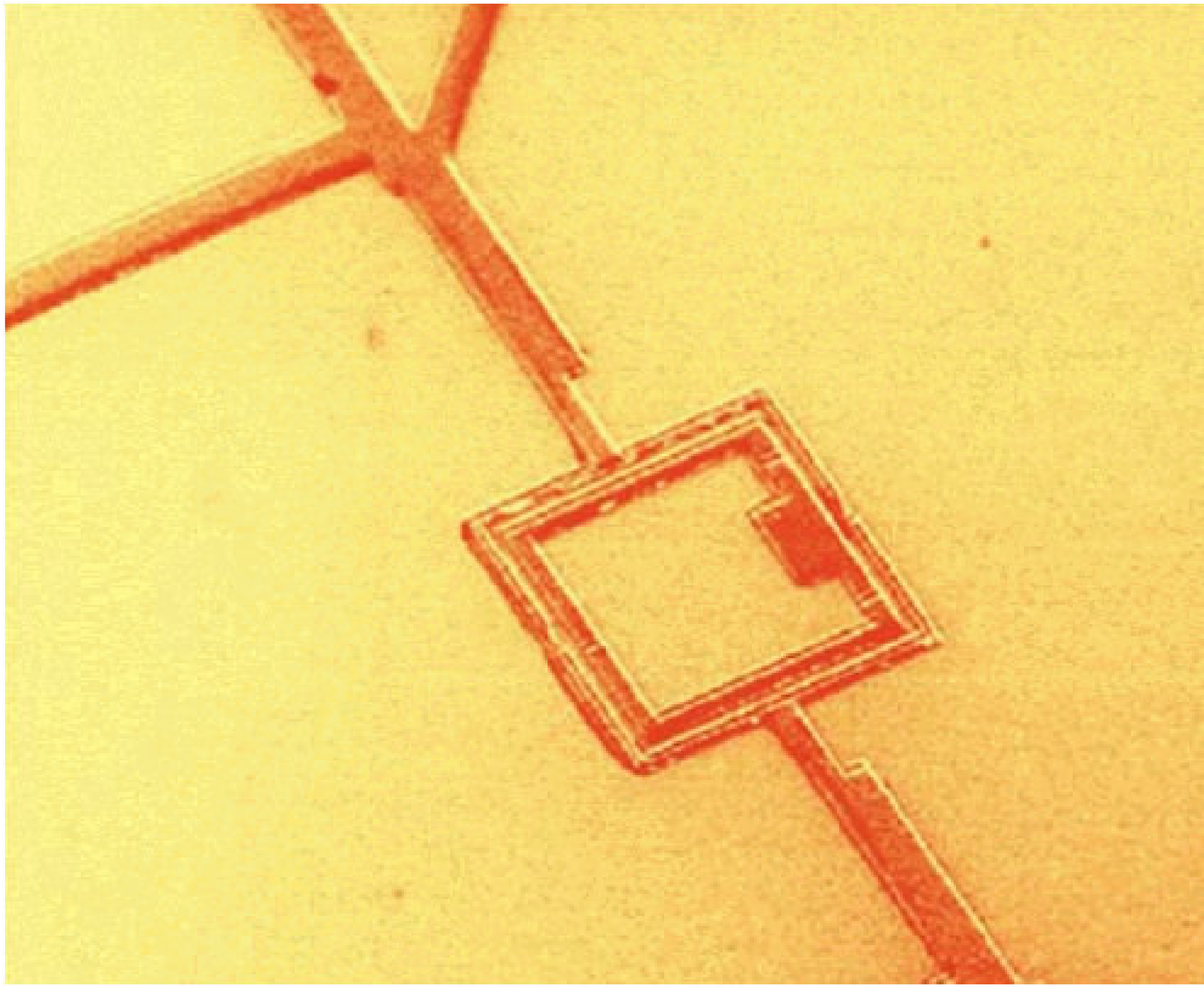


Addressing Qubits

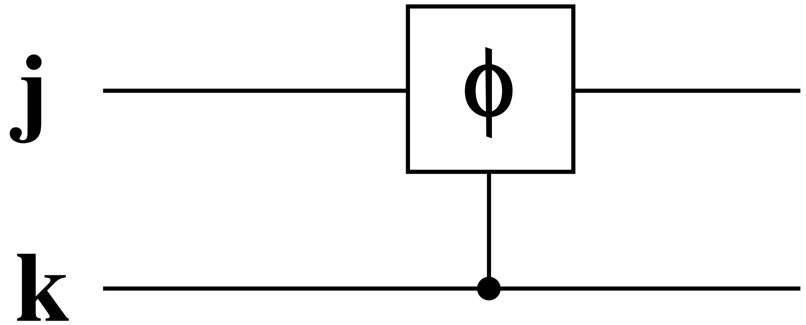
GaAs quantum dots (Purdue)



Squid (Delft)



2-Qubit Gates



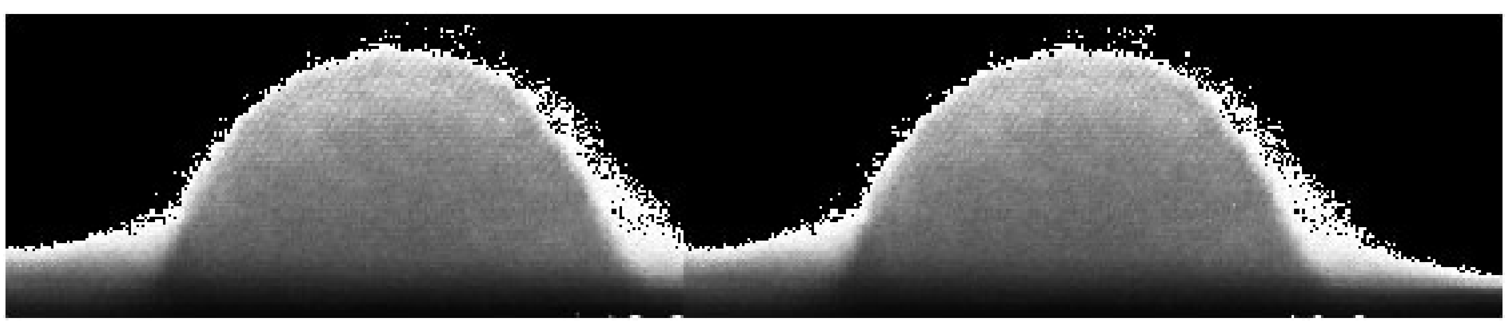
$U_{jk} = e^{i\phi(Z_j+Z_k-Z_jZ_k)}$

requires

coupling to fields

coupling between qubits

Typical couplings



Exchange: $H_E = J\,\vec{I}^1 \cdot \vec{I}^2$

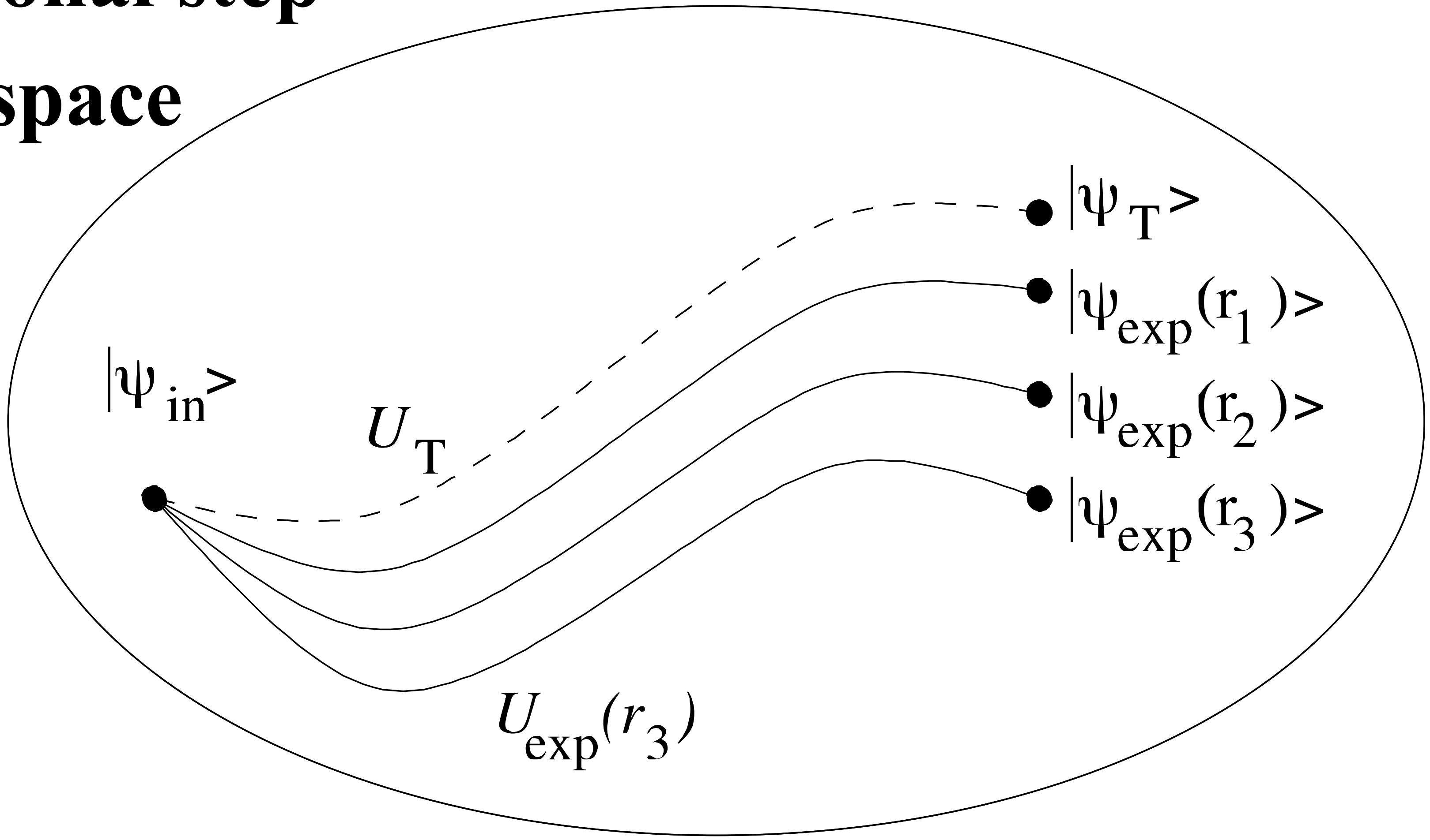
Required properties :

switchable

correct form

Gate Errors

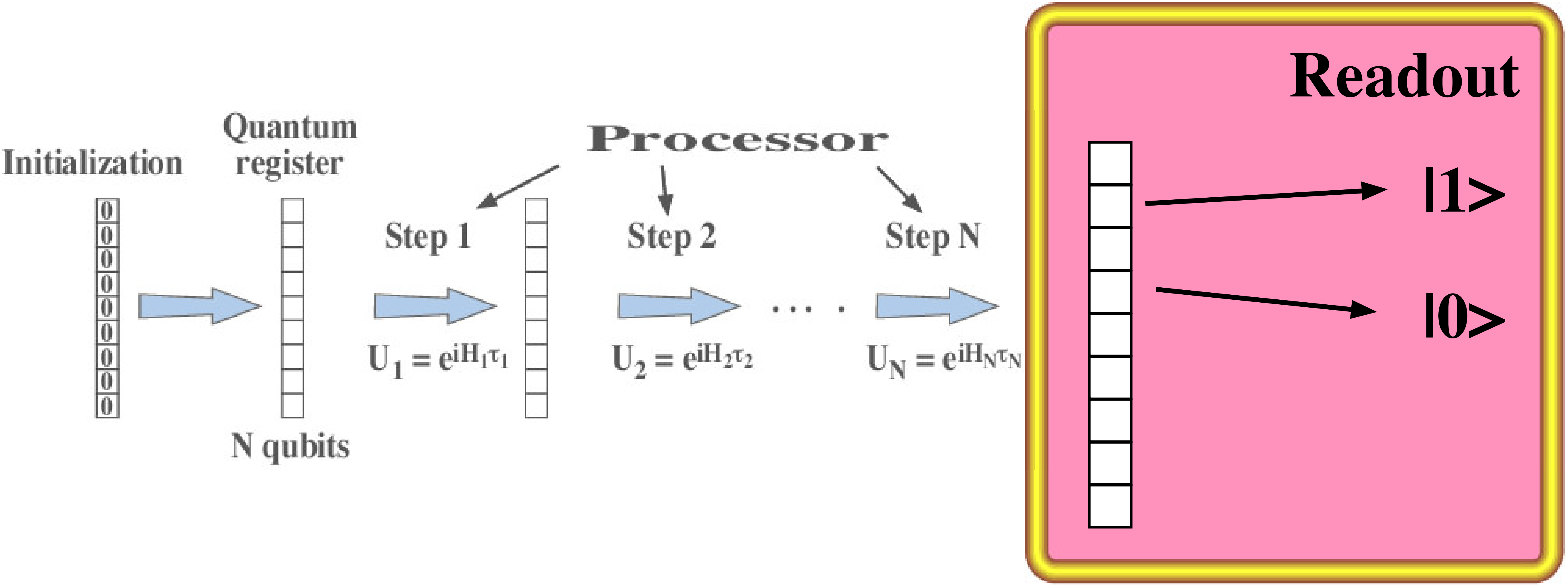
Computational step in Hilbert space



Gate fidelity: $$F = \frac{Tr\{U^{\dagger}_{exp}\ U_{ideal}\}}{Tr\{U^{\dagger}_{ideal}\ U_{ideal}\}}$$

Should reach error correction threshold

9.3 Readout



9.3.1 Principle and Strategies

9.3.2 Example: Deutsch-Jozsa algorithm

9.3.3 Effect of correlations

9.3.4 Repeated measurements

What is the Result ?

$$|\Psi_{fin}> = c_0\,|000\,..\,0> + c_1\,|000\,..\,1> + c_2\,|000\,..\,10> + \ldots$$

2^N terms

quantum

Readout

classical

Complete information:

2^N coefficients

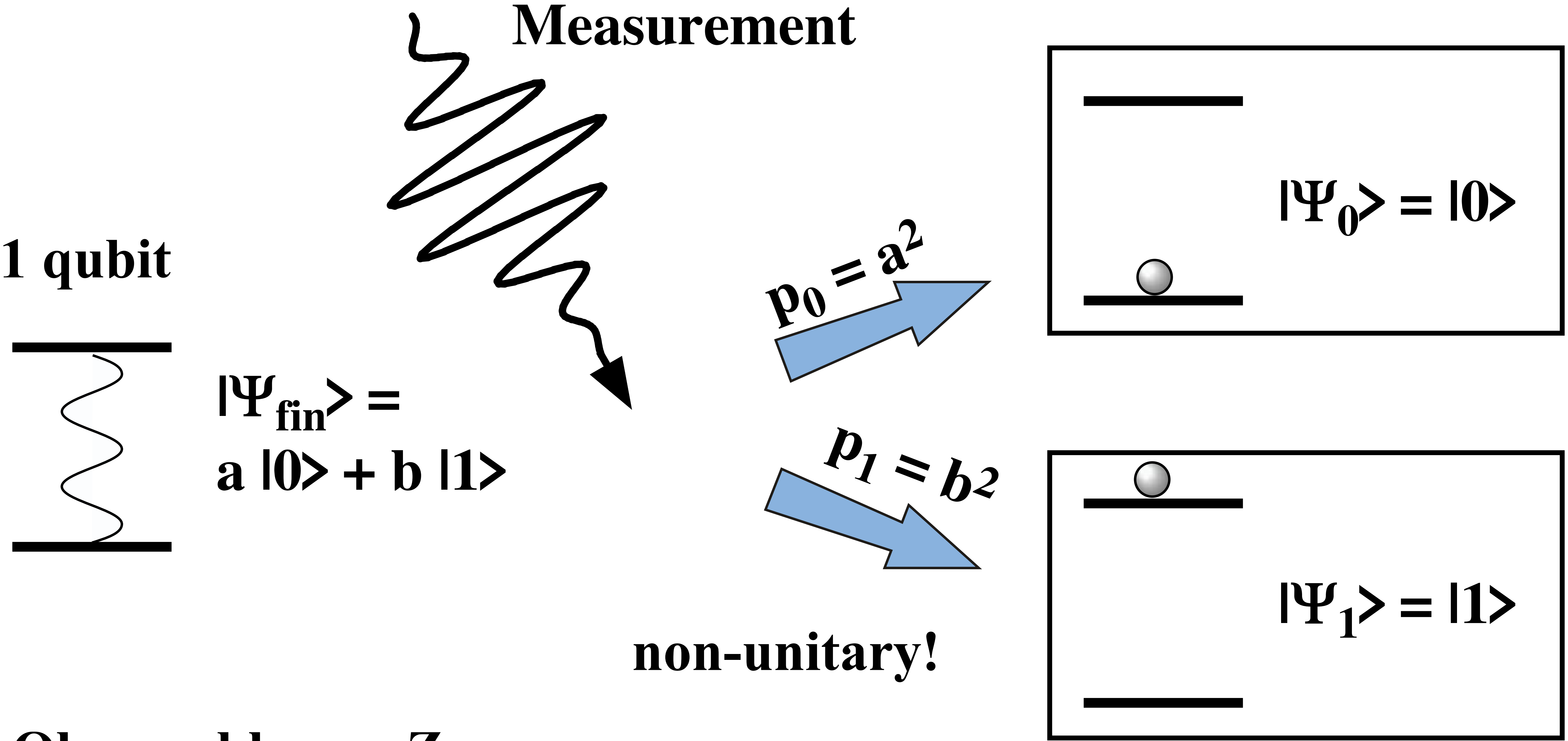
- inefficient
- usually not necessary

Desired results: True

17

. . .

QM Measurement



Observables:

Z_i

X_i

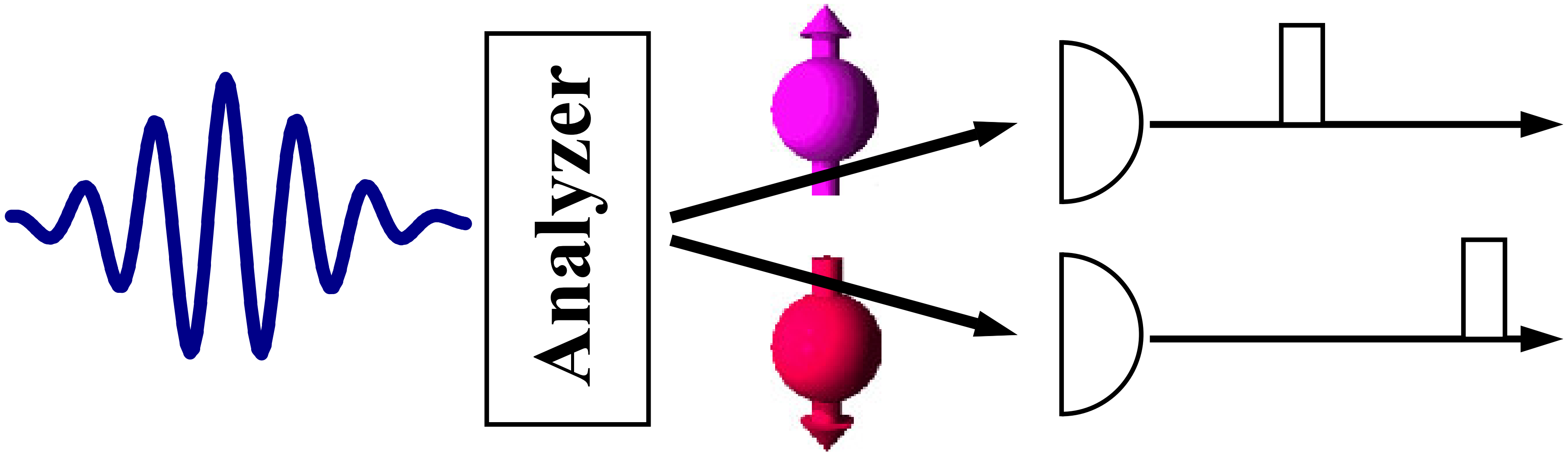
$\alpha X_i + \beta Y_i + \gamma Z_i$

Multi-qubit observables:

$Z_i Z_j$, $X_i Y_j$ ...

Reliable Readout

Errors



Efficiency < 100%

Dark counts

Possible solutions

QND readout

Read out copies

Example: Deutsch - Jozsa

Algorithm

$$|\Psi_0> = \sum_x |x,0> \quad \Rightarrow \quad |\Psi_{fin}> = \sum_x |x,f(x)>$$

Goal: distinguish 2 cases:

- all f(x) are the same (= constant function)
- same number of f(x) = 0 and f(x) = 1 (=balanced function)

Example: Deutsch - Jozsa

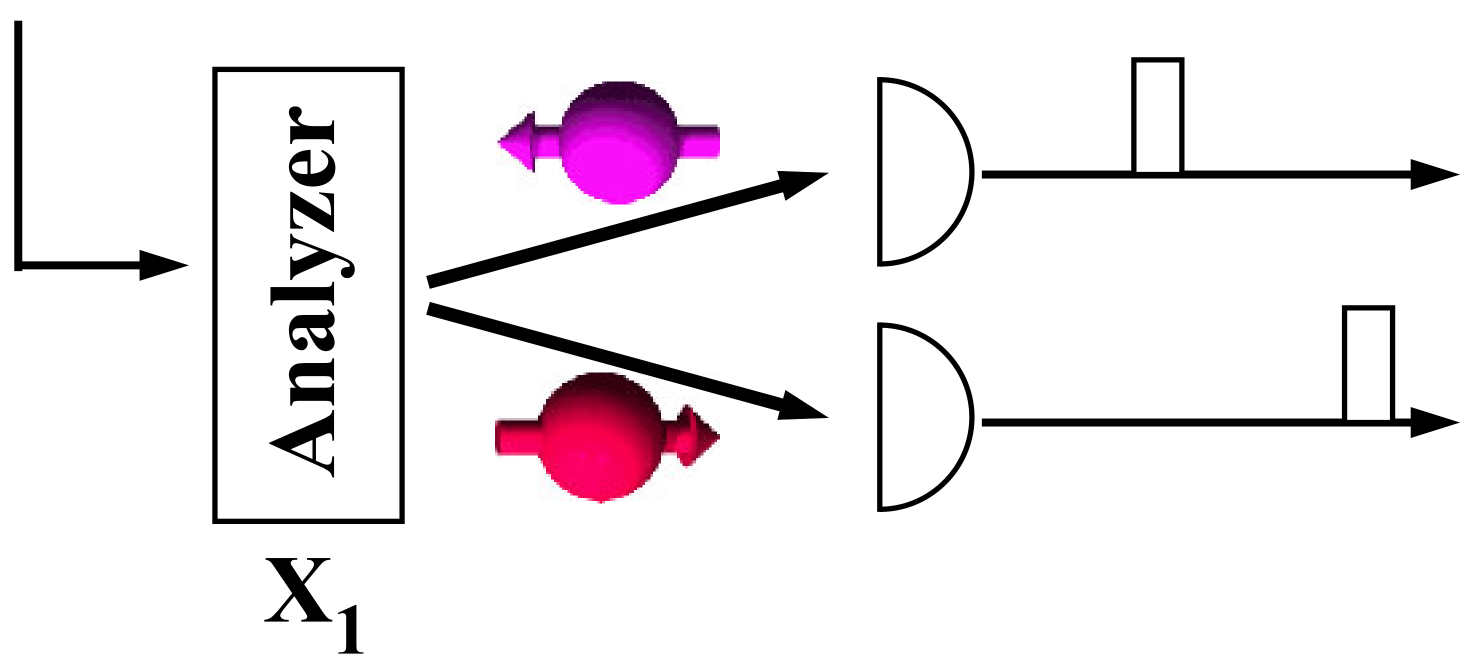
Algorithm

$$|\Psi_0\rangle = \sum_x |x,0\rangle \quad \Longrightarrow \quad |\Psi_{fin}\rangle = \sum_x |x,f(x)\rangle$$

Final quantum state

$$|\Psi_=\rangle = (|0\rangle - |1\rangle) \otimes (|f(0)\rangle - |\bar{f}(0)\rangle)$$

$$|\Psi_{\neq}\rangle = (|0\rangle + |1\rangle) \otimes (|f(0)\rangle - |\bar{f}(0)\rangle)$$



Correlations

Problem: Distinguish the states

	Z_1	Z_2
$\|\Psi_1\rangle = \frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$	50:50	50:50
$\|\Psi_2\rangle = \frac{1}{2}(\|00\rangle + \|01\rangle + \|10\rangle + \|11\rangle)$	50:50	50:50

Solution : Look at correlations:

$|\Psi_1\rangle$: 2 qubits are 100% correlated

$|\Psi_2\rangle$: 2 qubits are 0% correlated

1-qubit observables:

Z_1, Z_2

2-qubit observable :

$Z_1 Z_2$

Copy and Repeat

Copy operation :

CNOT

(a|0> + b|1>)|0> → a'|0>|0> + b'|1>|1>

Read ancilla

|0>|0> or |1>|1>

Reset ancilla

Repeat until result is clear

9.4

Alternative Schemes

9.4.1 Linear Optics and Measurements

9.4.2 Quantum Cellular Automata

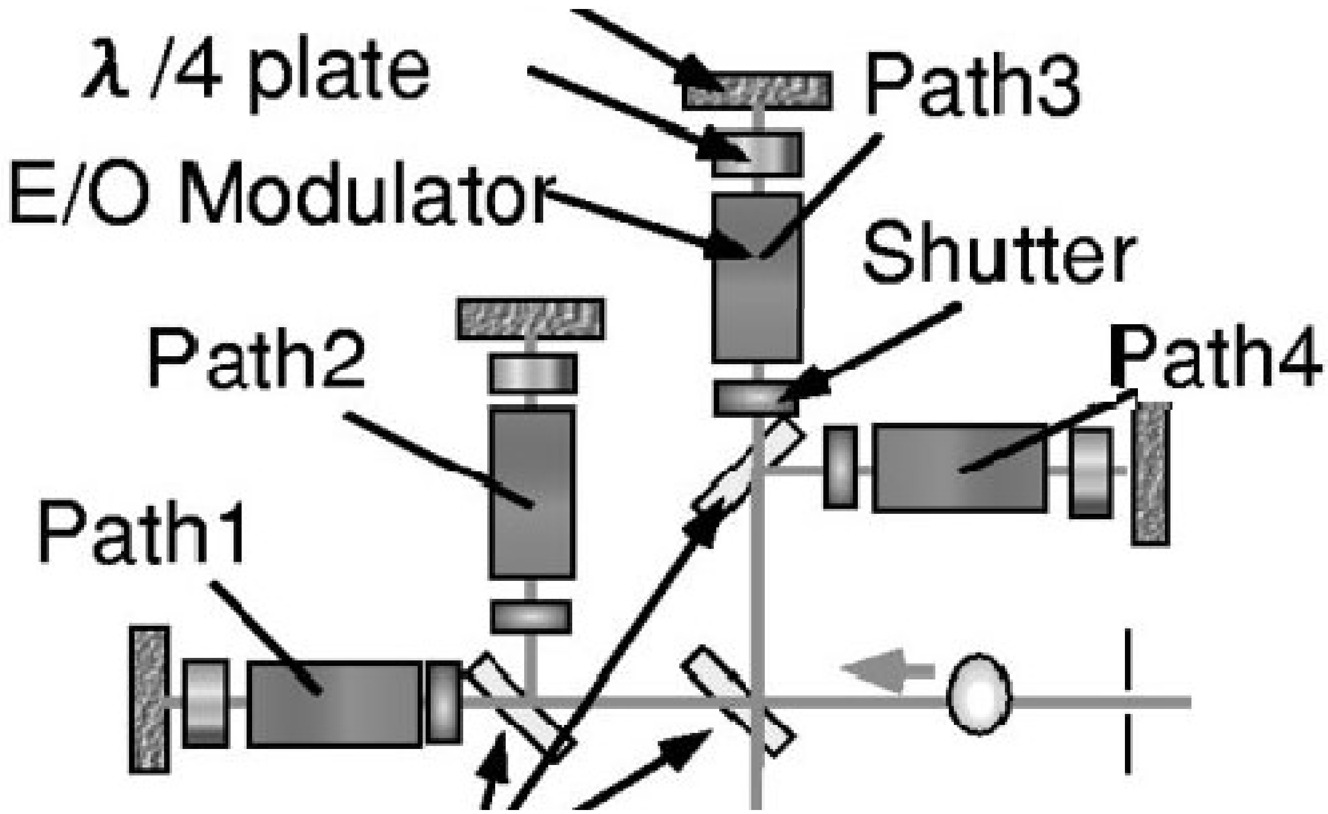
9.4.3 One-way Quantum Computer

9.4.4 Adiabatic Quantum Computer

Linear Optics Quantum Computer

S. Takeuchi, 'Experimental demonstration of a three-qubit quantum computation algorithm using a single photon and linear optics', Phys. Rev. A 62, 032301 (2000).

Deutsch-Jozsa



Beam splitters

2 input qubits = 4 paths

1 "accumulator" qubit

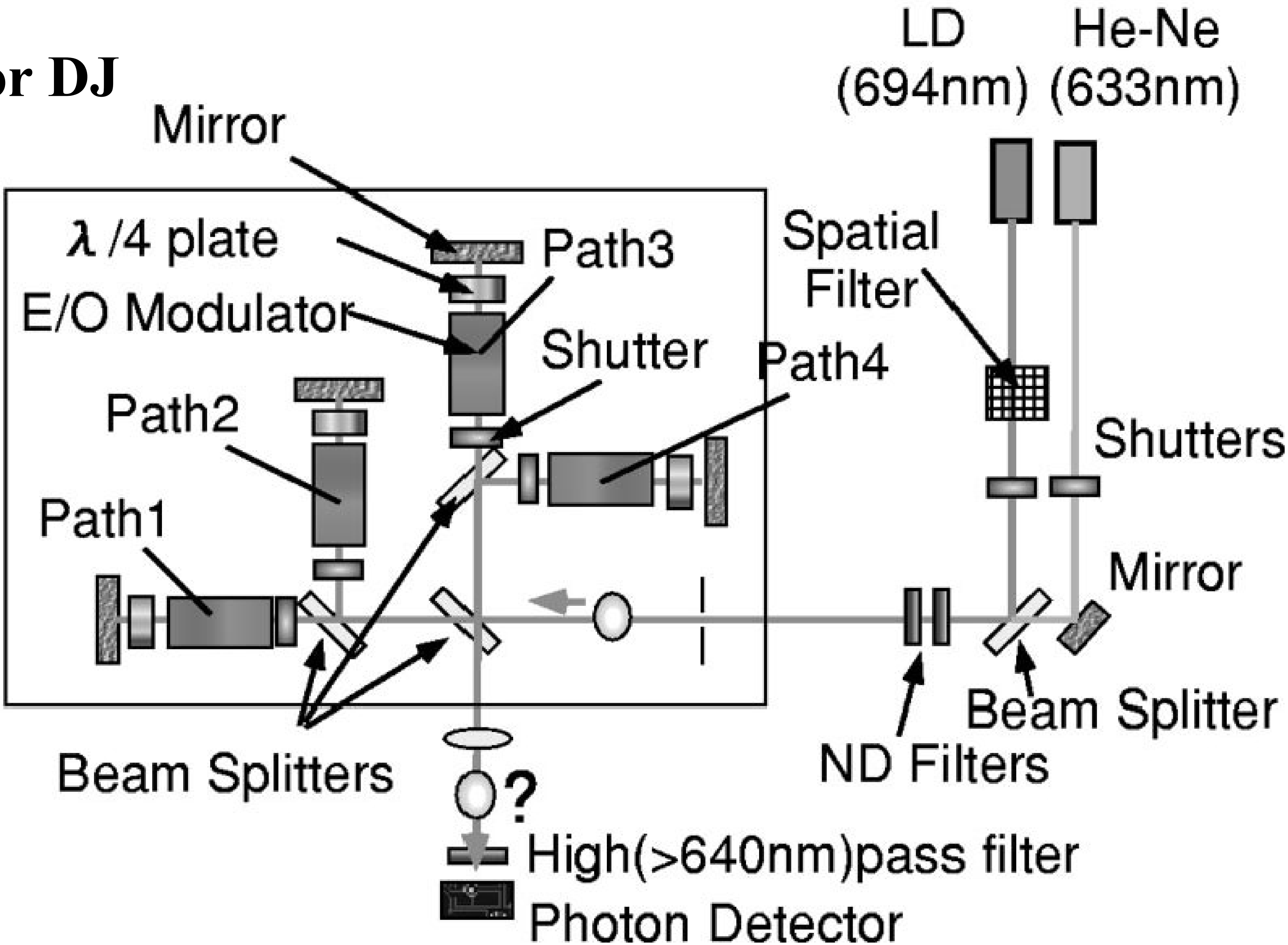
= polarization

Oracle: voltage of EOMs

Linear Optics Quantum Computer

S. Takeuchi, 'Experimental demonstration of a three-qubit quantum computation algorithm using a single photon and linear optics', Phys. Rev. A 62, 032301 (2000).

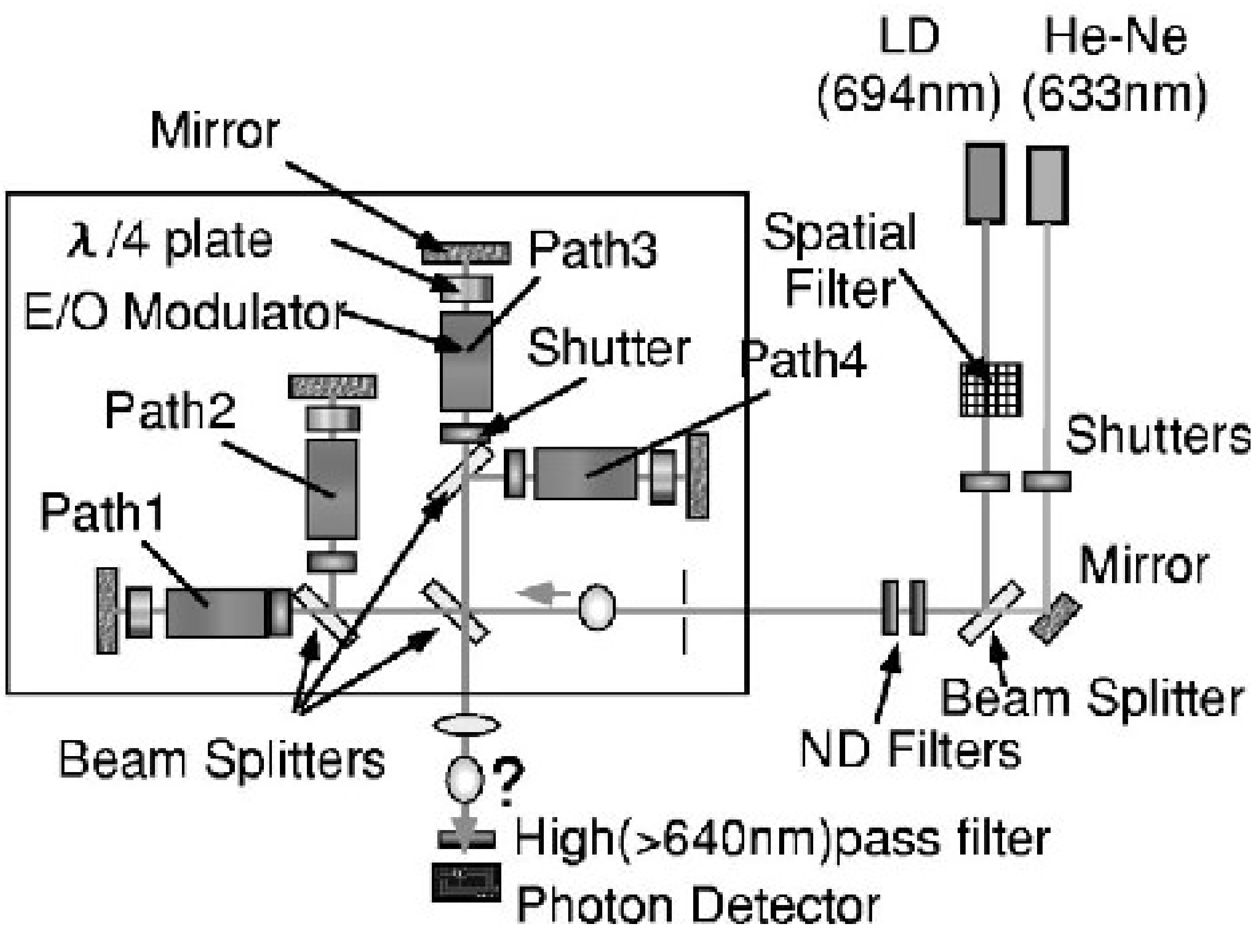
Setup for DJ



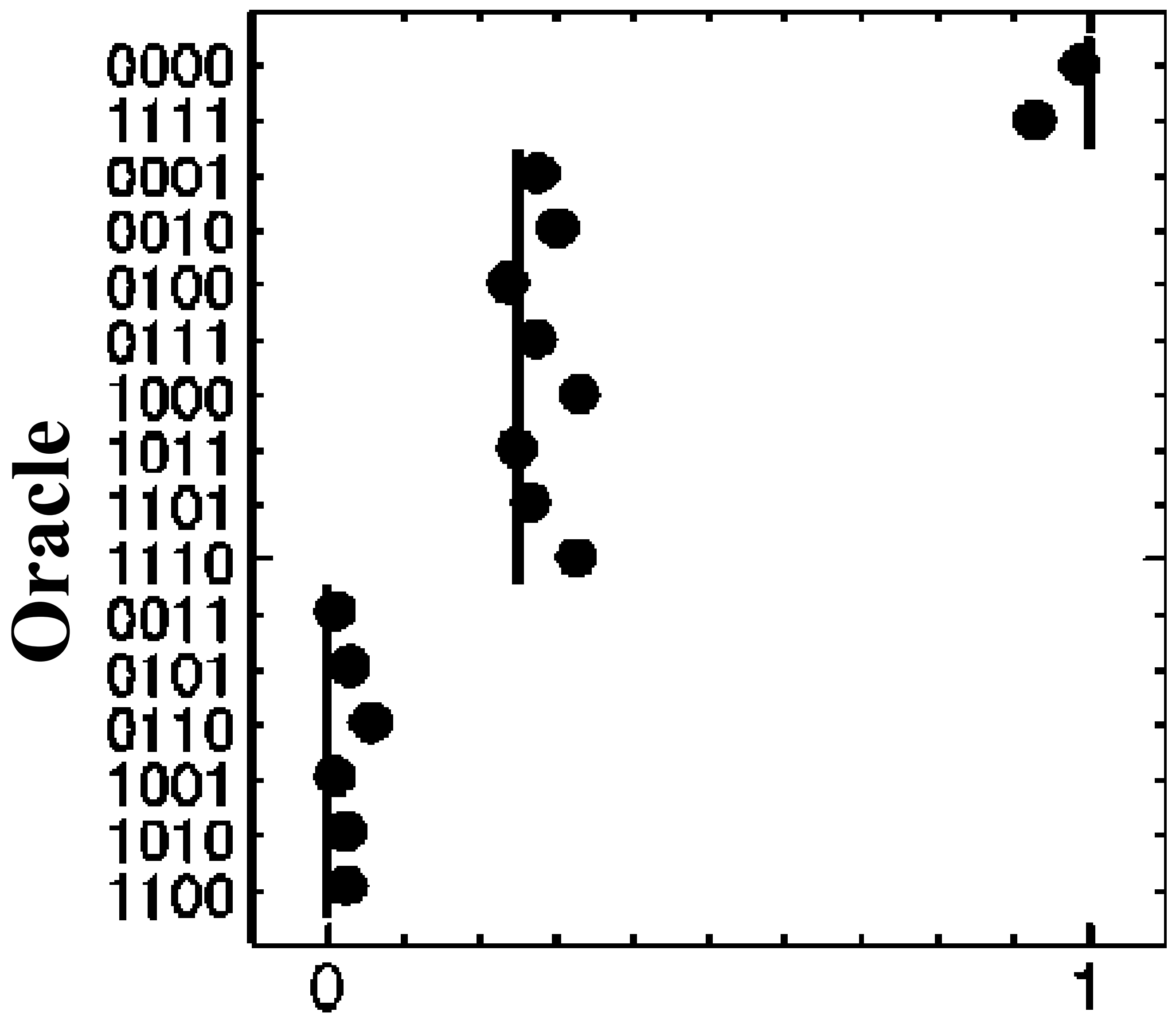
Linear Optics Quantum Computer

S. Takeuchi, 'Experimental demonstration of a three-qubit quantum computation algorithm using a single photon and linear optics', Phys. Rev. A 62, 032301 (2000).

Setup for DJ



Results



Linear Optics Quantum Computer

The Problem:

Size of setup increases exponentially with # qubits

The Solution: Nature 409, 46 (2000).

articles

A scheme for efficient quantum computation with linear optics

E. Knill*, R. Laflamme* & G. J. Milburn†

** Los Alamos National Laboratory, MS B265, Los Alamos, New Mexico 87545, USA*
† Centre for Quantum Computer Technology, University of Queensland, St. Lucia, Australia

Quantum computers promise to increase greatly the efficiency of solving problems such as factoring large integers, combinatorial optimization and quantum physics simulation. One of the greatest challenges now is to implement the basic quantum-computational elements in a physical system and to demonstrate that they can be reliably and scalably controlled. One of the earliest proposals for quantum computation is based on implementing a quantum bit with two optical modes containing one photon. The proposal is appealing because of the ease with which photon interference can be observed. Until now, it suffered from the requirement for non-linear couplings between optical modes containing few photons. Here we show that efficient quantum computation is possible using only beam splitters, phase shifters, single photon sources and photo-detectors. Our methods exploit feedback from photo-detectors and are robust against errors from photon loss and detector inefficiency. The basic elements are accessible to experimental investigation with current technology.

Linear Optics + Measurements

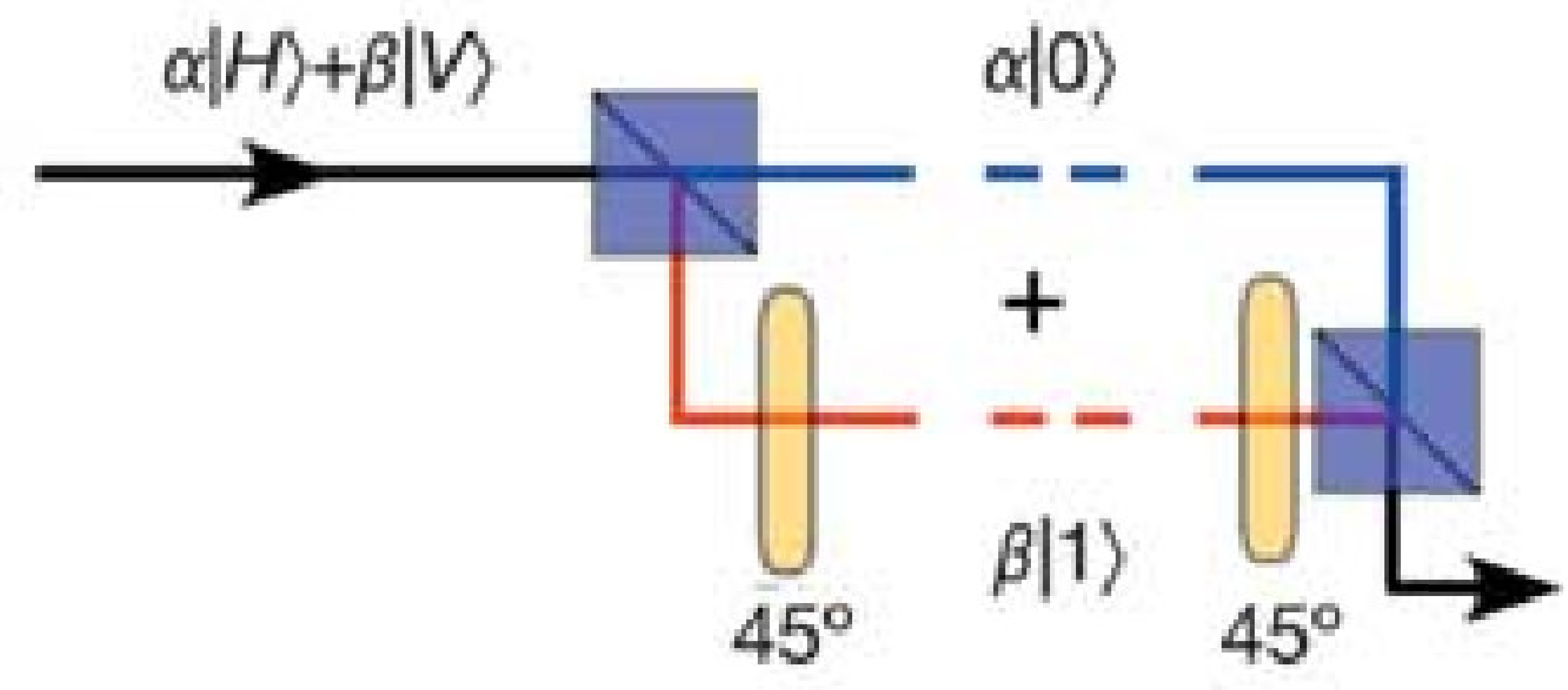
Optical mode |n>

qubit: Photon $|0_L\rangle = |01\rangle$

$|1_L\rangle = |10\rangle$

2 optical modes

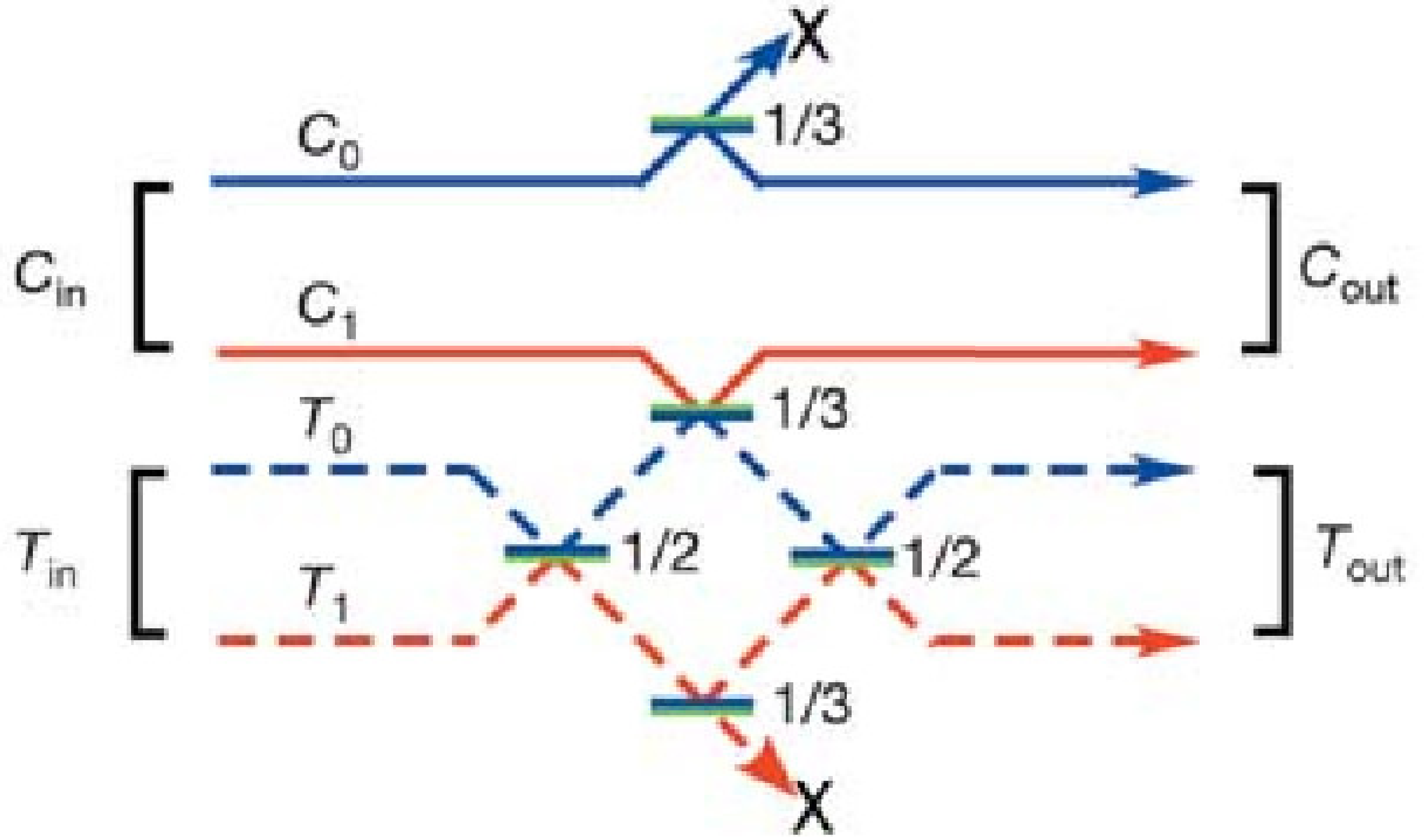
Conversion from polarization-qubit to spatially encoded qubit



Experimental Linear Optics QC

Basic idea : couple qubits by measurements and feed-forward

J.L. O'Brien, G.J. Pryde, A.G. White, T.C. Ralph, and D. Branning, 'Demonstration of an all-optical quantum controlled-NOT gate', Nature 426, 264 (2003).



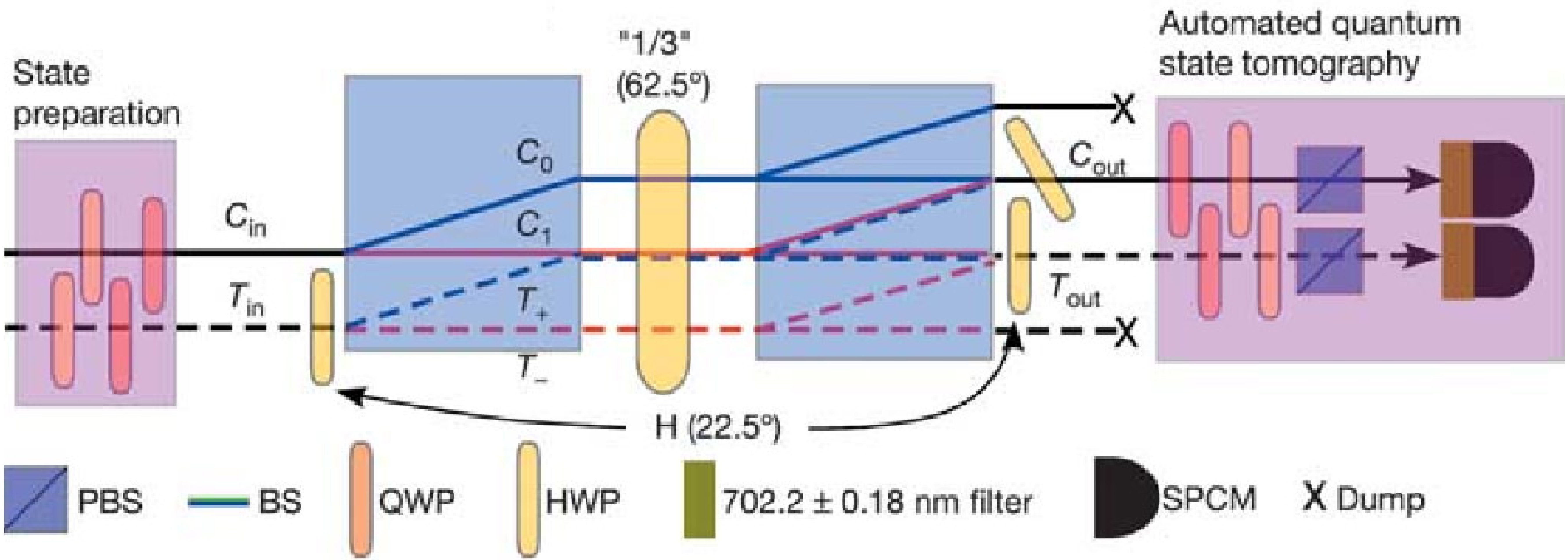
Experimental difficulties

Gates are probabilistic

Must store photons

Experimental Linear Optics QC

J.L. O'Brien, G.J. Pryde, A.G. White, T.C. Ralph, and D. Branning, 'Demonstration of an all-optical quantum controlled-NOT gate', Nature 426, 264 (2003).

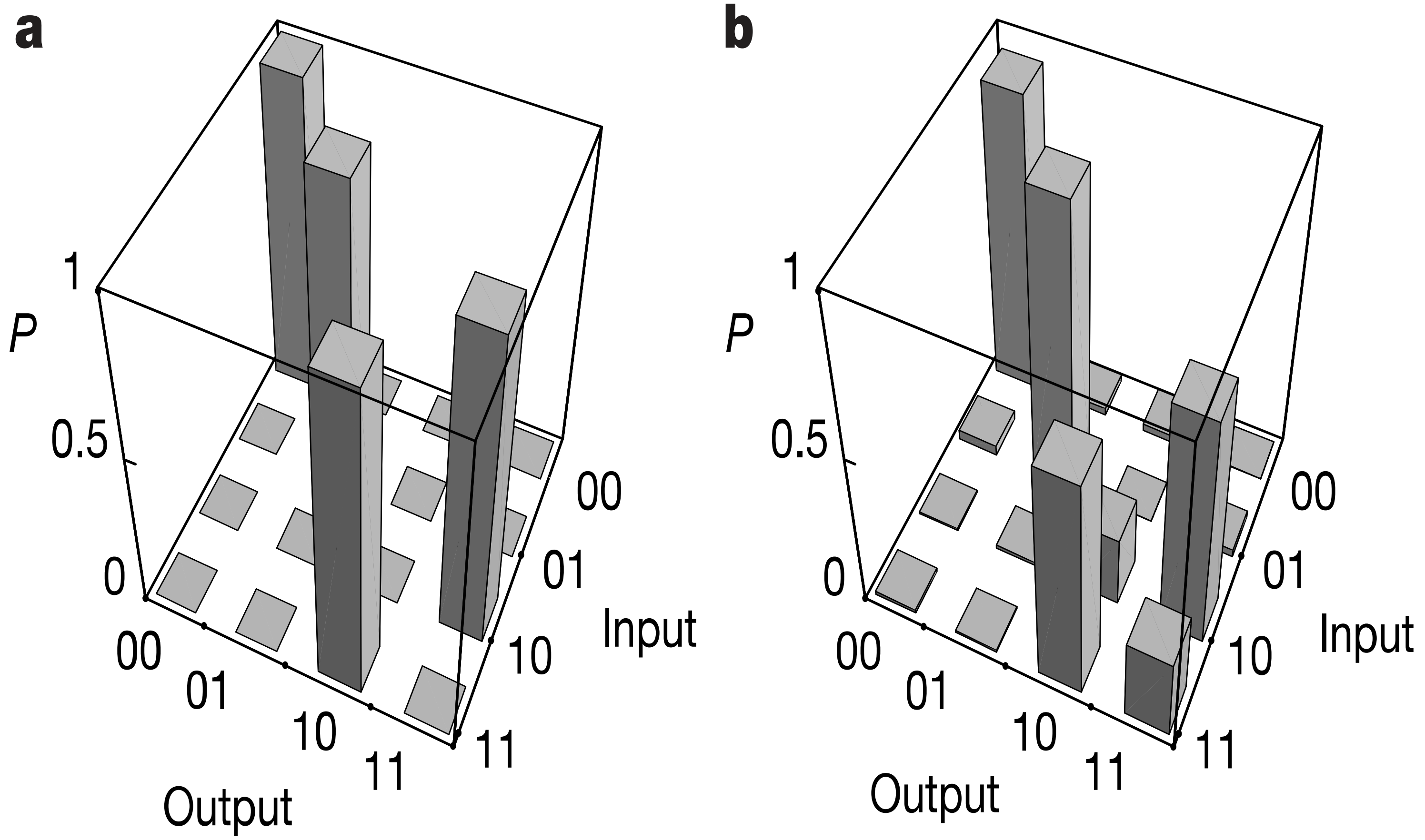


Input : degenerate photon pairs

Experimental Linear Optics QC

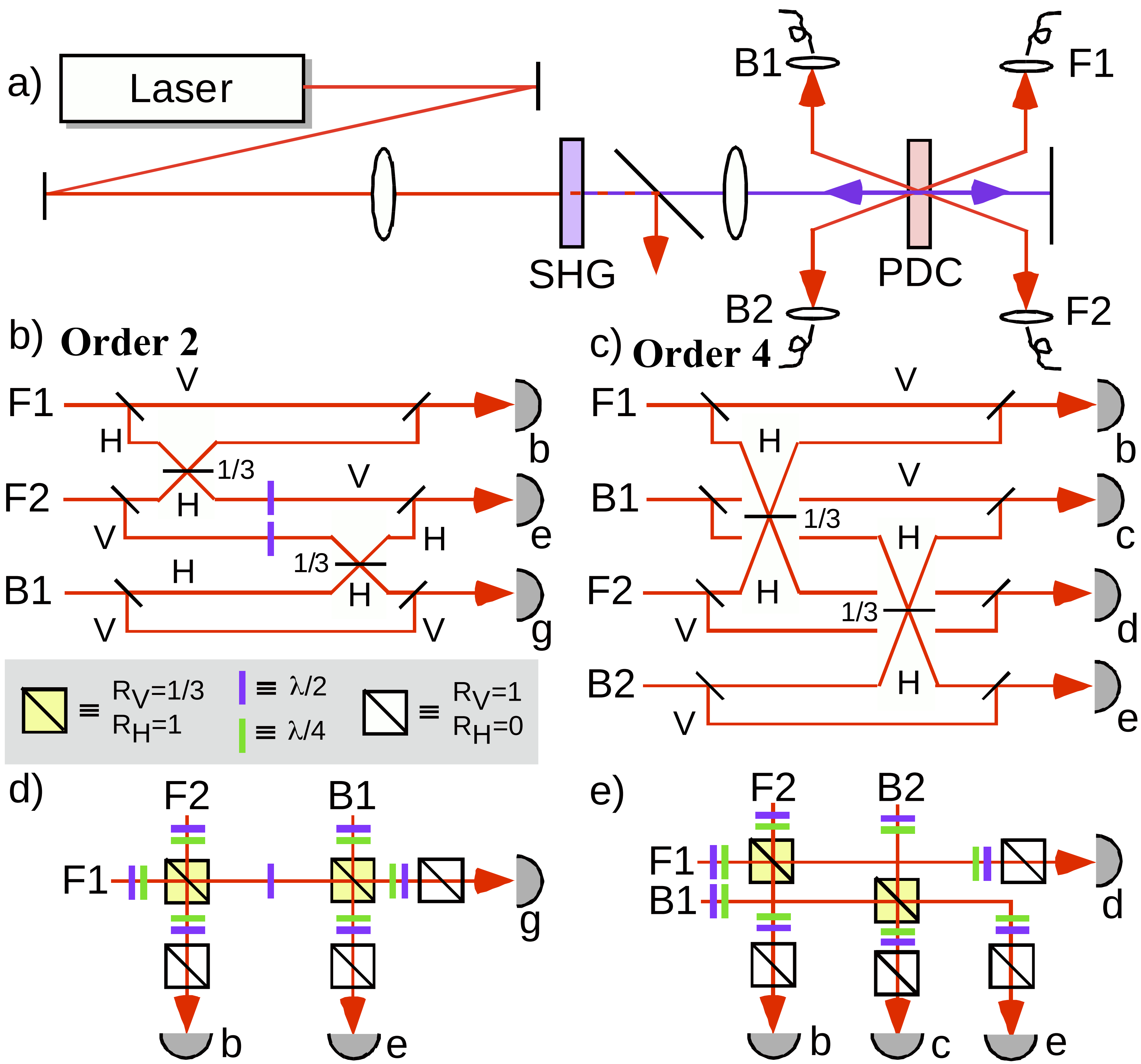
J.L. O'Brien, G.J. Pryde, A.G. White, T.C. Ralph, and D. Branning, 'Demonstration of an all-optical quantum controlled-NOT gate', Nature 426, 264 (2003).

Result



Theory (ideal)   Experiment

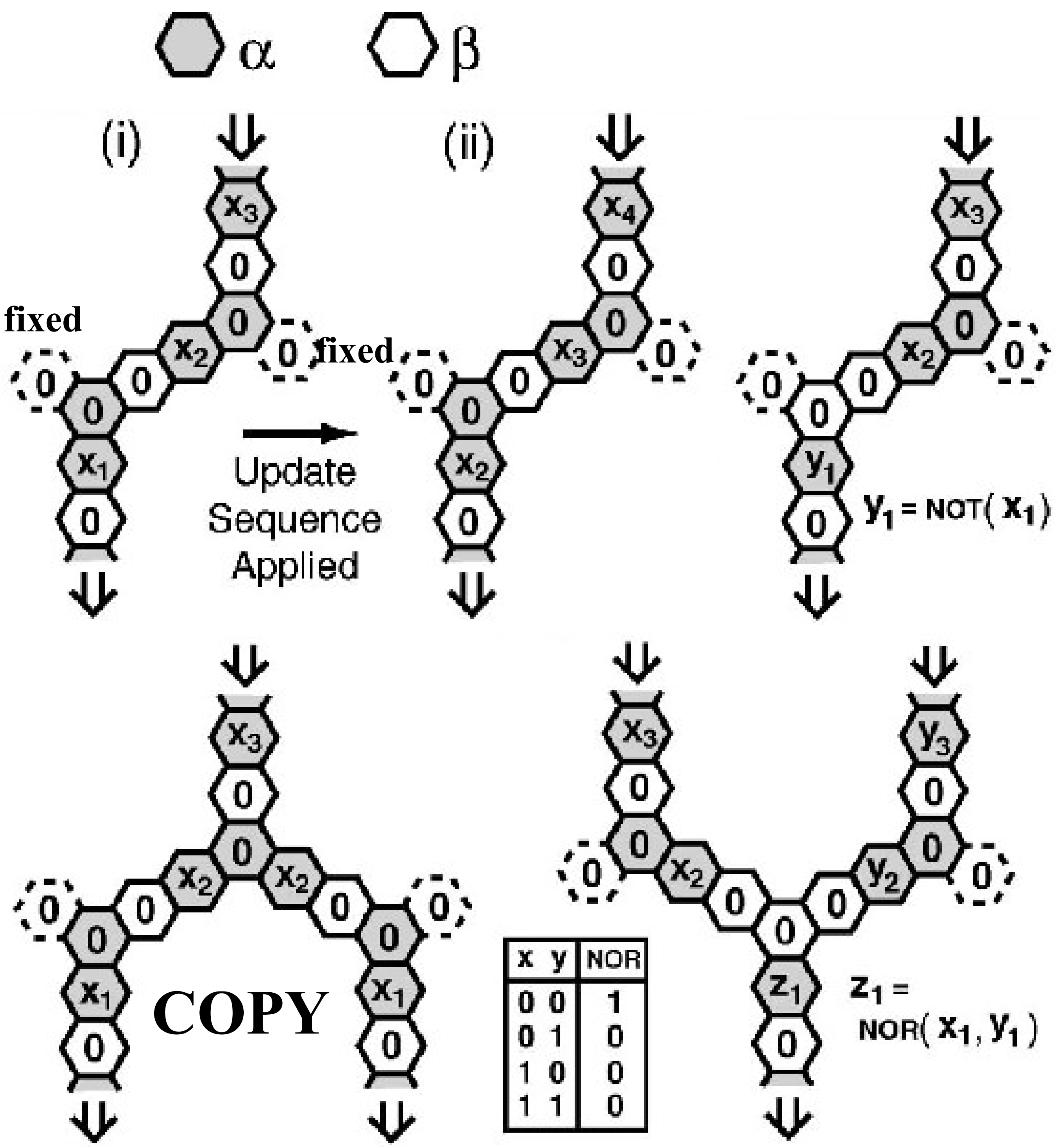
Experimental Linear Optics QC



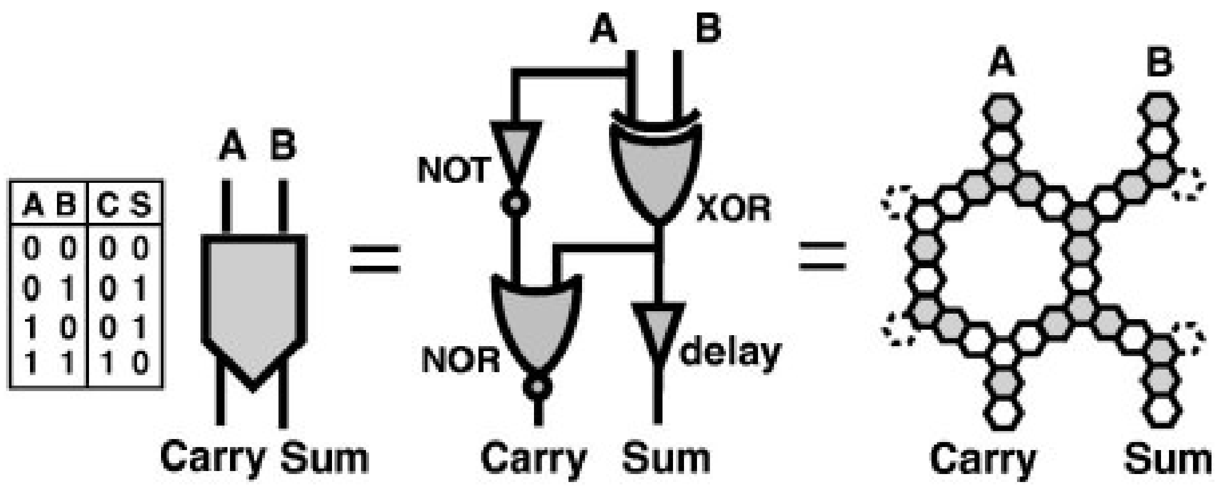
Lanyon et al. Experimental demonstration of a compiled version of Shor's algorithm with quantum entanglement Phys. Rev. Lett. 99, 250505 (2007).

Quantum Cellular Automata

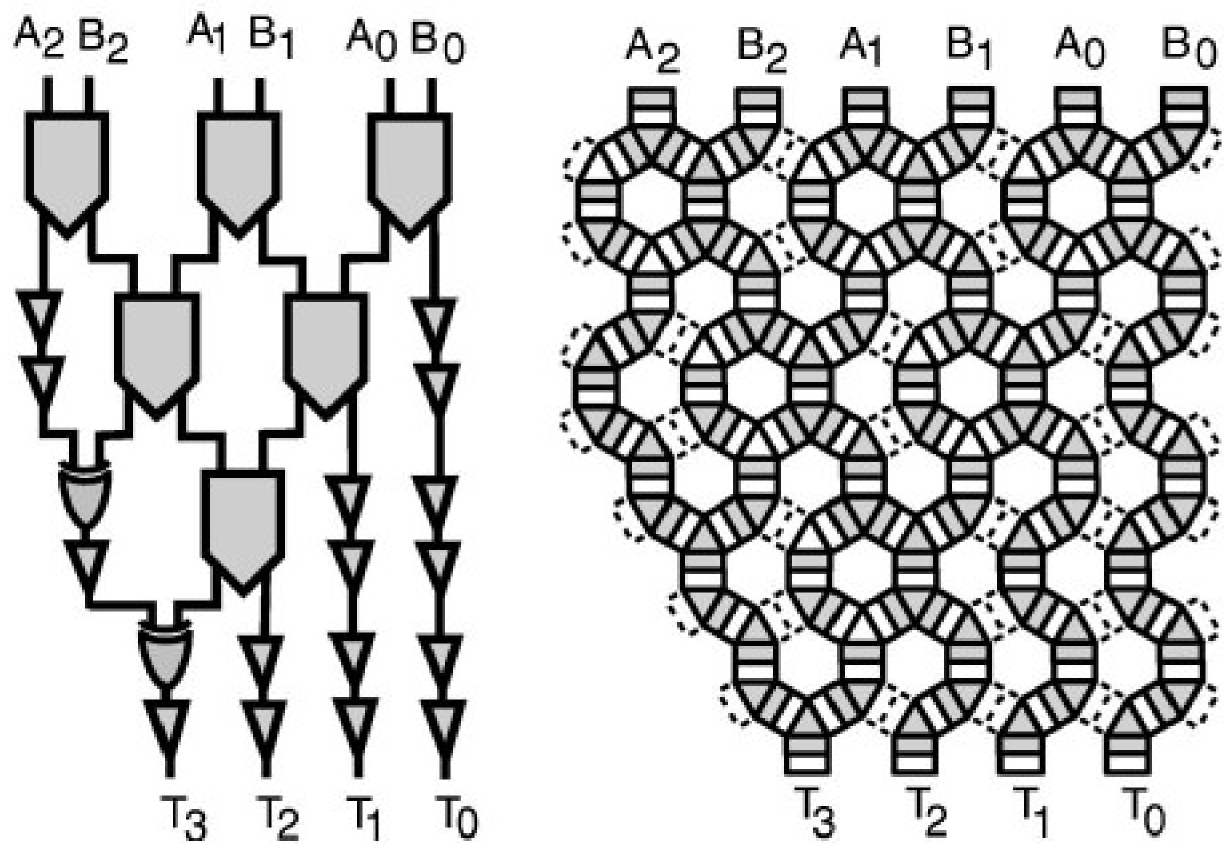
S.C. Benjamin, and N.F. Johnson, 'Cellular structures for computation in the quantum regime', Phys. Rev. Lett. 60, 4334 (1999).



1-qubit half-adder



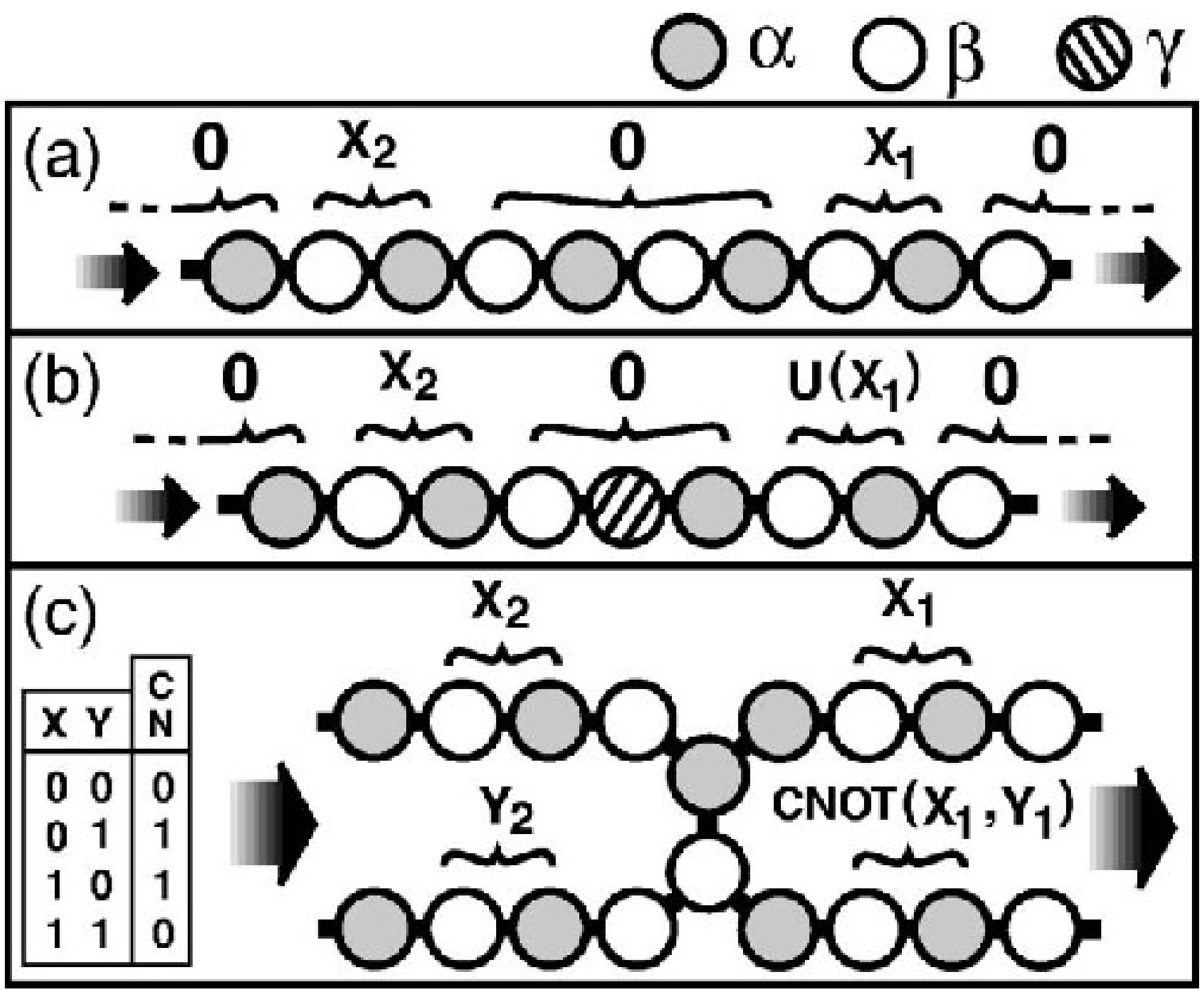
3-qubit full adder



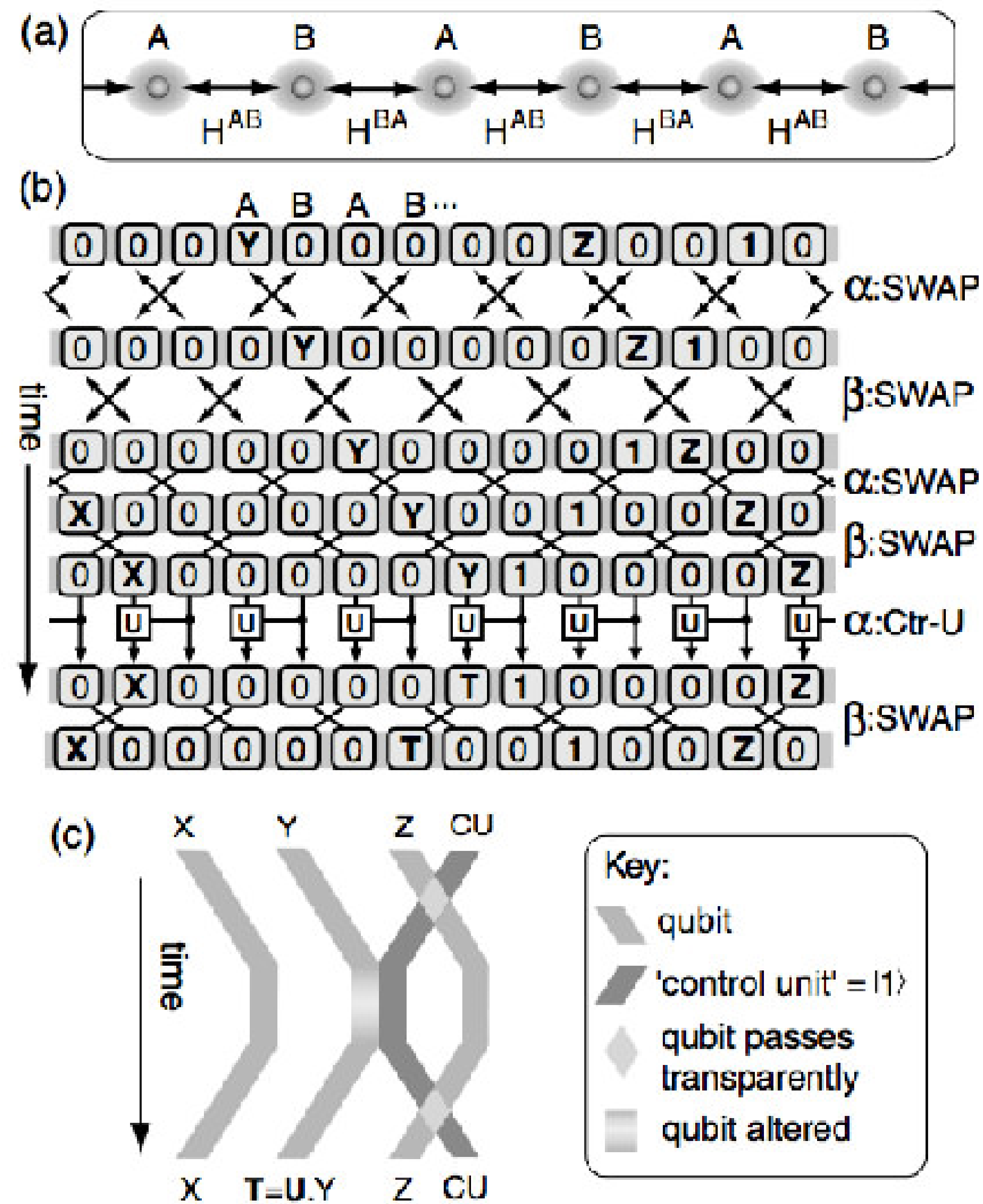
Quantum Cellular Automata

S.C. Benjamin, and N.F. Johnson, 'Cellular structures for computation in the quantum regime', Phys. Rev. Lett. 60, 4334 (1999).

3 physical qubits



2 physical qubits



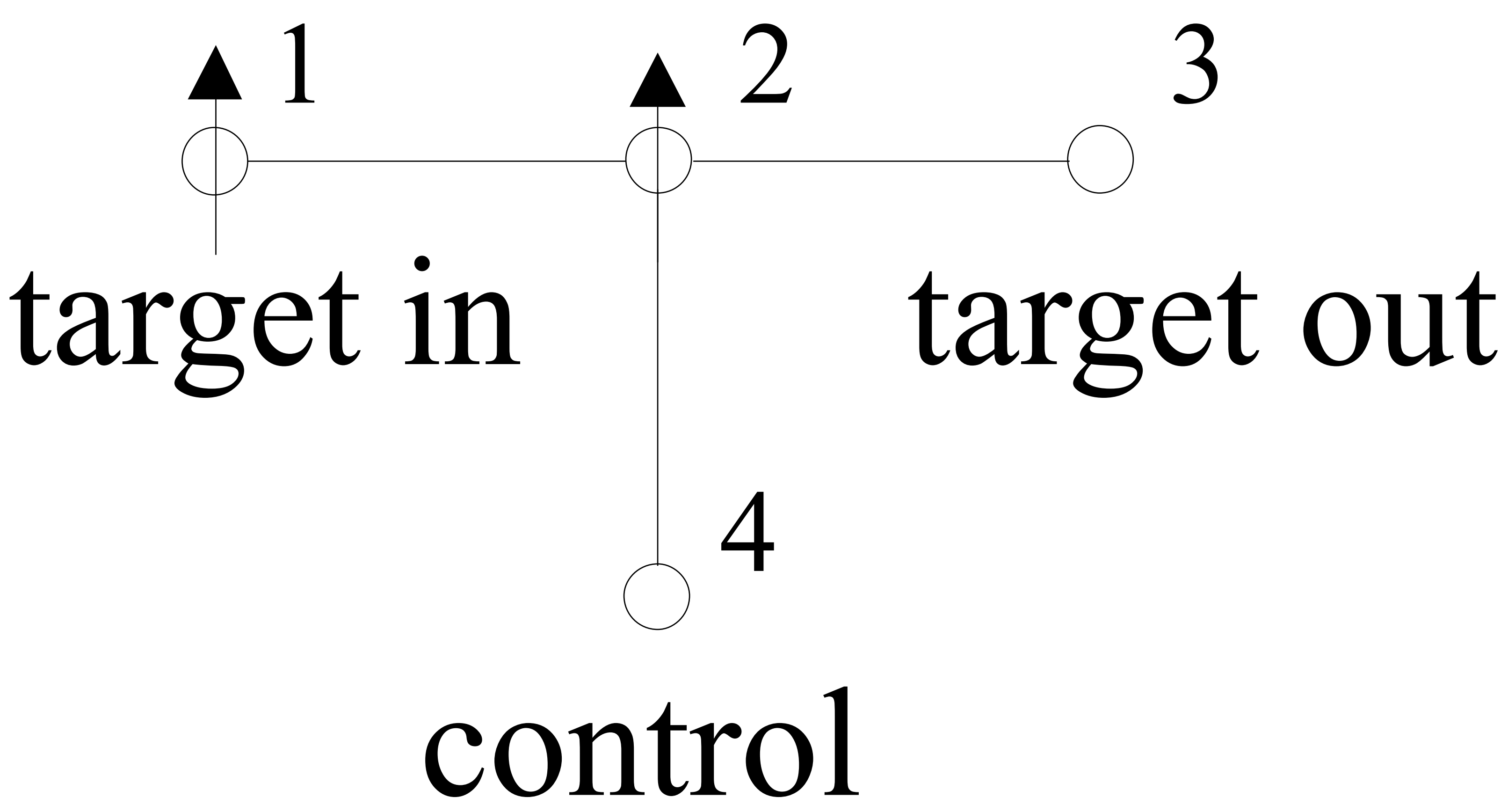
One-Way Quantum Computer

- Prepare "Cluster state"
- Perform measurements on individual qubits

R. Raussendorf, and H.J. Briegel, 'A One-Way Quantum Computer', Phys. Rev. Lett. 86, 5188 (2001).

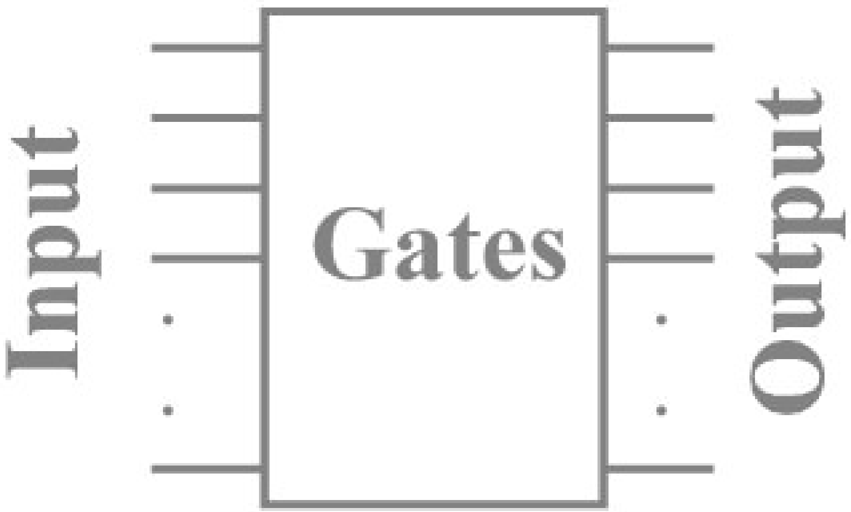
R. Raussendorf, D.E. Browne, and H.J. Briegel, 'Measurement-based quantum computation on cluster states', Phys. Rev. A 68, 022312 (2003).

Example: CNOT



Adiabatic Quantum Computing

Network model

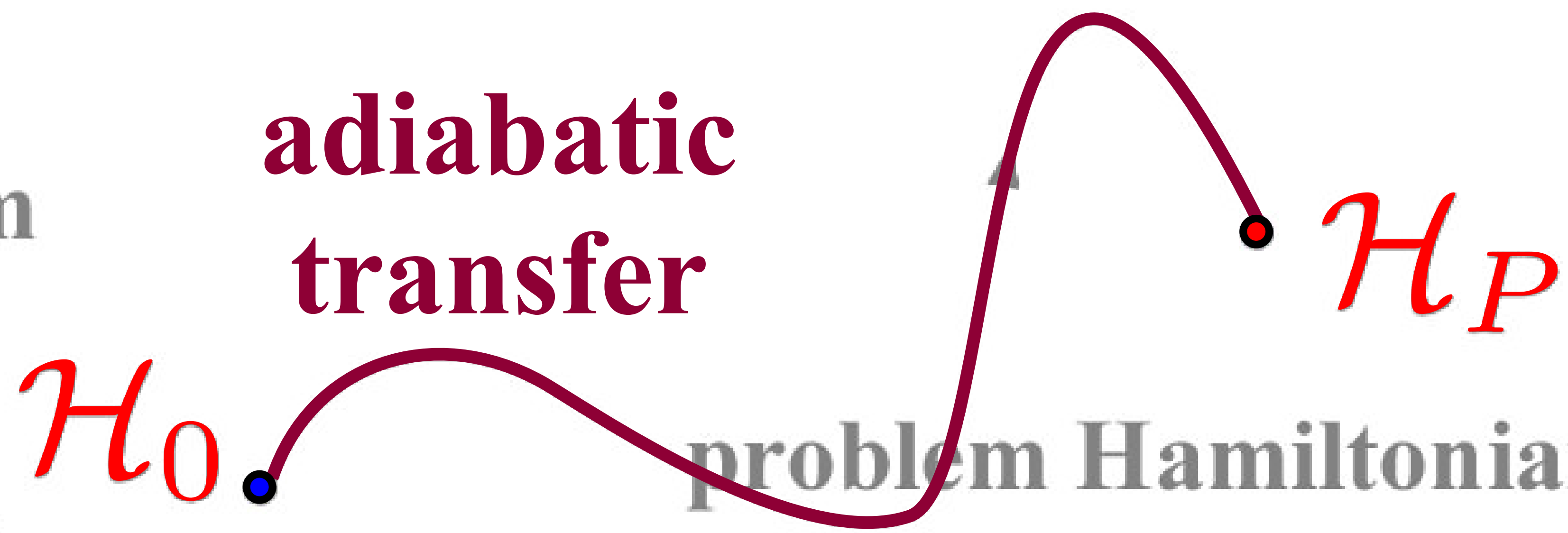


e.g. Shor algorithm

Adiabatic model

Solution = ground state

$$\mathcal{H}|\psi_g\rangle = E_0|\psi_g\rangle$$



initial Hamiltonian

accessbile ground state

problem Hamiltonian

ground state = solution

Farhi et al., Science 292, 472 (2001).

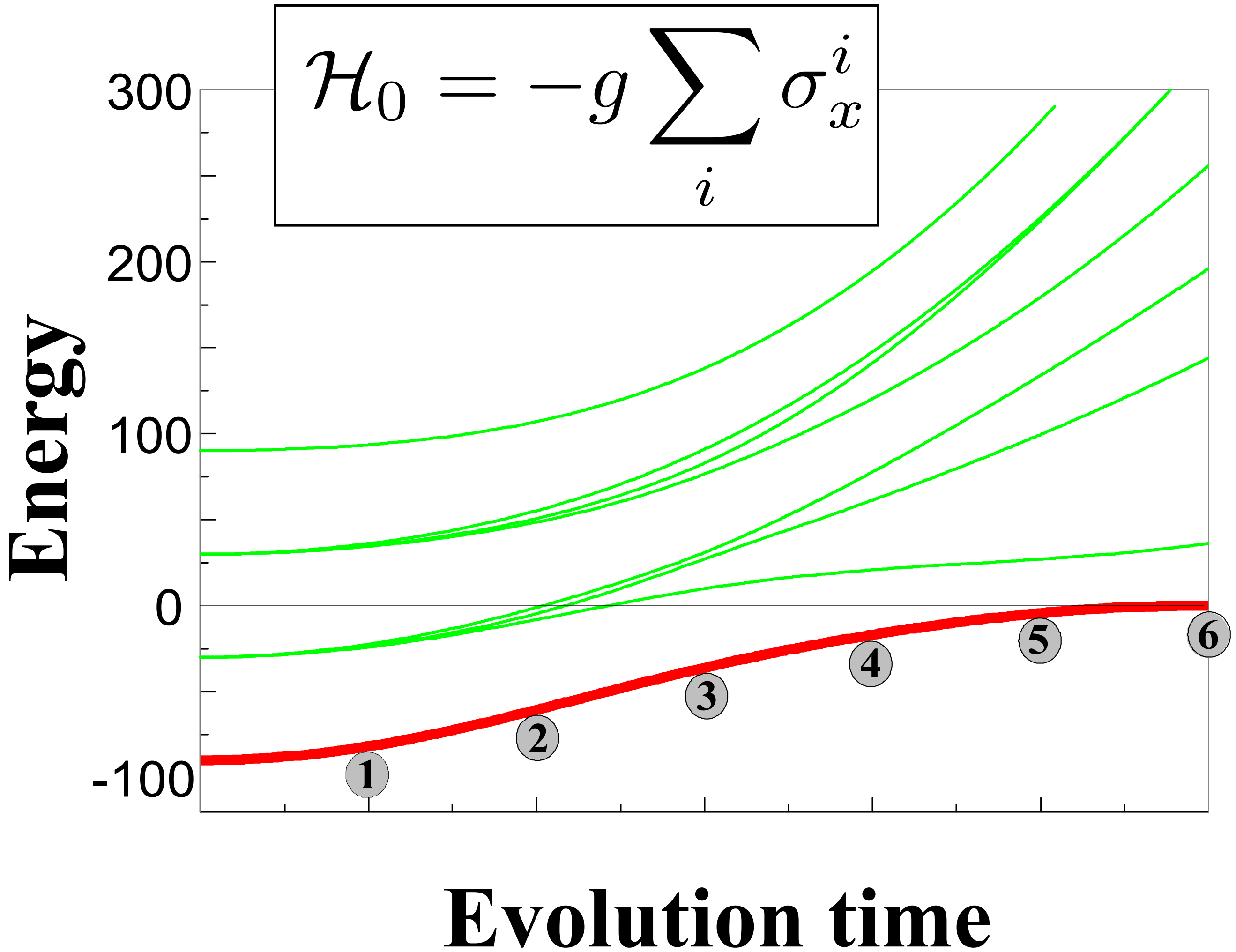
Adiabatic Factoring

trial factors

$$H_P = \sum_{x,y} f(x,y)|x,y\rangle\langle x,y|$$

$$f(x,y) = (N - xy)^2$$

$$21 = \square \times \square \ ?$$



Change of state during adiabatic evolution