Bell States

$$\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)$$

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)$$

John Stewart Bell
1928-1990

Entangled 3-Qubit States

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\rangle \pm |\downarrow\downarrow\downarrow\rangle)$$

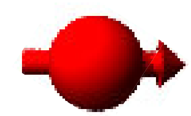
$$|\mathrm{W}_{001}\rangle = \frac{1}{\sqrt{3}}(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\downarrow\rangle)$$

Quantum mechanics is nonlocal !

$$|\psi_1\rangle = |\uparrow\rangle \otimes |\uparrow\rangle$$

$$|\psi_2\rangle = |\uparrow\rangle \otimes |\downarrow\rangle$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |\downarrow\rangle$$



$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\uparrow\rangle) + (|\downarrow\rangle \otimes |\downarrow\rangle)$$

$$\rho_4 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

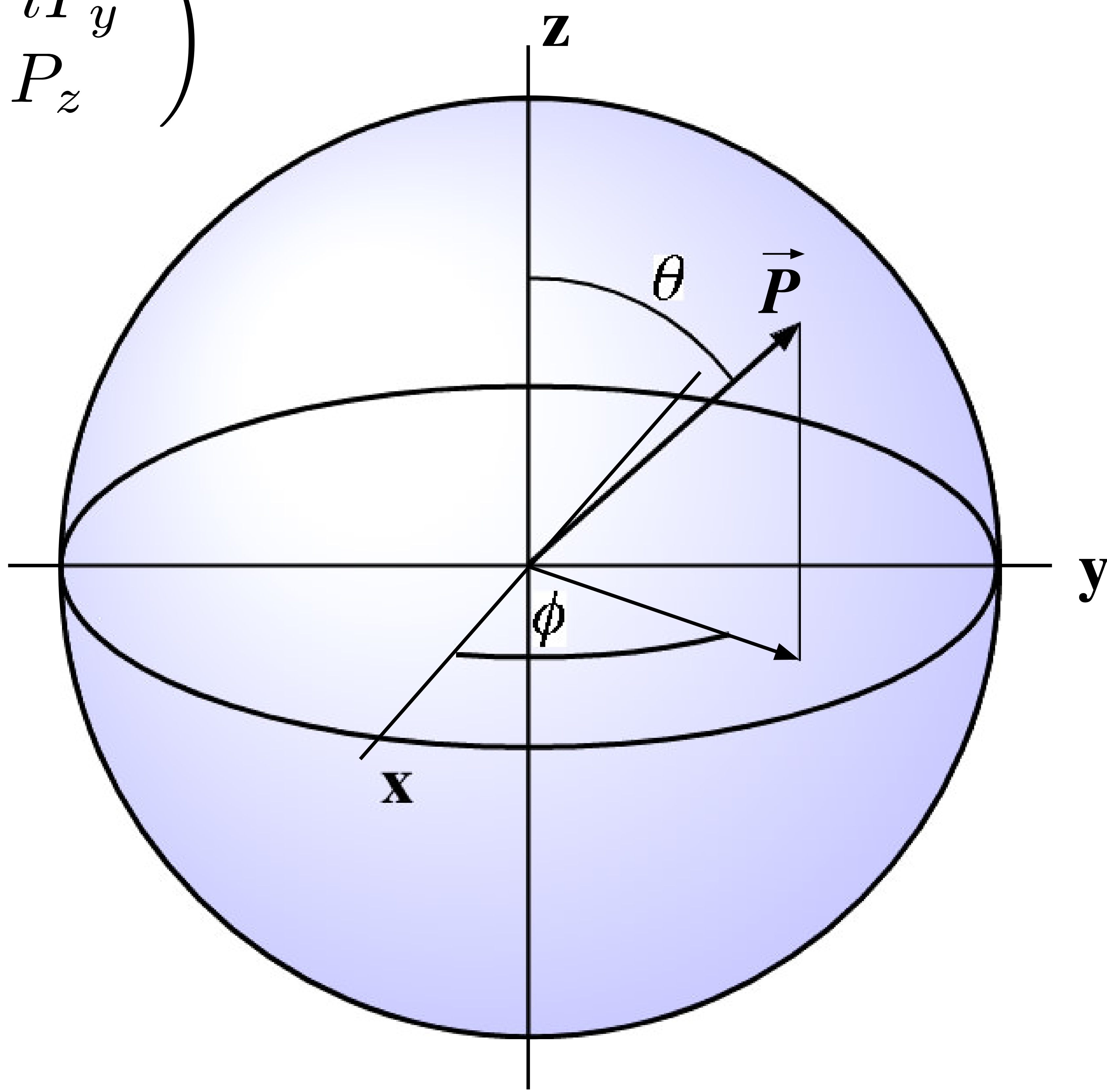
Maximally mixed state

The Bloch Sphere

arbitrary qubit state

$$\frac{1}{2}\begin{pmatrix} 1+P_z & P_x - iP_y \\ P_x + iP_y & 1-P_z \end{pmatrix}$$

$$= \frac{1}{2}\left(\mathbf{1} + \frac{2}{\hbar}\vec{P}\cdot\vec{\mathbf{S}}\right)$$



Hidden Variables

1935 EPR: Is QM complete ?

A. Einstein, B. Podolsky, and N. Rosen
Can quantum mechanical description of physical reality be considered complete ?
Phys. Rev. 47, 777 (1935).

Possible solutions :

- hidden variables
- faster than light interactions

John Bell

J.S. Bell
On the Einstein Podolsky Rosen paradox
Physics <u>1</u>, 195–200 (1964).

J. Bell
On the problem of hidden variables in quantum mechanics
Rev. Mod. Phys. <u>38</u>, 447 (1966).

Hidden variables are not compatible with QM

"Bell inequalities" allow quantitative test

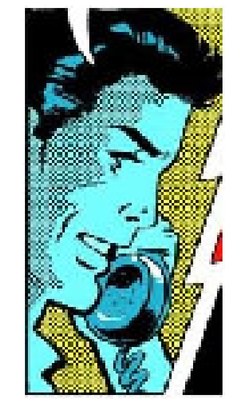
John Stewart Bell
1928 – 1990

according to

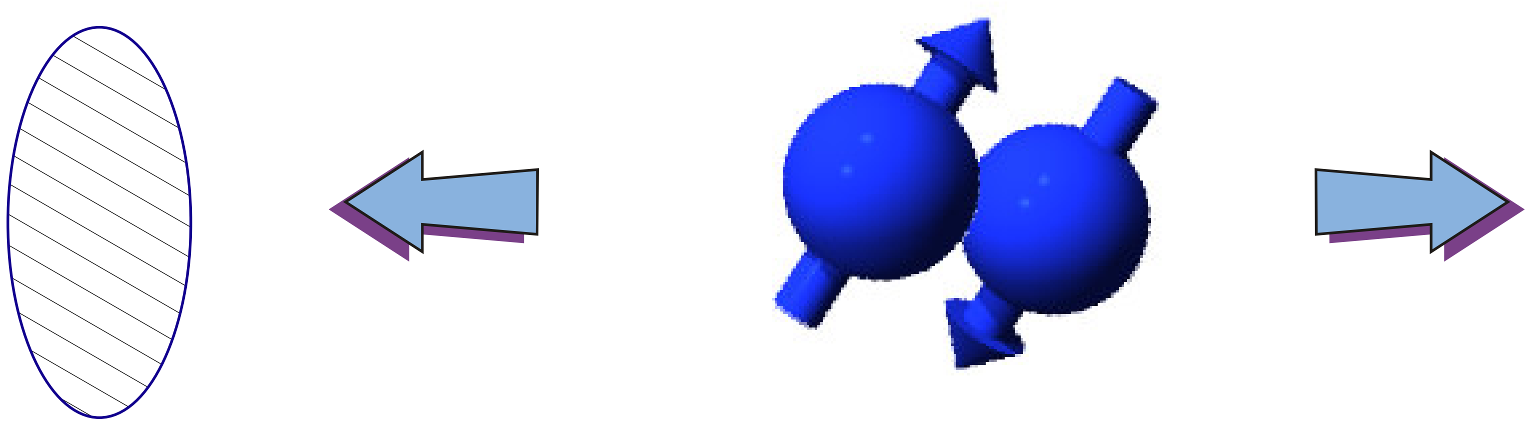
Bell Inequalities

J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt
Proposed experiment to test local hidden-variable theories
Phys. Rev. Lett. <u>23</u>, 880–884 (1969).

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



Polariser A **Polariser B**



Orientation a, a' **Orientation b, b'**

$$a = \frac{2}{\hbar}\vec{S}_A \cdot \vec{a}$$

"Classical" Analysis

Possible measurement outcomes of a, a', b, b' : ±1

Consider f := (a+a')b - (a-a')b' .

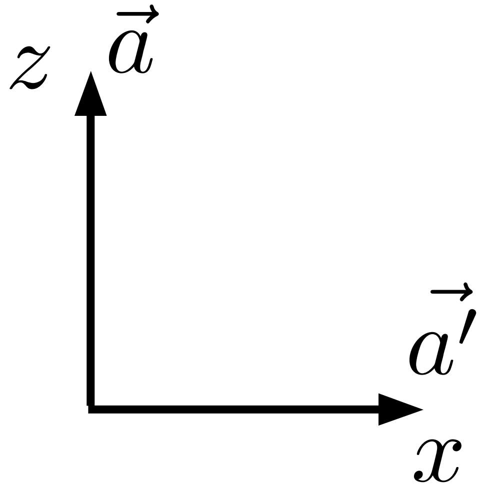
$$\left.\begin{array}{lll} (a+a') = 0 & \text{and} & (a-a') = 2 \\ & \text{or} & \\ (a+a') = 2 & \text{and} & (a-a') = 0 \end{array}\right\} |f| = 2$$

$$\bar{f} = \sum_{a,a',b,b'} p(a,a',b,b') f \leq 2$$

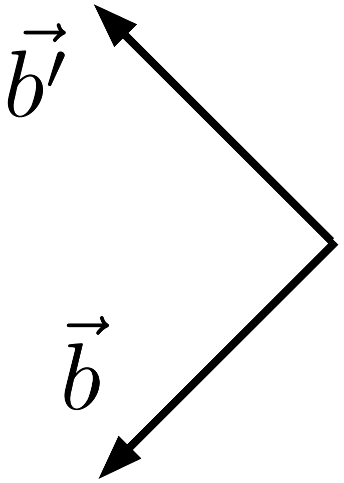
Must hold if probability density exists

$$\overline{ab} + \overline{a'b} - \overline{a'b} + \overline{a'b'} \leq 2$$

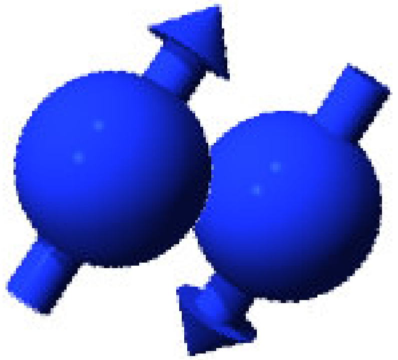
QM Analysis



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



$$\vec{a} = z$$
$$\vec{a'} = x$$



$$\vec{b} = \frac{1}{\sqrt{2}}(-z - x)$$
$$\vec{b'} = \frac{1}{\sqrt{2}}(z - x)$$

$$\langle ab\rangle = \langle\psi|ab|\psi\rangle = \frac{1}{\sqrt{2}}$$

$$\langle a'b\rangle = \frac{1}{\sqrt{2}} \quad \langle ab'\rangle = -\frac{1}{\sqrt{2}} \quad \langle a'b'\rangle = \frac{1}{\sqrt{2}}$$

violates CHSH

$$\langle ab\rangle + \langle a'b\rangle - \langle ab'\rangle + \langle a'b'\rangle = 2\sqrt{2} > 2$$

??????????

Bell inequalities :

Quantum mechanics is incompatible with local realism

Experiments must decide !

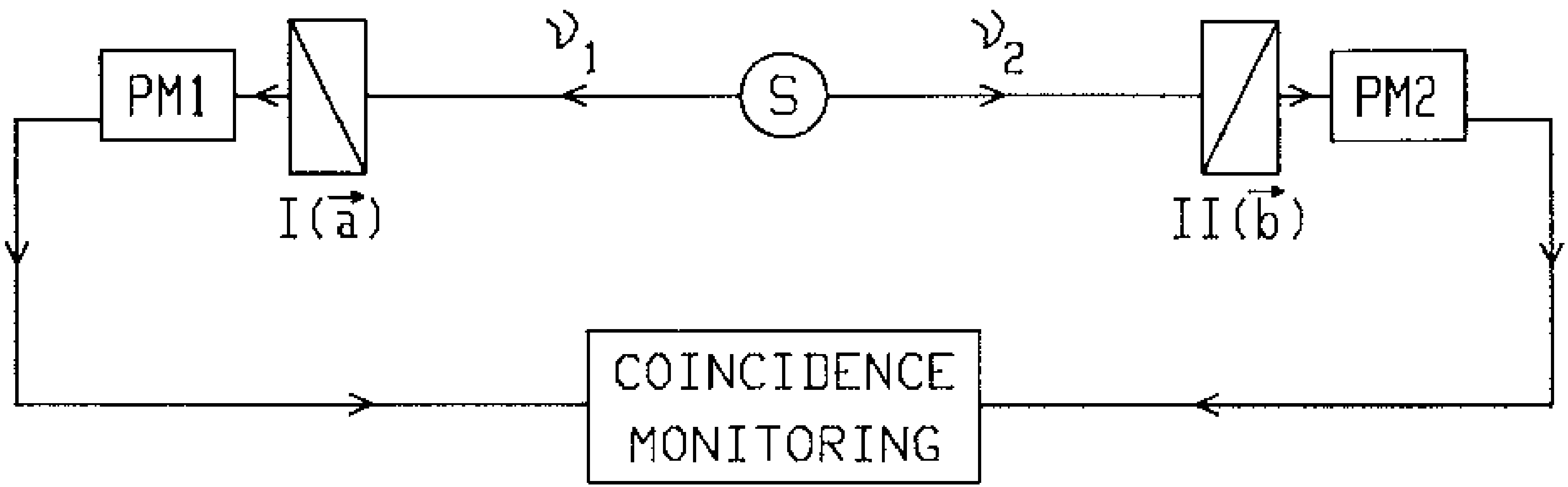
The Aspect Experiments

VOLUME 47, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1981

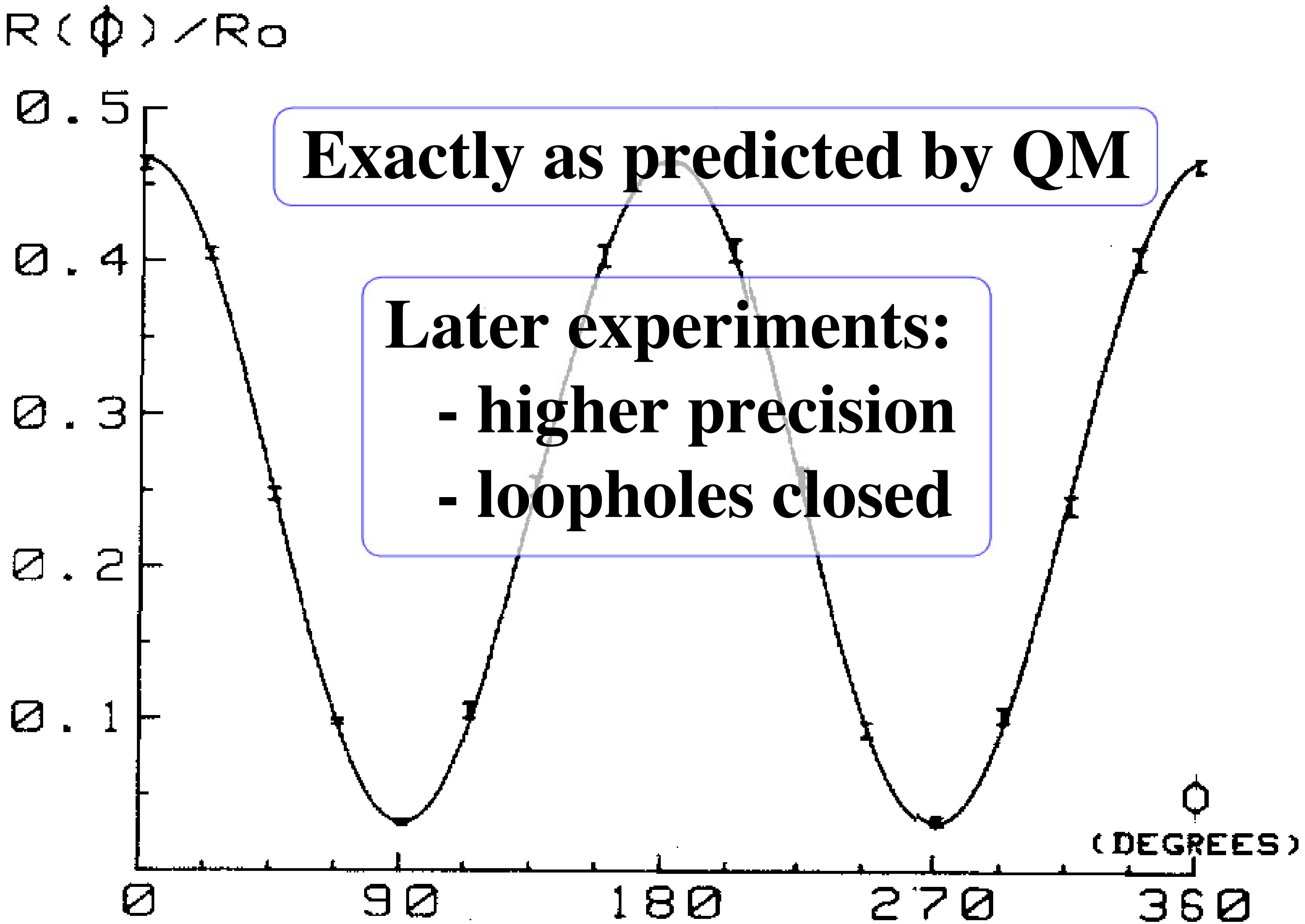
Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France

(Received 30 March 1981)



The Aspect Experiments



Exactly as predicted by QM

Later experiments:
- higher precision
- loopholes closed

Cloning

Cloning

Dolly

Classical Cloning

Classical CNOT

$$(x, y) \longrightarrow (x, x \text{ XOR } y)$$

$$(x, 0) \longrightarrow (x, x)$$

Quantum Cloning

Quantum CNOT

$$|00\rangle \xrightarrow{\text{CNOT}} |00\rangle \qquad |10\rangle \xrightarrow{\text{CNOT}} |11\rangle$$

$|0\rangle, |1\rangle$ **can be ~~cloned~~ ! copied**

Cloning refers to general, unknown states

General state of source qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Try cloning:

$$|\psi 0\rangle \xrightarrow{\text{CNOT}} \alpha|00\rangle + \beta|11\rangle$$

Target state

$$\neq$$

$$|\psi\psi\rangle = \alpha^2|00\rangle + \beta\alpha|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$$

General Proof

General cloning operator U

$$U|\psi s\rangle = |\psi\psi\rangle \qquad U|\phi s\rangle = |\phi\phi\rangle$$

Scalar product of input states | **Scalar product of rhs**

$$\langle\psi s|\phi s\rangle = \langle\psi|\phi\rangle\langle s|s\rangle \qquad (\langle\psi|\phi\rangle)^2$$

equal only if
$\langle\psi|\phi\rangle$ **= 0 or 1**

No cloning theorem :

U does not exist !!

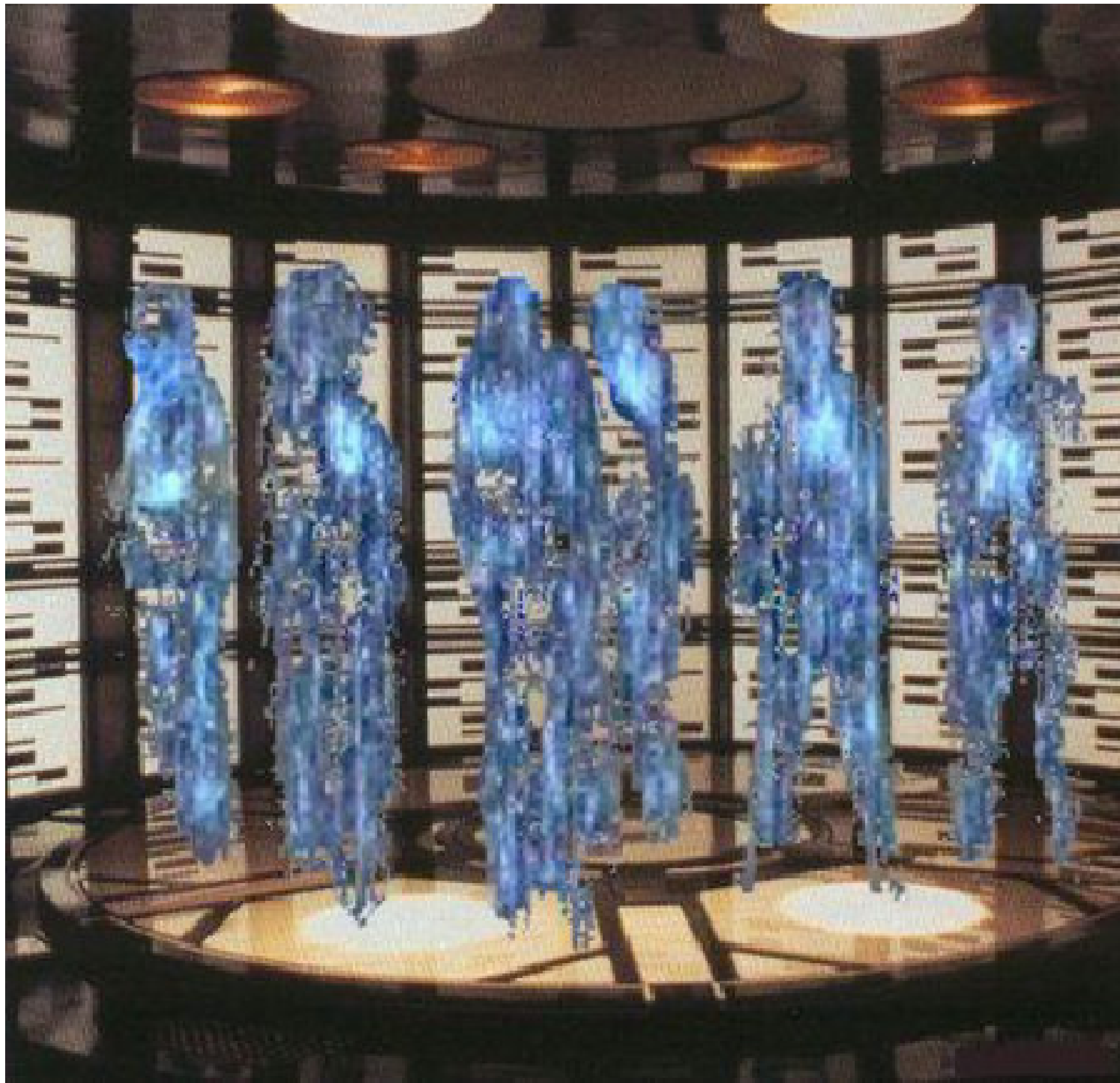
- **makes quantum communication safe**
- **makes quantum error correction difficult**

Quantum Teleportation

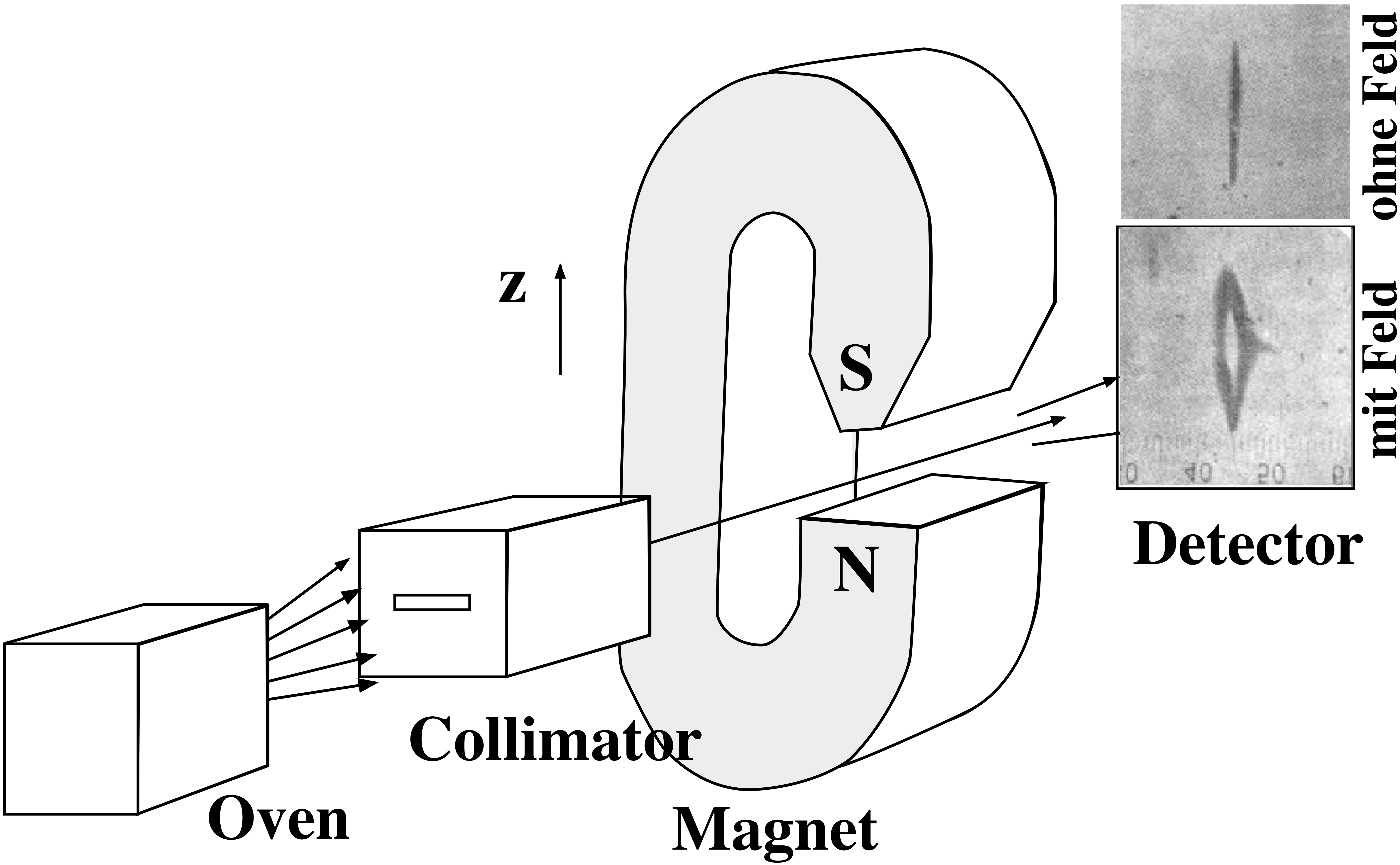
Impossible : $$U|\psi s\rangle = |\psi\psi\rangle$$

Possible : $$U|\psi s\rangle = |s'\psi\rangle$$

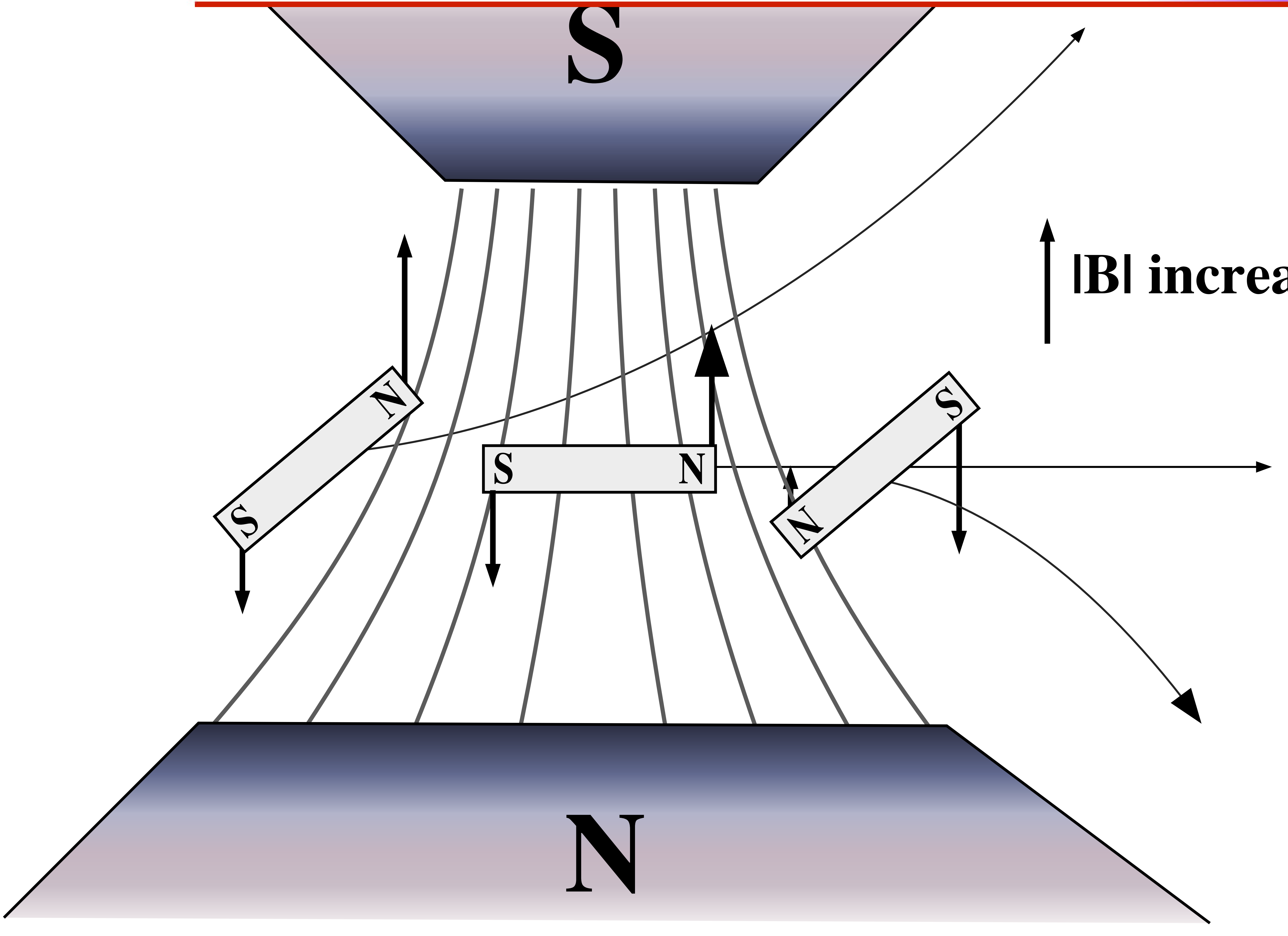
State is "teleported"



The Stern-Gerlach Experiment



The Stern-Gerlach Experiment



The Copenhagen Interpretation

Bohr, Heisenberg 1927

N. Bohr
Das Quantenpostulat und die neuere Entwicklung der Atomistik.
Die Naturwissenschaften 16, 245–257 (1928).

Bohr - Einstein debate (EPR)

The Copenhagen Interpretation

- Quantum mechanics describes individual systems
- Quantum mechanical probabilities are primary
  i.e. they cannot be derived from a deterministic theory
  (like classical statistical mechanics).
- The world must be divided into two parts:
  - the system (described quantum mechanically)
  - the measurement apparatus (classical)

  The division between system and measurement apparatus can be made at an arbitrary position.
- The measurement process is irreversible.
- Complementary properties cannot be measured simultaneously

Critique

Two different types of evolution:

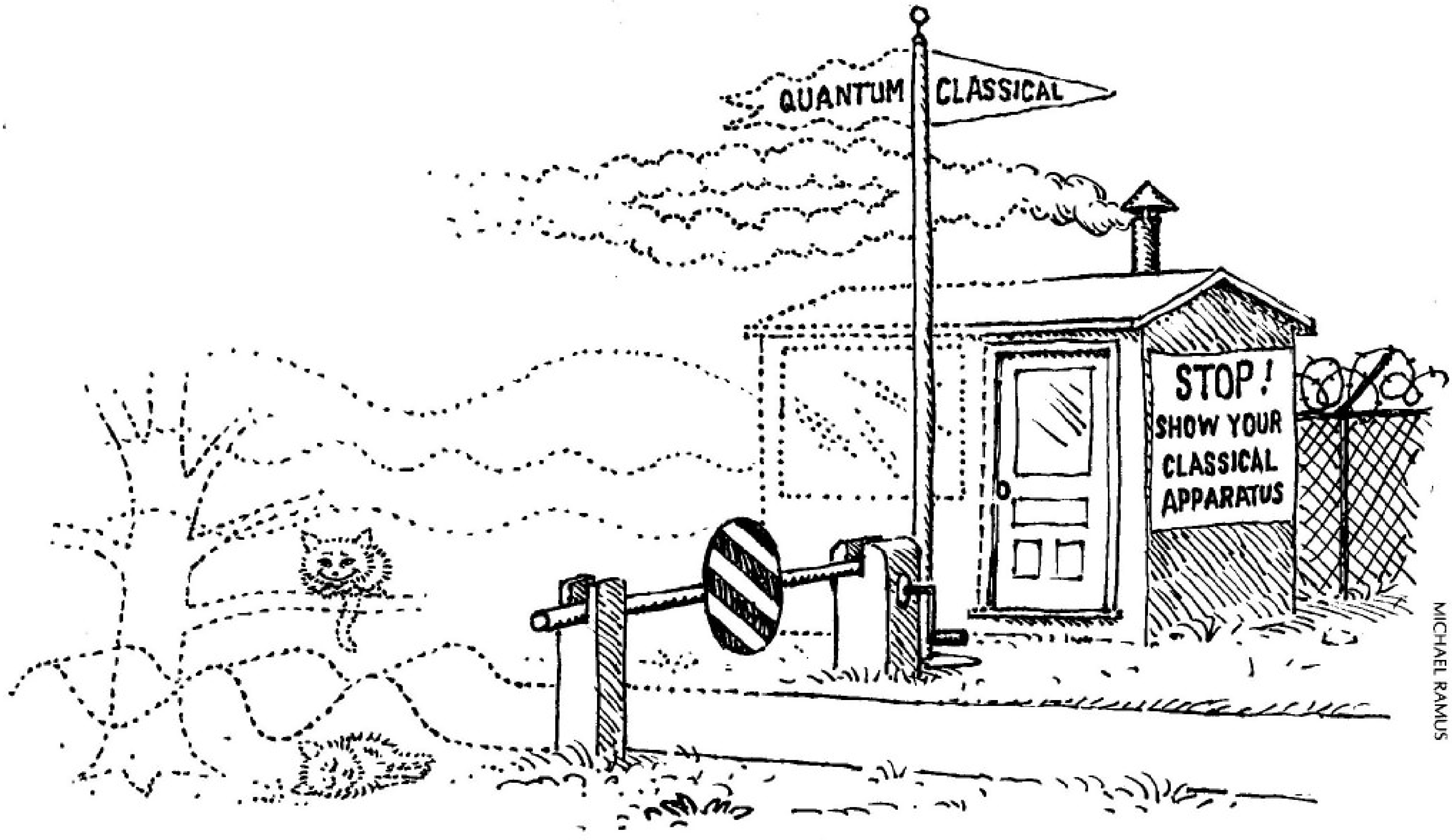
- the 'normal' unitary evolution according to Schrödinger's equation

- the nonunitary measurement process

Is the measurement process a physical process ?

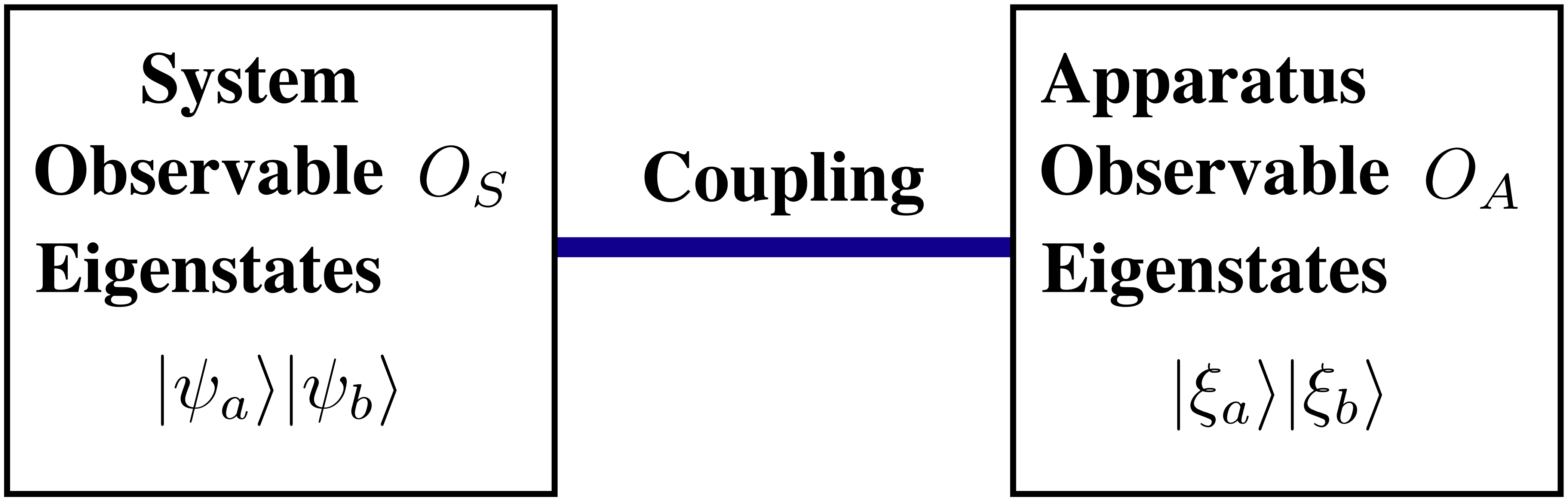
→ von Neumanns' model

The World is Divided into 2 Parts



W. Zurek, “Decoherence and the transition from quantum to classical, Physics Today, October 1991.

von Neumann's Model



Measurement : coupling system - apparatus

$$\mathcal{H}_{int} = O_S B_A$$

Desired result of interaction:
eigenvalues of O_S can be read off a "pointer variable" of the apparatus

Example: Stern-Gerlach experiment:

$$O_S = S_z$$

Pointer variable = position z

von Neumann's Model

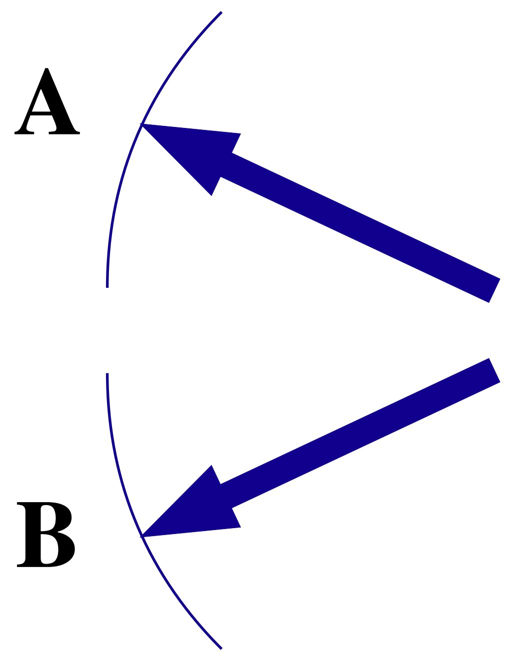
Initially, system and apparatus are uncorrelated

$$|\psi\rangle_S \otimes |\xi\rangle_A = (c_a|\psi_a\rangle_S + c_b|\psi_b\rangle_S) \otimes |\xi\rangle_A$$

Interaction must be such that apparatus evolves as follows:

$$\psi_a\rangle_S \otimes |\xi\rangle_A \rightarrow \psi_a\rangle_S \otimes |\xi_a\rangle_A$$

$$\psi_b\rangle_S \otimes |\xi\rangle_A \rightarrow \psi_b\rangle_S \otimes |\xi_b\rangle_A$$



Evolution of superposition state

$$(c_a|\psi_a\rangle_S + c_b|\psi_b\rangle_S) \otimes |\xi\rangle_A$$

$$\rightarrow c_a|\psi_a\rangle_S \otimes |\xi_a\rangle_A + c_b|\psi_b\rangle_S \otimes |\xi_b\rangle_A$$

von Neumann's Model

$$c_a|\psi_a\rangle_S \otimes |\xi_a\rangle_A + c_b|\psi_b\rangle_S \otimes |\xi_b\rangle_A$$

Combined system is now in an entangled superposition state

No state reduction / collapse of wavefunction

Let apparatus be classical → **reduction occurs**

$$c_a|\psi_a\rangle_S \otimes |\xi_a\rangle_A + c_b|\psi_b\rangle_S \otimes |\xi_b\rangle_A$$

Result is λ_a ↙ ↘ **Result is λ_b**

$$c_a|\psi_a\rangle_S \otimes |\xi_a\rangle_A \qquad c_b|\psi_b\rangle_S \otimes |\xi_b\rangle_A$$

System and apparatus no longer entangled

Pointer variable indicates system state

System state has collapsed

Discussion

- Does not resolve measurement problem
- Realistic description of actual measurements
- Collapse of system state is ‘explained’
- Can be iterated, e.g. until collapse occurs in conscious brain
- Wigner’s friend