III. QC IMPLEMENTATION BY ION TRAPS

A. Introduction

Any implementation has to define a quantum mechanical system that provides the quantum register containing N qubits.
In the case of ion traps, the qubits are represented by stationary states of atomic ions stored in an electromagnetic trap. Transitions between these states can be effected by laser pulses or by resonant magnetic fields (similar to NMR quantum computers).

B. Trapping Ions

Earnsnshaw's theorem states that static electromagnetic fields cannot trap a charge. However, using a combination of static and alternating electromagnetic fields it is possible to confine ions. The two traditional traps are the Paul and Penning traps.

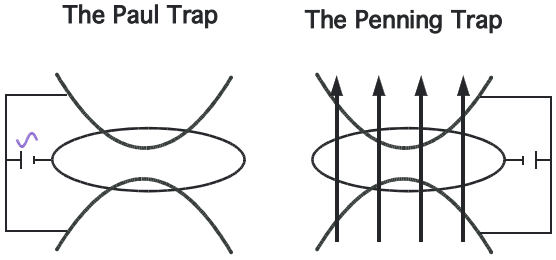

FIG. III.1: Two classical ion traps.

Both consist of a set of electrodes that generate a quadrupolar field. In the case of the Paul trap, the voltage on the electrodes varies sinusoidally. The ion is therefore successively attracted to the polar end caps or to the ring electrode. On average, it experiences a net force that pushes it towards the center of the trap. In the exact center, the field is zero and any deviation results in a net restoring force. The Penning trap has the same electrodes, but the electric field is static: it is repulsive for the end caps. The ions are prevented from reaching the ring electrode by a longitudinal magnetic field.

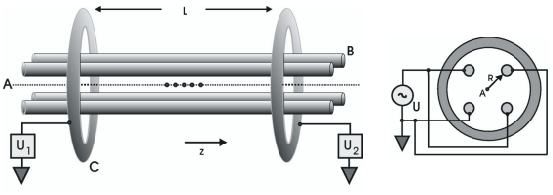

FIG. III.2: Linear quadrupole trap.

The Paul Trap can also be made into an extended linear trap. In this design, a radio frequency potential is applied to the four rods to confine the ions in the radial direction. A static potential applied to the end caps prevents the ions from escaping along the axis. The resulting effective potential (averaged over an rf cycle) can be written as

$$V = \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2,$$

where ω_α, $\alpha = x, y, z$ are the vibrational frequencies along the three orthogonal axes. By design, one has $\omega_x = \omega_y \gg \omega_z$, i.e. strong confinement perpendicular to the axis and week confinement parallel to the axis.

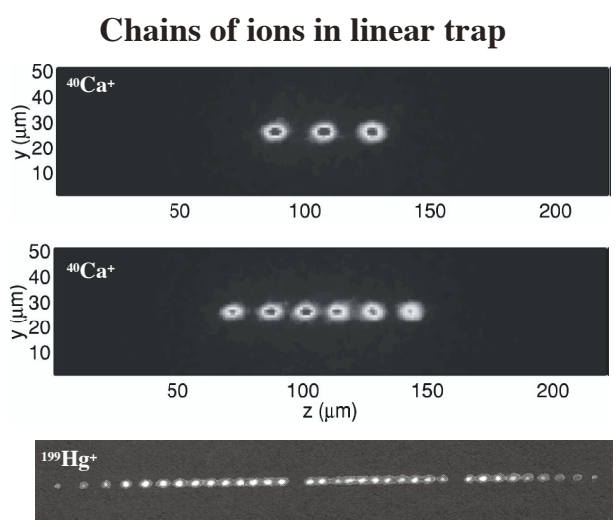

FIG. III.3: Strings of ions in linear traps.

Ions that are placed in such a trap will therefore preferentially order along the axis, essentially corresponding to a linear quantum register. The distance between the ions is determined by the equilibrium between the confining potential $\omega_z^2 z^2$ and the Coulomb repulsion between the ions. This type of trap has two important advantages for quantum computing applications: it allows one to assemble the ions in a linear chain where they can be addressed by laser beams and the equilibrium position of the ions (on the symmetry axis) is field free. This is in contrast to the conventional Paul trap where the Coulomb repulsion between the ions pushes them away from the field-free point. As a result, ions in a Paul trap perform a micromotion driven by the rf potential. In the linear Paul trap, the field-free region is a line where a large number of ions can remain in zero field and therefore at rest.
When more than one ion is confined in such a trap, the system has multiple eigenmodes of the atomic motion. The lowest mode is always the center of mass motion of the full system, in full analogy to the motion of atoms in a crystal. A change of the fundamental vibrational mode can be compared to the Mössbauer effect, where the recoil from the photon is shared between all atoms in the crystal. The higher vibrational modes, which correspond to phonons with nonzero wave vector, as well as the vibrational modes that include wave vector components perpendicular to the axis, will not be relevant in this context.

To use such systems as quantum computers, one needs a way to apply quantum gates to the individual ions.

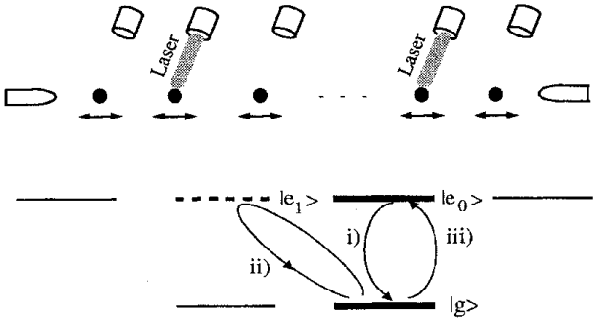

FIG. III.4: Laser-operated quantum gates in stored ions.

One possibility is to implement them by laser pulses. Since the separation of the ions is typically of the order of 10 optical wavelengths, diffraction-limited laser beams are sufficient for individual addressing of the qubits. The interaction between neighboring ions, which is required for the implementation of 2-qubit gates is via the Coulomb repulsion.

C. The Need for Laser Cooling

Motional processes shift the transition frequency through the Doppler effect:

$$\omega = \omega_0 + \vec{k} \cdot \vec{v},$$

where $\vec{k}$ is the wave vector of the laser field and $\vec{v}$ the atomic velocity. In free atoms, the velocity can have arbitrary values, with the probability of a specific velocity determined by the Boltzmann distribution. The optical spectra of ensembles of atoms are therefore broadened and/or shifted according to their motional state.
In trapped ions, the motional energy is quantized. Depending on the trap potential, the motional states can often be approximated by a collection of harmonic oscillators. For each degree of freedom, one finds in the spectrum sidebands, shifted from the central transition by the vibrational frequency.

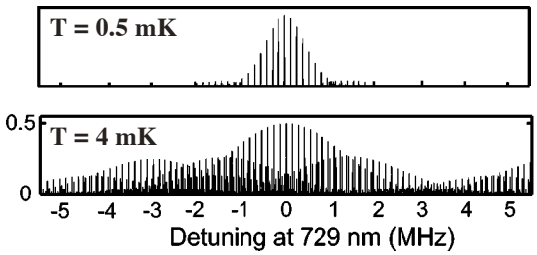

FIG. III.5: Spectrum of a Ca^+ ion in a trap showing motional sidebands.

In all techniques suggested to date for quantum computing with trapped ions, the spatial coordinates of the qubit ions play an important role either as a qubit or as a variable used for coupling different qubits. This is only possible if the ions are in a well defined motional state. The only state that allows well defined initialization is the ground state. The ions must therefore be cooled into their ground state as a part of the initialization process.
The technique to bring them into the ground state is laser cooling, which was developed in the 1980's. One uses lasers to reduce the velocity to very low values.

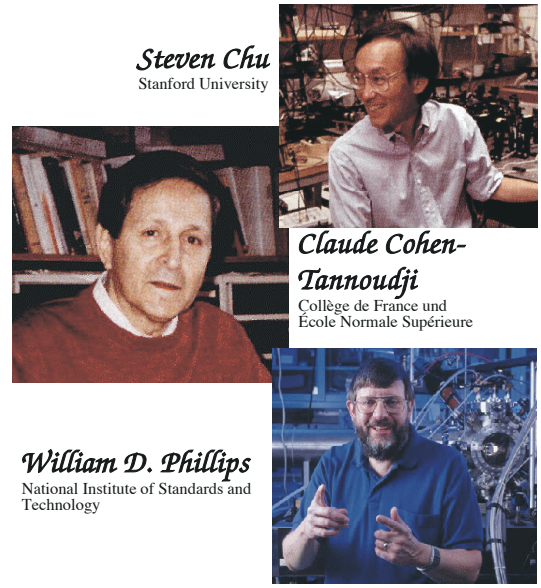

FIG. III.6: In 1997, the physics Nobel prize was awarded for the development of laser cooling.

The development of laser cooling was awarded with the Nobel prize in 1997.
Thermal atoms (T 300 K) have velocities of the order of 1000 m/s. While atomic ions can be trapped with electromagnetic fields, this is not possible for neutral atoms. Instead, they can be trapped with laser beams. Ions as well as neutral atoms can be cooled with laser beams to temperatures of less than $1\mu K$.
The basic principle is that atoms that absorb photons also absorb the momentum of the photons. Suitable arrangements allow one to use this momentum transfer for creating extremely strong forces that push the atoms in the direction of the laser beam. Adjusting the experimental parameters properly, these forces can be conservative (i.e. they form a potential) or they can be dissipative friction forces.
The frictional forces are the ones that are needed to reduce atomic velocities (i.e. temperatures). This type of optical forces are often called optical molasses, since the atoms move like in a highly viscous medium.
One effect of slowing the atoms down is that the de Broglie wavelength of these atoms becomes large. Simultaneously, the velocity spread decreases, corresponding to an increase in the coherence length.
If the atom moves towards the light, the light must have a lower frequency than that required for a stationary atom to be resonant the atom. Assume that

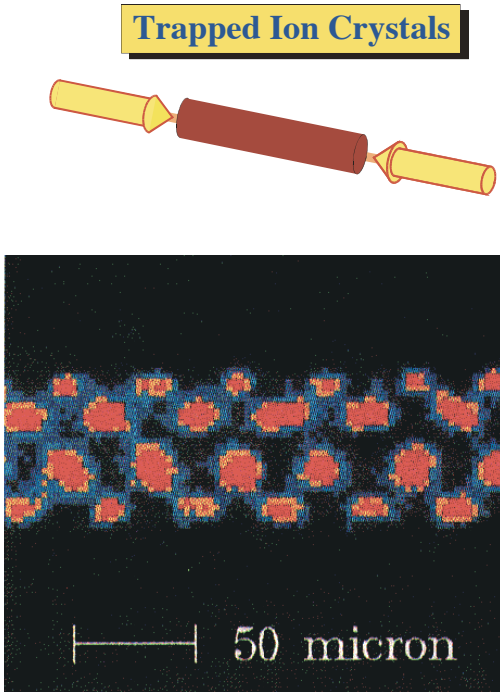

FIG. III.7: Crystalline arrangement of stored ions.

the atom is moving in the opposite direction of the light at a considerable speed and is struck by a stream of photons. If the photons have the right energy the atom will be able to absorb one of them and take over its energy and its momentum. The atom will then be slowed down somewhat. After an extremely short time, normally around a 10 nanoseconds, the retarded atom emits a photon. The atom can now immediately absorb a new photon from the oncoming stream. The emitted photon also has a momentum, which gives the atom a certain small recoil velocity. But the direction of the recoil varies at random, so that after many absorptions and emissions the speed of the atom has diminished considerably. To slow down an atom an intensive laser beam is needed. Under the right conditions the resulting acceleration of the atom can reach 100000 g.
A follow-up development that uses laser cooling is the generation of Bose-Einstein condensates, which was again awarded a Nobel prize in 2001.

D. Laser-induced Forces on Atoms and Ions

Light can generate strong forces on atoms and ions. One way to understand this effect starts from a classical picture, by calculating the force as the gradient of the potential energy,

$$F = -\overrightarrow{\nabla}(U) = -\overrightarrow{\nabla}(-\overrightarrow{E}\cdot\overrightarrow{d})$$

While atoms have no permanent dipole moment, they have an induced dipole moment generated by the resonant light. The induced dipole moment is shifted in phase relative to the laser field. On resonance, the phase shift is $\pi/2$. For a red detuned laser beam ($\nu_{las} < \nu_{at}$), the induced dipole is in phase with the laser beam and the interaction energy is negative, i.e. the atom can reduce it's energy by the interaction with the laser beam; as a result it is pulled into the region of intense light.

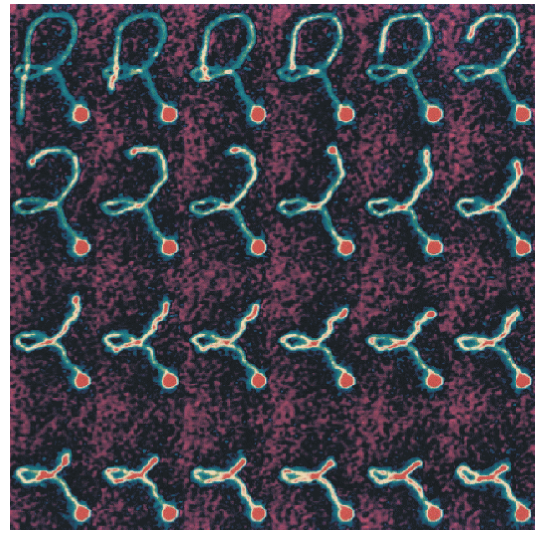

FIG. III.8: Manipulation of macromolecules with optical tweezers.

This allows one to use a tightly focused laser beam for trapping particles, if the laser frequency is red-detuned with respect to the relevant transition(s). While this has first been demonstrated for neutral atoms, it has become important also for atomic ions, but even molecules, macromolecules and small solid particles can be manipulated with lasers. The technique has become popular as "optical tweezers".
For a blue-detuned laser beam, the phase difference is larger than $\pi/2$. The interaction energy is then positive and the atom is pushed out of the laser beam.

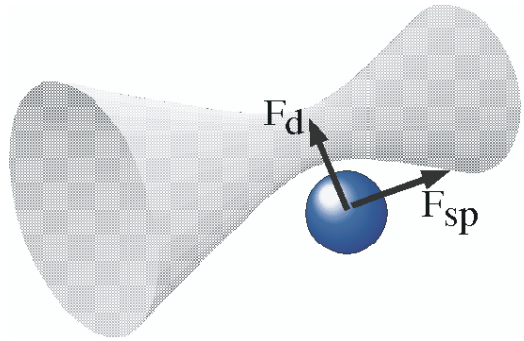

FIG. III.9: Forces acting on an atom close to the focus of a laser beam.

The forces induced in this way can become large when the laser intensity varies over a short distance. The fastest variation is of the order of the optical wave-

length. This can occur in a tightly focused beam, for a standing wave, or for an evanscent wave.

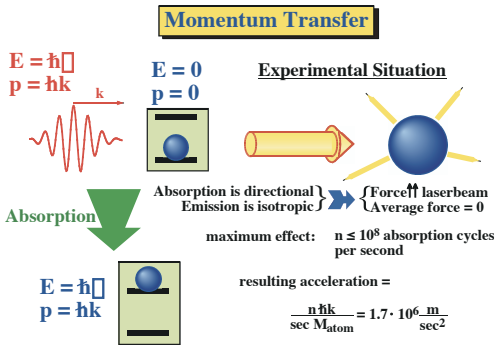

FIG. III.10: Principle of laser cooling.

Another picture of the light-induced forces is quantum mechanical: When a photon is absorbed, its momentum is transfered to the atom. The momentum transfered per unit time is equal to the forces acting on the atom.
The momentum change due to the transfer of a single photon momentum is relatively small; it corresponds to a change of the atomic velocity by a few cm/s. As an example, we calculate the momentum transfered by a single photon at a wavelength of 589 nm:

$$\Delta p = \frac{h}{\lambda} = \frac{6.626 \cdot 10^{-34} Js}{589 \cdot 10^{-9} m} = 1.125 \cdot 10^{-27} \frac{mkg}{sec}.$$

Given the atomic mass $m_{Na} = 3.818 \cdot 10^{-26} kg$ of sodium, this corresponds to a change of its velocity of

$$\frac{\Delta p}{m_{Na}} = 2.95 \frac{cm}{sec}.$$

This estimate was first made by Einstein in 1917 [15] and verified experimentally by Frisch 1933 [16] with a classical light source. Since the atoms scattered less than three photons in his experiment, the effect was very small.
However, if an allowed atomic transition is excited by a laser, the atom reemits the photon with a few nanoseconds (16 ns for Na) and is ready to absorb another atom. It can therefore scatter up to 10^8 photons per second, and the momentum transferred by them adds up to a force

$$F = \frac{\Delta p}{\tau} = \frac{2.95 \frac{cm}{sec}}{16 ns} = 3.52 \cdot 10^{20} N$$

corresponding to an acceleration of

$$a = \frac{F}{m_{Na}} = \frac{3.52 \cdot 10^{20} N}{3.818 \cdot 10^{-26} kg} = \frac{3.52 \cdot 10^{-20} N}{3.818 \cdot 10^{-26} kg}$$

$$= 9.21 \cdot 10^5 \frac{m}{sec^2} \AA 100'000 g.$$

This implies that an atom arriving with the velocity of a jet place at the laser beam is stopped over a distance of a few cm.

End of week 3, May 3.

E. Laser Cooling

The absorption probability for light depends on the laser frequency like a Lorentzian. The rate r at which an atom scatters (i.e. aborbs and emits) photons depends on the laser detuning $\Delta\omega$as

$$r = \Gamma_1 \frac{\omega_x^2}{\Gamma_1^2 + 4\Delta\omega^2}$$

where Γ_1 is the spontaneous emission rate from the excited state and ω_x^2 represents the Rabi frequency, which is proportional to the strength of the laser field and the transition dipole moment.
As long as the atoms are not at rest, the resonance frequency depends on the atomic velocity through the Doppler effect. This can be used to generate a force that depends on the velocity - a requirement for cooling, i.e. reducing the velocity. For a moving atom, the spontaneous scattering rate changes to

$$r = \Gamma_1 \frac{\omega_x^2}{\Gamma_1^2 + 4(\Delta\omega_0 + kv)^2}$$

where k is the laser wave vector and v the atomic velocity. The force due to these scattered photons is

$$r = \hbar k \Gamma_1 \frac{\omega_x^2}{\Gamma_1^2 + 4(\Delta\omega_0 + kv)^2}$$

I is maximized for atoms whose velocity matches the condition

$$v = -\Delta\omega_0 / k$$

i.e. when the Doppler shift brings them into exact resonance with the laser frequency.
Such velocity-dependent forces are useful to selectively reduce the velocity of fast atoms. To bring the atoms to a standstill, i.e. reduce their velocity to zero, it is necessary to use a setup that distinguishes this particular velocity from all others.

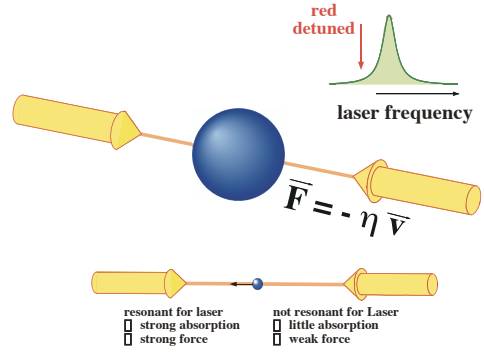

FIG. III.11: Optical molasses with 2 counterpropagating, red-detuned laser beams.

This can be achieved by superimposing two laser beams propagating in opposite directions. In the rest frame of the laboratory system, these laser beams have

the same frequency. An atom at rest in the laboratory frame therefore absorbs the same number of atoms from both laser beams and the forces due to these laser beams cancel.

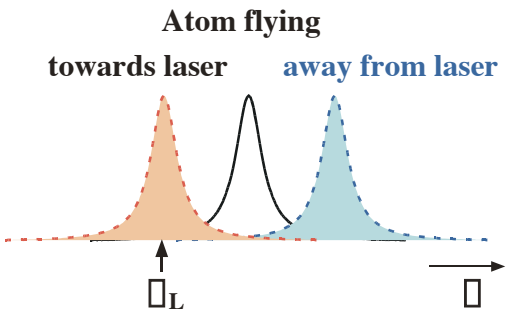

FIG. III.12: Effect of Doppler shift on absorption probability.

An atom moving towards one of the red-detuned laser beams absorbs a larger number of photons from this beam and therefore experiences a net force that is opposed to its direction of propagation. The individual laser beams exert forces

$$F_{\pm} = \pm\hbar k r_{\pm}$$

with the scattering rates

$$r_{\pm} = \Gamma_1 \frac{\omega_x^2}{\Gamma_1^2 + 4(\Delta\omega_0 \pm kv)^2}$$

The force resulting from the two laser beams is

$$F_{om} = \hbar k \Gamma_1 \omega_x^2 \{ \frac{1}{\Gamma_1^2 + 4(\Delta\omega_0 + kv)^2} - \frac{1}{\Gamma_1^2 + 4(\Delta\omega_0 + kv)^2} \}$$

are for the two laser beams.

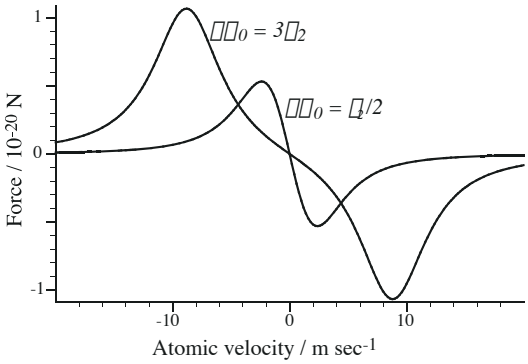

FIG. III.13: Velocity dependent force in 1D optical molasses.

Since the resulting force reduces the atomic velocity, one can consider the effect of the two laser beams as creating a viscous medium in which the atoms move. Thus the expression "optical molasses". Close to zero velocity, the resulting force is proportional to the velocity,

$$F_{om} \propto -\eta v,$$

where η is the effective viscosity. It's value can be calculated as the first derivative of the scattering force with respect to the velocity:

$$\eta = -\frac{dF_{om}}{dv}|_{v=0}$$

Introducing the rate r_0 at which an atom absorbs photons from an atom at rest,

$$r_0 = \frac{\Gamma_1 \omega_x^2}{\Gamma_1^2 + 4(\Delta\omega_0 + kv)^2}$$

we obtain for the viscosity

$$\eta = 16 r_0 \hbar k^2 \frac{\Delta\omega_0}{\Gamma_1^2 + 4(\Delta\omega_0 + kv)^2}$$

Being proportional to the laser detuning (for small detunings), it changes sign at resonance. This implies that the atoms are cooled down for red detuned lasers but heated for blue detuning.

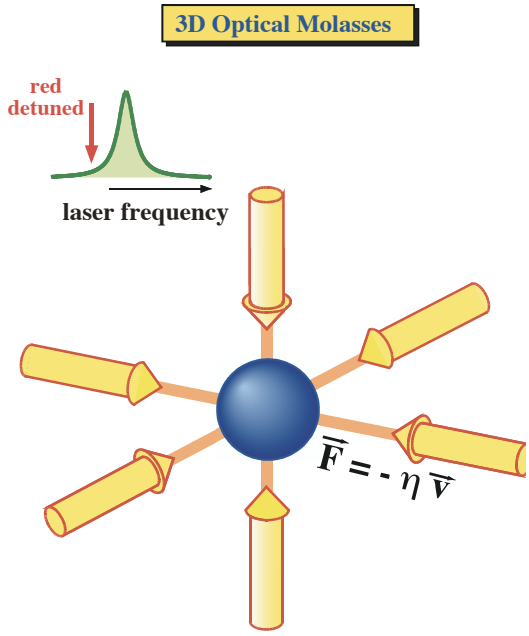

FIG. III.14: 3D optical molasses with 3 pairs of counter-propagating, red-detuned laser beams.

The same principle can easily be extended to three dimensions by using counterpropagating beams in three mutually orthogonal directions.
In the case of trapped ions, the main difference is that the motional degrees of freedom are quantized. The ions will therefore only absorb light at specific frequencies, which correspond to sidebands of the transition from the ion at rest. The sidebands are displaced by multiples of the vibration frequency.

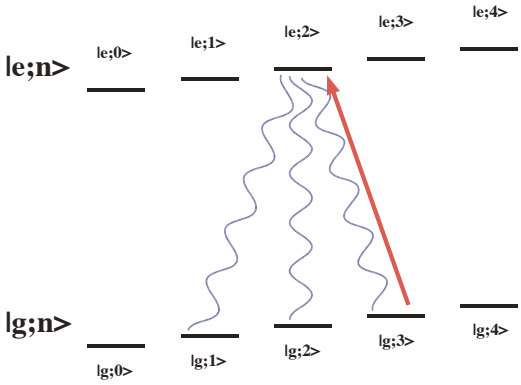

FIG. III.15: Schematics of sideband cooling for a single degree of freedom.

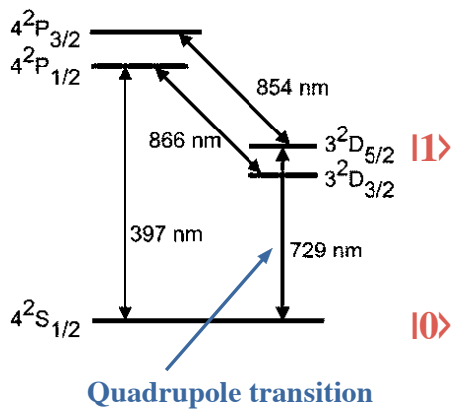

FIG. III.16: Possible qubit implementation using a metastable state in Ca^+.

Cooling of trapped ions is therefore achieved by irradiating the lower-frequency sidebands, as shown in the figure. In reality, the laser drives not only the $\langle g,3| \leftrightarrow \langle e,2|$ transition, but all $\langle g,n| \leftrightarrow \langle e,n-1|$ transitions for n ¿ 0. For each absorption event, the vibrational quantum number is reduced by one unit, since the photon energy is smaller than the energy difference of the two internal states. The emission process occurs with roughly equal probabilities into the different ground states, thus not affecting the average vibrational energy. The only state that is not coupled to the laser is the $\langle g,0|$ state. As a result, all atoms eventually are driven into this state in the absence of heating mechanisms.

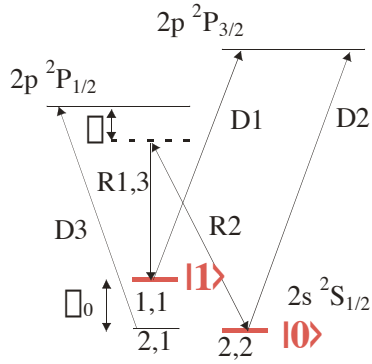

FIG. III.17: Possible qubit implementation using 2 hyperfine states $^9Be^+$.

F. Qubits

Since the atomic ions stored in traps have a large number of states, there are many distinct possibilities to define qubits. Since spontaneous decay rates through allowed transitions are of the order of a few nanoseconds, the requirement of long decoherence times implies that both states of the qubits must either be sublevels of the electronic ground state or metastable states.

A typical example of a a qubit implementation is the Ca^+ ion. Here the transition between the $4^2S_{1/2}$ ground state and the $3^2D_{5/2}$ excited state was suggested as a qubit. Since the angular momentum changes by two quanta between these states, the transition if forbidden in the dipole approximation. It is weakly allowed as a quadrupole transition, but the lifetime of the excited state is long enough for quantum computing applications.

The second common choice is the encode the quantum information in sublevels of the electronic ground state. Since the spontaneous transition rate between ground states is very small, the lifetime is again long compared to all relevant timescales.

The intialization of the qubits is achieved through optical pumping and sideband cooling as described above.

G. Gate Operations on Atomic Ions

If the upper state is a metastable state, single qubit gates can be implemented by laser pulses on the forbidden transition that connects the state to the lower qubit state. The Hamiltonian that describes the coupling between the atom and laser field is typically written as

$$H_1 = -\mu \cdot E = \hbar\Omega[S_+ e^{i(\omega t)} + S_- e^{-i(\omega t)}],$$

where Ω quantifies the interaction strength, which is given by the product of the transition dipole matrix element and the electric field strength of the laser field. Since forbidden transitions have a small dipole moment, high laser intensities are required to drive these transitions.

To see how such a Hamiltonian implements a quantum gate, it is convenient to transform it into an interaction representation with respect to the unperturbed Hamiltonian of the qubit:

$$H_1^r = \hbar\frac{\Omega}{2}[S_+ e^{i(\delta\omega t)} + S_- e^{-i(\delta\omega t)}],$$

where $\delta\omega = \omega_0 - \omega$ is the frequency detuning between the energy separation of the two levels forming the

qubit and the laser frequency ω. If the laser is set to resonance, the interaction Hamiltonian becomes

$$H_1^{res} = \hbar\Omega S_x.$$

As discussed separately, this allows one to construct, e.g., NOT operations as

$$NOT = e^{-i\pi S_x},$$

i.e. by letting H_1^{res} act for a time $\tau = \pi/\Omega$.
Different coupling Hamiltonians can be generated by shifting the phase ϕ of the laser field. The general form of the Hamiltonian is then

$$H_1 = \hbar\frac{\Omega}{2}(S_+ e^{i\phi} + S_- e^{-i\phi}.$$

$\phi = 0$ corresponds to the operator given above, for $\phi = \pi/2$ we obtain

$$H_1^y = \hbar\Omega S_y.$$

All necessary single qubit operations can be generated from two such Hamiltonians, typically S_x and S_y. However, it is often more convenient to use different phases also.
If the qubit is defined by two hyperfine states that are connected by a magnetic dipole transition, the gate operations can be implemented by microwave pulses [17], as discussed in the section on magnetic resonance implementations. Since the wavelength of microwave radiation is large compared to the distance between the ions, microwaves will interact with all qubits simultaneously. Addressing of individual qubits therefore requires a magnetic field gradient to separate the transition frequencies of the ions.
The second possibility for addressing hyperfine qubits is to use Raman laser pulses.

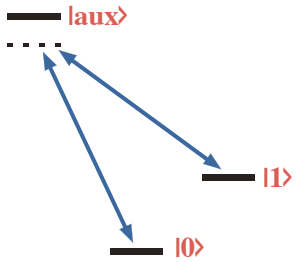

FIG. III.18: Optical readout of a single qubit.

For this purpose, one uses two laser fields, whose frequency difference matches the energy level separation of the two qubit states. The laser frequency is close to a transition to an auxiliary state. Choosing an appropriate set of parameters (frequencies, field strengths), it is possible to generate laser pulses that effectively drive the transition between the two qubit states, with negligible excitation of the auxiliary state.
Two qubit gates can be constructed in different ways. One elementary gate is the phase-flip gate

$$P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Depending on how the qubits are stored, this gate can be implemented in different ways. The technique that is mostly being considered now is to use the common motion of the ions as a bus qubit, which is not used to store information, but to couple the qubits to each other. To generate a two-qubit operation, one couples one ion to the vibrational motion. A subsequent operation on the second ion, which is conditional on its vibrational state, creates a two-qubit gate between the two ions.

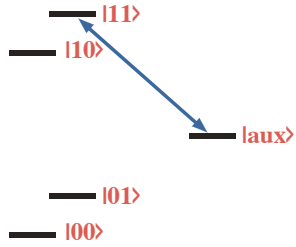

FIG. III.19: Selective laser pulse to generate a phase shift of state $|11\rangle$.

The entanglement of the ion with the vibrational motion can be created by tuning a laser to a transition between a vibrational substate and an auxiliary state. If a 2π pulse is applied to the transition marked in the figure, the stae $ket11$ is returned to itself modulo a π phase shift that marks the spinor character of the transition. The result is the phasegate P_4 given above. Another important element is the SWAP $_j$ between the internal qubit and the vibrational state

$$\text{SWAP }_j = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

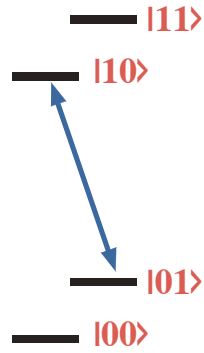

FIG. III.20: Laser tuning for swapping information between internal and vibrational states.

It can be created by tuning the laser to the red sideband of the qubit transition and applying a π_x pulse. A $CNOT_{jk}$ operation between the two qubits j and k can be constructed from these elements by the sequence

$$\text{CNOT }_{jk} = H_k \text{ SWAP }_k C_j \text{ SWAP }_k H_k,$$

where C_j is the phase shift gate applied to the j'th qubit and H_k the Hadamard gate applied to qubit k.

H. Readout

One of the important advantages of trapped ion quantum computers is the possibility to read the result with a very high selectivity and success probability by using an optical cycling transition from the state that is to be detected. It can be performed by irradiation of an allowed transition connected to the qubit states and detecting the fluorescence. For an efficient readout, this transition should be a "cycling transition", i.e. the ion should always fall back into the state that is to be read out.

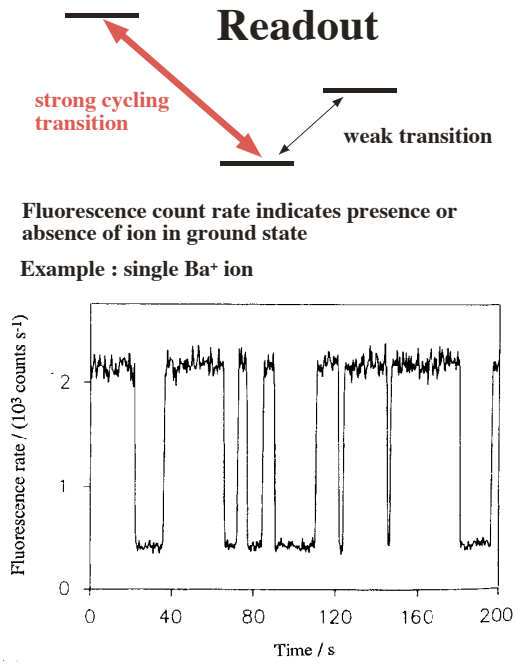

FIG. III.21: Optical readout of a single qubit.

As shown in the example data, the fluorescence level is an excellent indicator if the ion is in the state being interrogated [18]. The sudden drops in the fluorescence level indicate that the ion jumps into a different state, which is not coupled to the transition being irradiated. These transitions are referred to as "quantum jumps".
Using such a cycling transition, it is possible to scatter a large number of photons, thus providing high probabilities of detecting at least one of them.

I. Historical

The possibility to use laser for cooling purposes was first suggested by Hänsch and Schawlow [19] for cooling atomic vapors. Independently and almost simultaneously, Wineland and Dehmelt [20] suggested the use of laser cooling for trapped ions. Initial results were published 1978 by the group of Dehmelt and Wineland [21, 22]. The first three dimensional optical molasses were demonstrated by Chu and coworkers in 1985 [23].

J. Experimental Aspects

The most popular ion for quantum information studies is currently the Ca^+ ion. For laser cooling, excitation of resonance fluorescence and optical pumping of the ground state, different transitions are used. The experiment therefore requires laser sources at the wavelengths 397 nm, 866 nm, and 854 nm. If the E2 transition between the ground state and the metastable $D_{5/2}$ state is used as the qubit, a fourth laser with a wavelength of 729 nm is required. It's frequency stability must be better than 1 kHz
In most cases, the qubit consists of two ground state hyperfine sublevels. Transitions between them can be driven either by stimulated Raman transitions or by microwave excitation. A typical example is the Be^+ ion. For this system, five lasers with different wavelengths are required.
Tight confinement of the ions is advantageous as it increases the separation between the vibrational levels and therefore fascilitates cooling into the motional ground state. In addition, the vibrational frequencies are involved in the logic operations. Accordingly higher vibrational frequencies imply faster clocks.

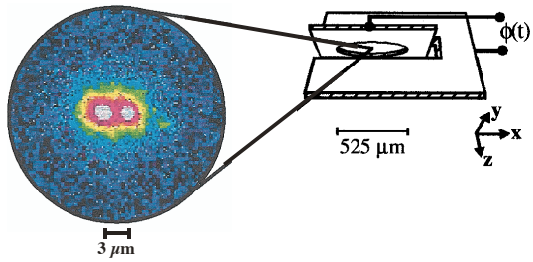

FIG. III.22: Two ions in a small elliptical trap.

Tight confinement can be achieved mainly by miniaturization of the traps. However, miniaturization is not without difficulties: it increases, e.g., the effect of uncontrolled surface charges in the trap and it makes addressing of the ions more difficult.

K. Achievements

The earliest quantum logic operation was reported by the group of Wineland [24].
The used a Be^+ ion where one of the qubits was a pair of internal states, two hyperfine sublevels of he electronic ground state, the $|F = 2, m_F = 2\rangle$ and $|F = 1, m_F = 1\rangle$ states with an energy difference of 1.25 GHz. This qubit represented the target qubit. The control qubit was defined by two lowest harmonic oscillator states, which are separated by 11 MHz.

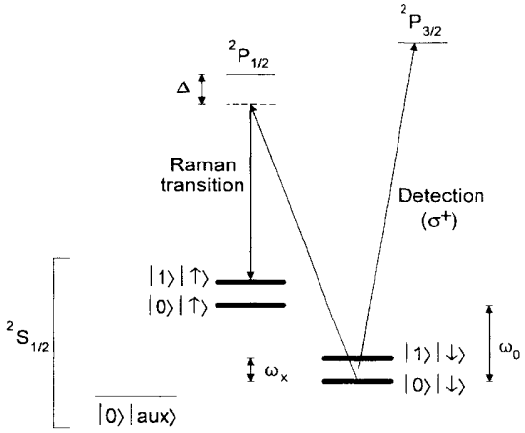

FIG. III.23: Level scheme used for implementing a CNOT gate.

Gates were implemented by Raman pulses: two laser fields, whose frequency difference matches the energy difference between the qubit states, drive the transition between them without populating electronically excited states. The Raman pulses have a relatively narrow frequency spread. They can be applied to the "carrier" at frequency ω_0 or to one of the two sidebands $\omega_0 \pm \omega_x$. The two sidebands correspond to simultaneous transitions of both qubits. In non-QIV terms, a sideband transition corresponds to a simultaneous change of the internal state and the motional state of the ion.

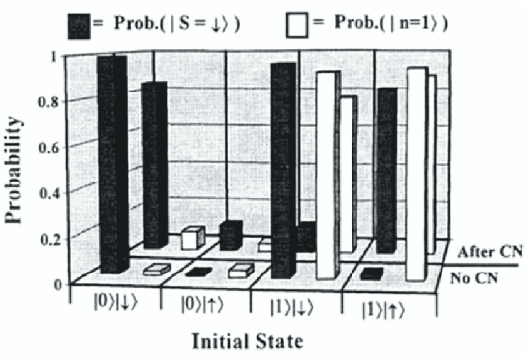

FIG. III.24: Experimental test of the CNOT gate.

They could show that a sequence of three Raman pulses implemented a CNOT gate.
Cooling of two ions into the vibrational ground state and their entanglement was achieved by the group of Wineland [25, 26]. The same group also demonstrated the creation of an entangled state between two ions. For this purpose they did not address the ions individually, but modified the effective Rabi frequency through fine-tuning of their micromotion. The resulting state was not a singulet state (but close to it) and the scheme is not directly applicable to quantum computing.
Using Ca^+ ions in a linear trap, optical addressing of individual ions was demonstrated, and in a chaing of 3 I-ions, coherent excitation of ions was demonstrated [27, 28].

L. Problems

One of the biggest problems of ion traps is that the ions, as charged particles, are relatively sensitive to stray fields in the vicinity. These fields can adversely affect the motion of the ions and, if they are time dependent, they heat the ions. Typical heating rates are of the order of 1 ms [25] for two ions in a trap. With increasing number of ions, heating rates are expected to increase: not only the number of particles that couple to these stray fields, but also the number of degrees of freedom that can be driven increases.
Like all other implementations of quantum computers, ion traps will have to demonstrate that they can perform a sufficiently large number of gate operations. If excited states are used for the qubits, they must be metastable to prevent spontaneous emission. This implies that the optical transitions to drive these states are weak and the corresponding Rabi frequencies low (or the laser intensities very high).
As the number of ions in a trap increases, a number of difficulties (like limited trap frequency, heating) increase, and it appears unlikely that traps will be able to accept a sufficiently large number of ions. This problem may be circumvented if the total number of qubits is stored in multiple traps. As it has been shown [29], it is possible to couple these separate traps through photons, thus creating an arbitrarily large quantum register with a linear overhead.
Addressing of qubits by laser must be achieved in the far field diffraction-limited regime, where the sepration between the ions must be large compared to an optical wavelength. This requirement sets a lower limit on the distance between the ions and therefore on the strength of the axial confinement potential. Since this potential also determines the vibrational frequency that enters the clock speed, it is obvious that ion traps cannot be operated with arbitrary speed. While direct microwave pulses can distinguish between the ions through their frequency-separation in an inhomogeneous magnetic field [17], it is not clear that this will allow significantly tighter confinement.

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