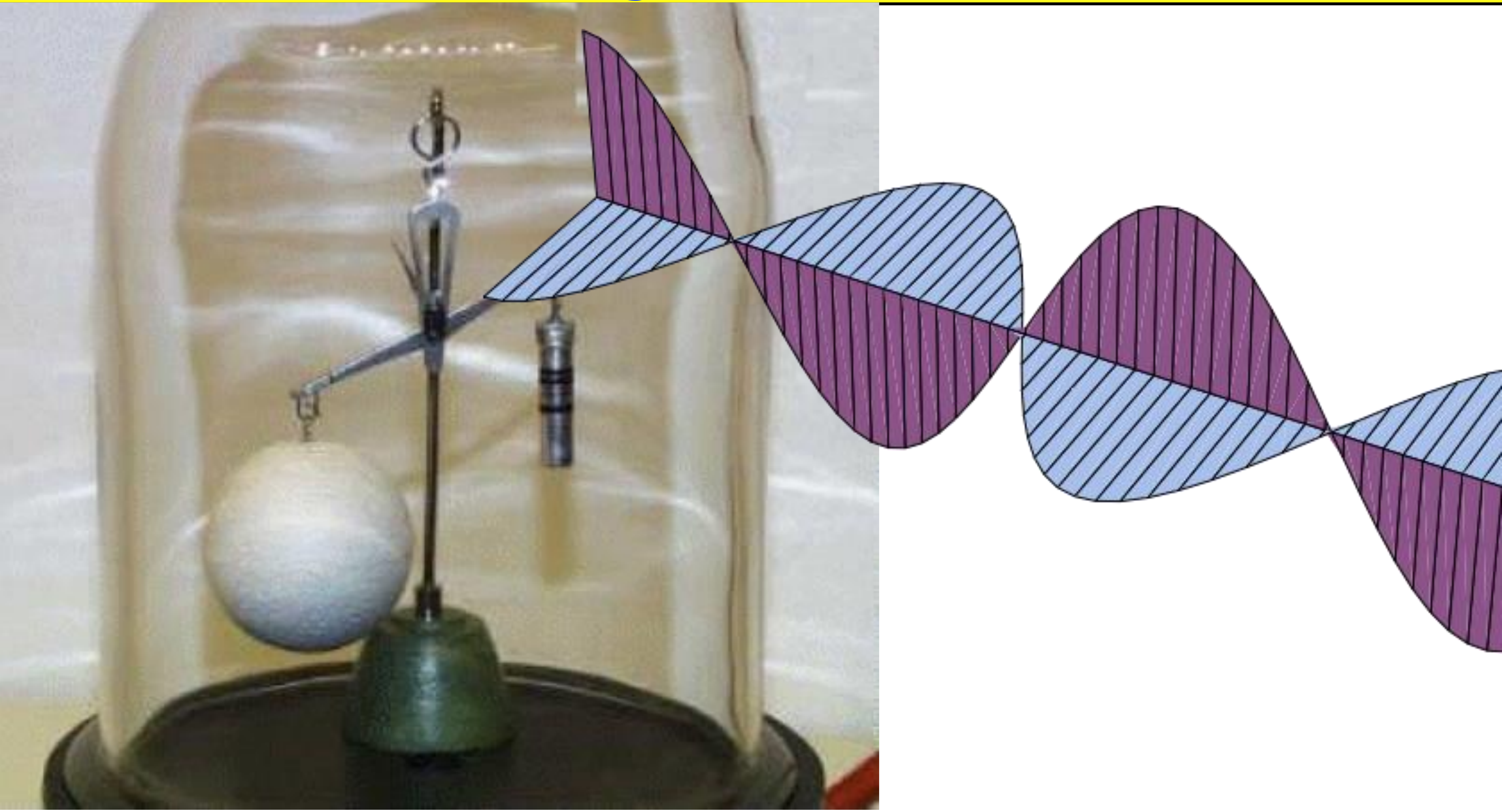
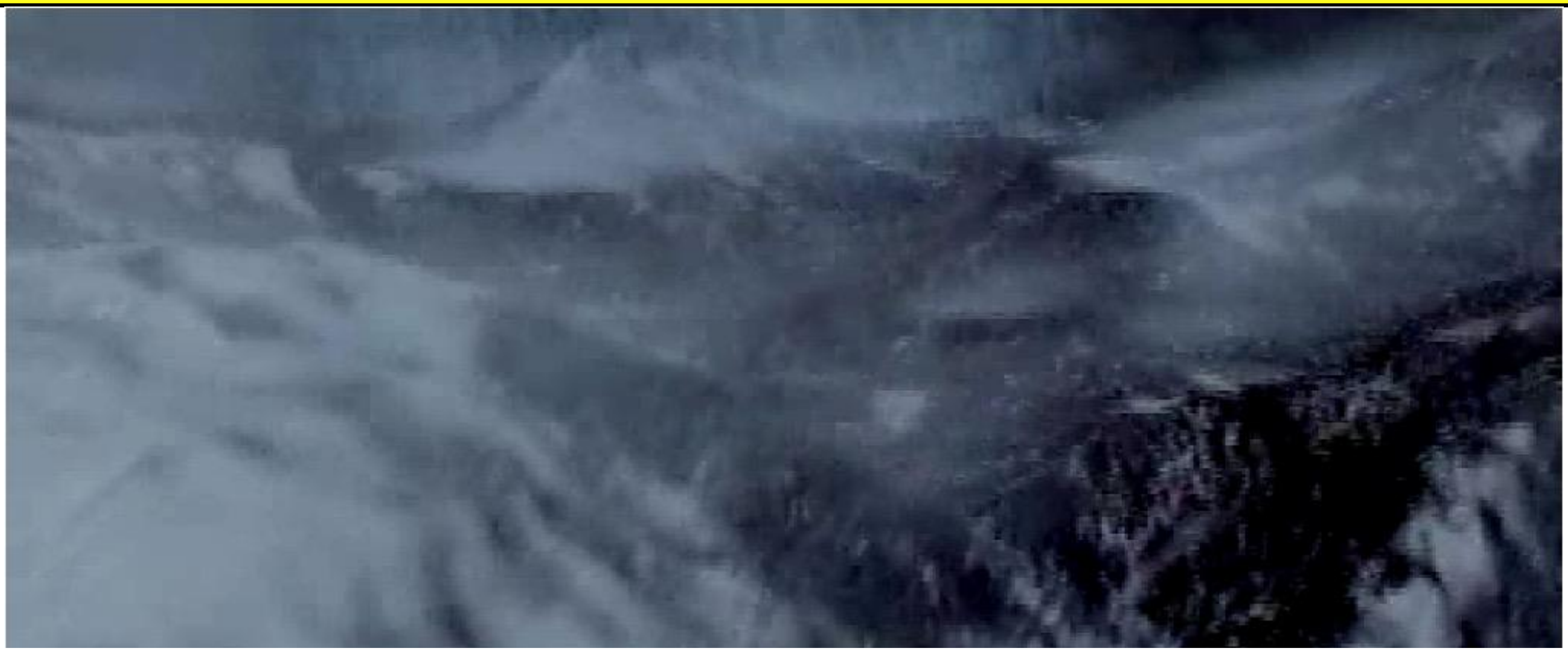
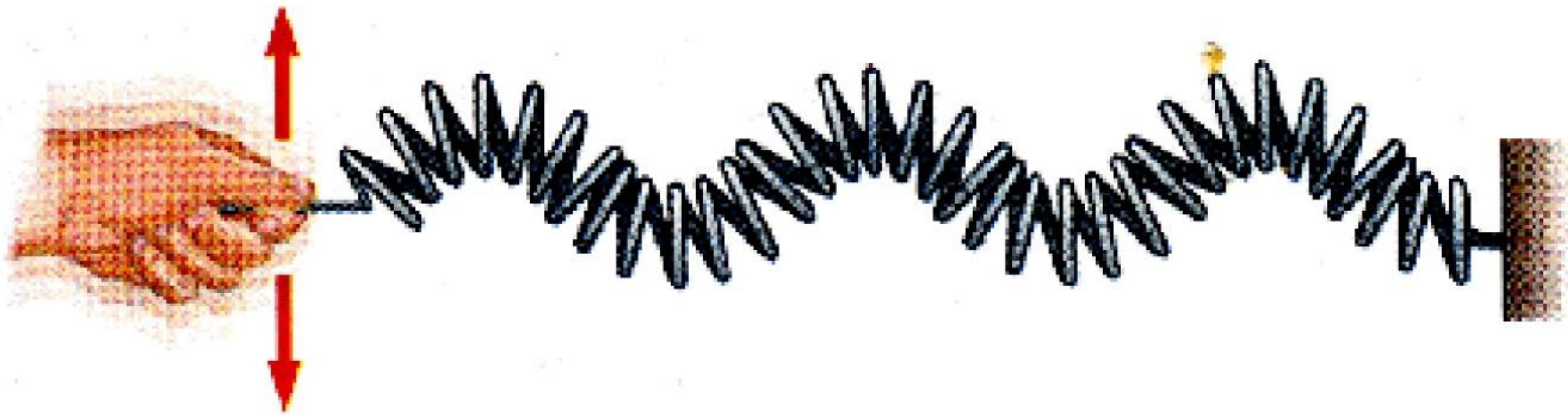
Physik B2



https://e3.physik.tu-dortmund.de/~suter/Vorlesung/Physik_A2_WS17/Physik_A2_WS17.html

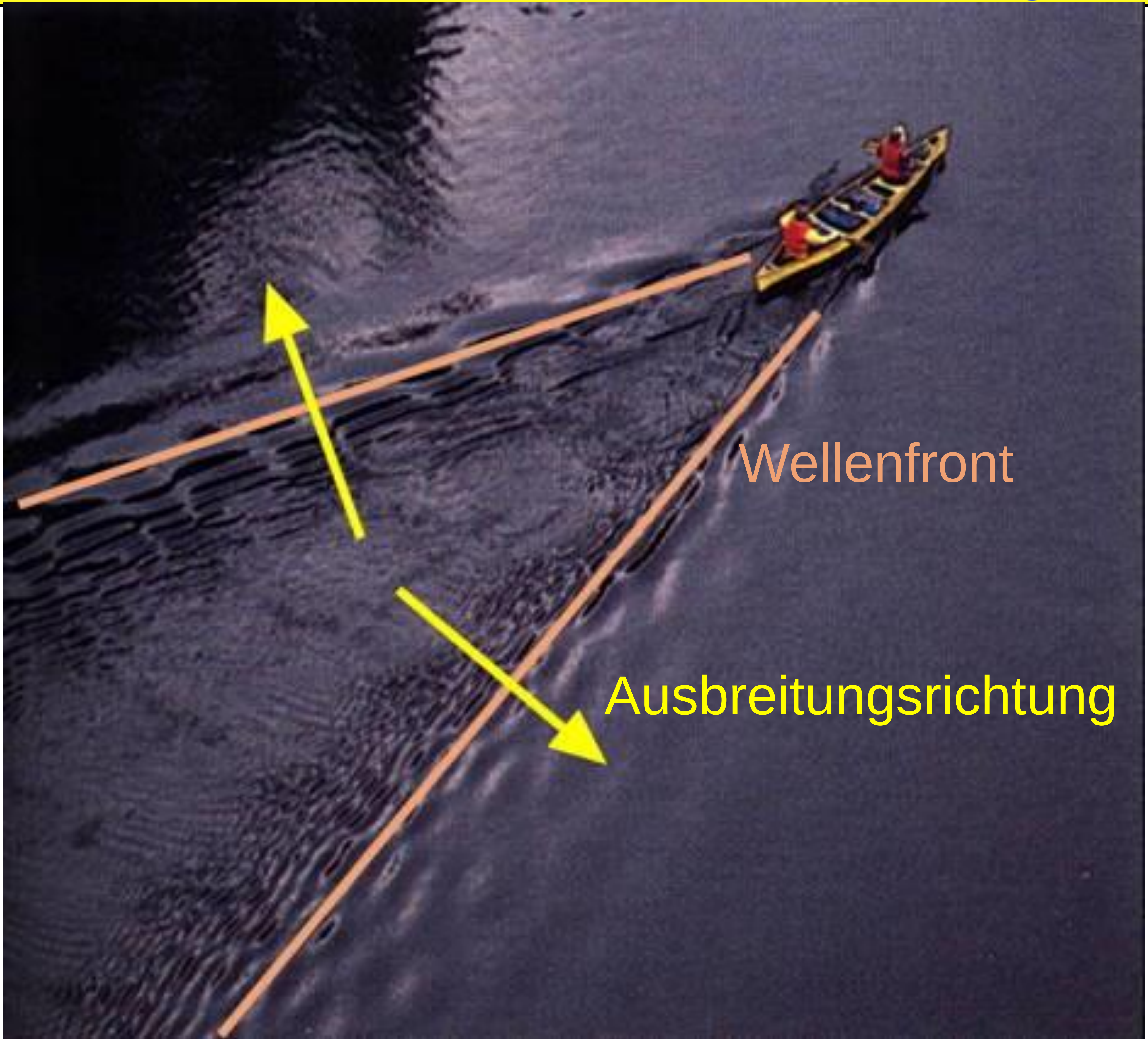
Wellen



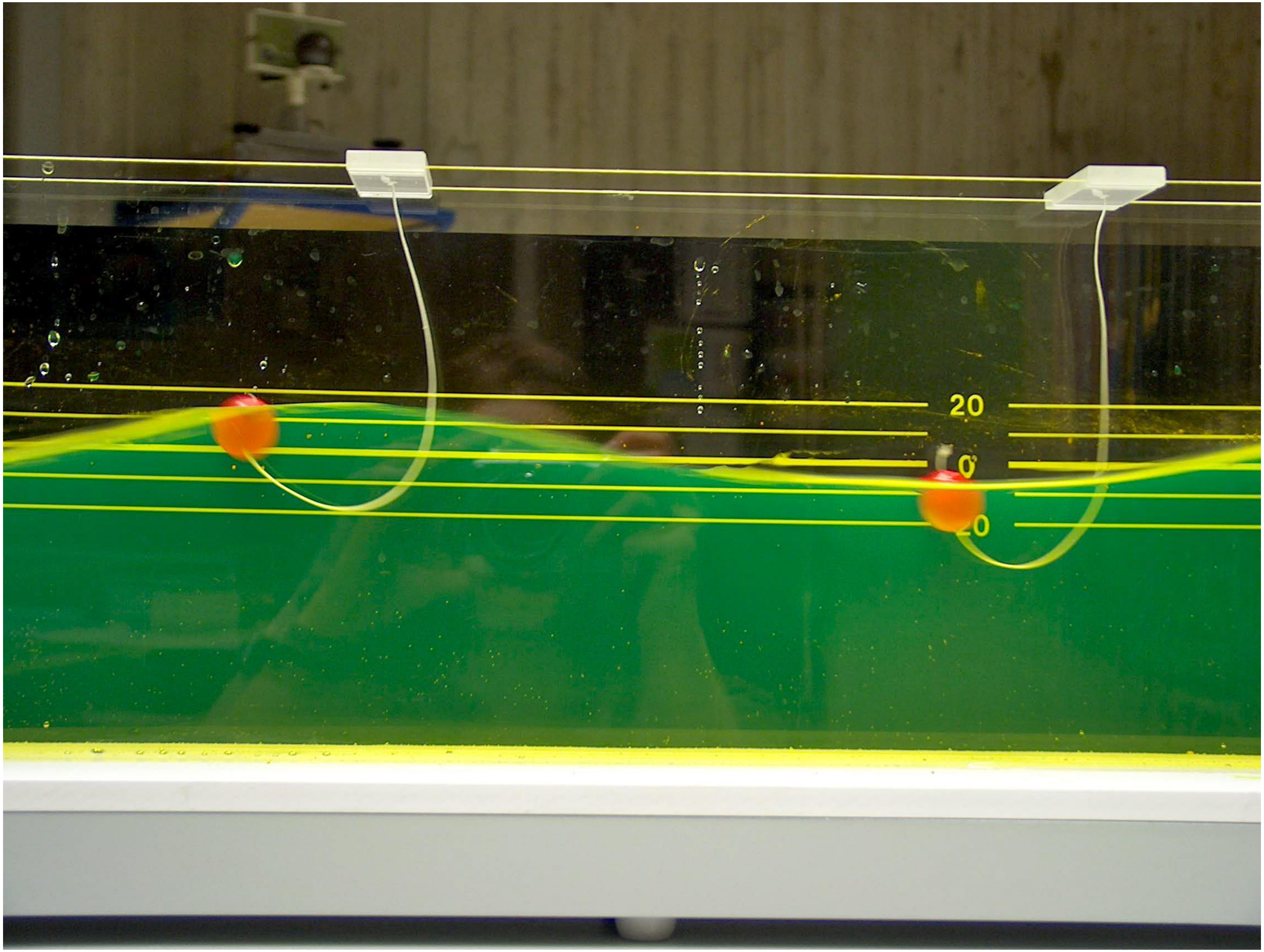


Welle = Ausbreitung einer Störung in einem kontinuierlichen Medium oder einer räumlich periodischen Struktur

Wellenfronten einer Bugwelle



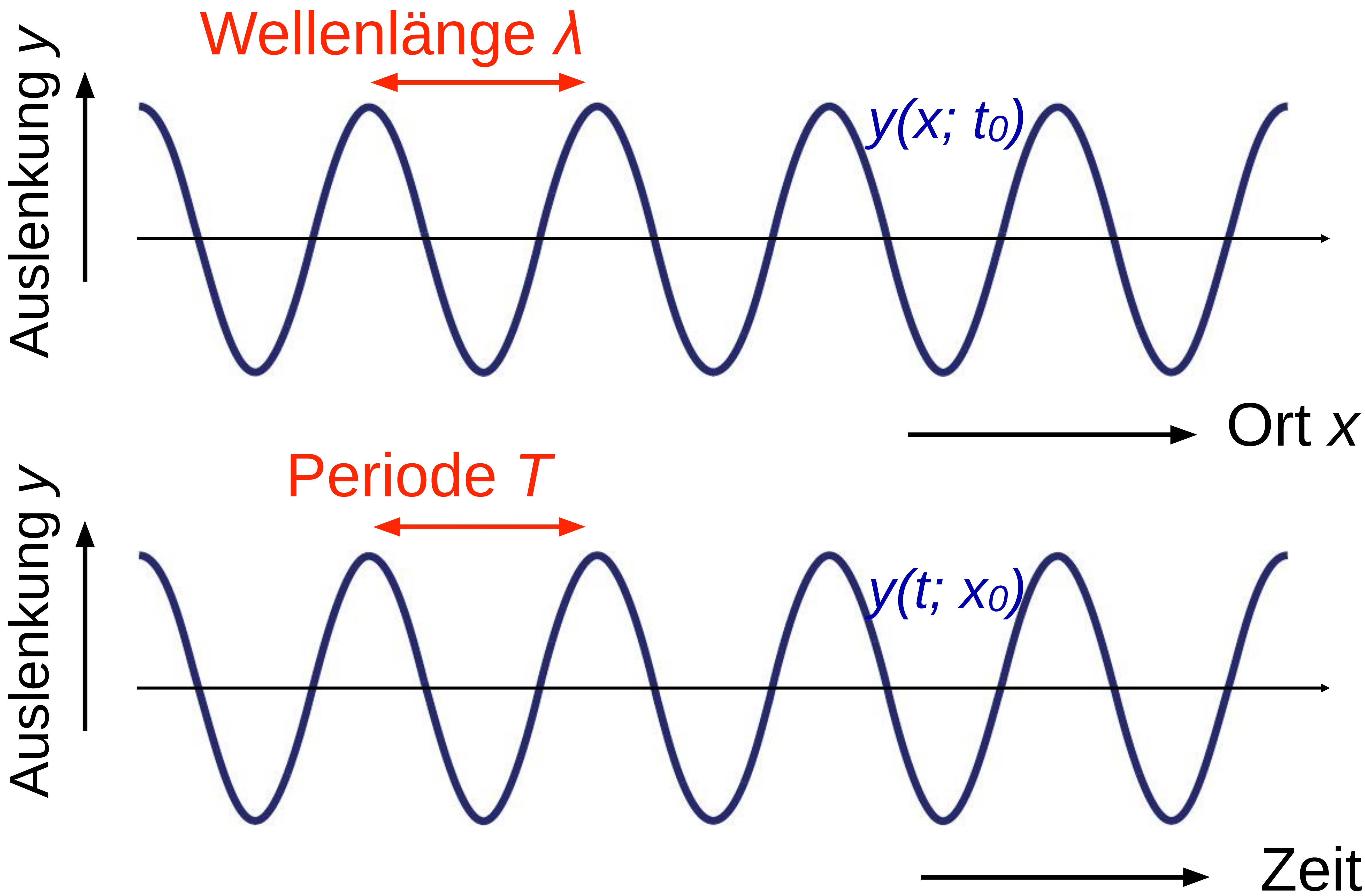
Wasserwelle



Welle in Zeit und Raum

Ebene harmonische Welle in 1 Dimension:

$$y = y_0 \cos\left(\omega t - kx + \phi\right)$$



Welle in Zeit und Raum

$$y(x,t) = y_0 \cdot \cos(\omega t - kx + \varphi)$$

Kreisfrequenz: ω

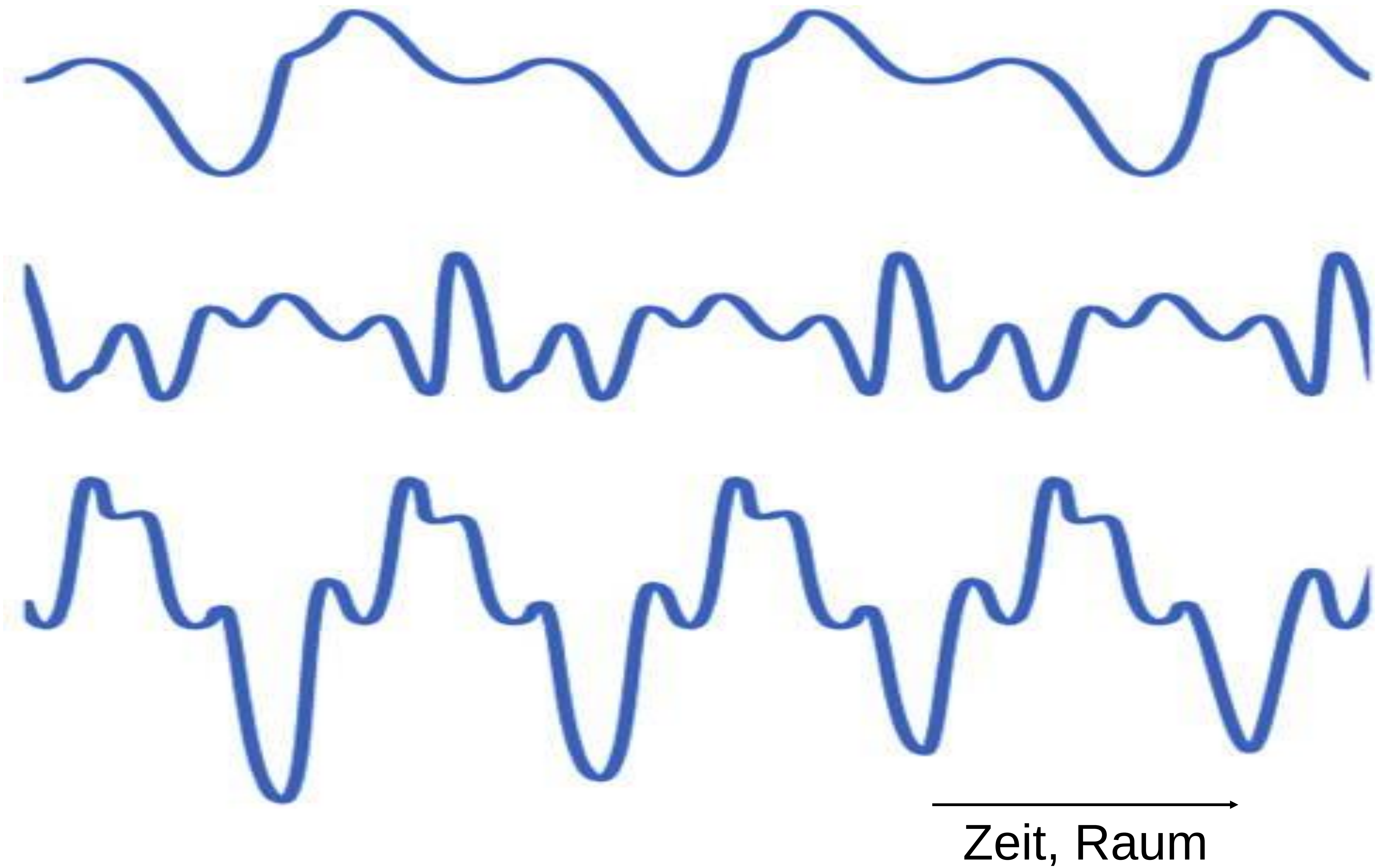
Wellenzahl: k

Phase: φ

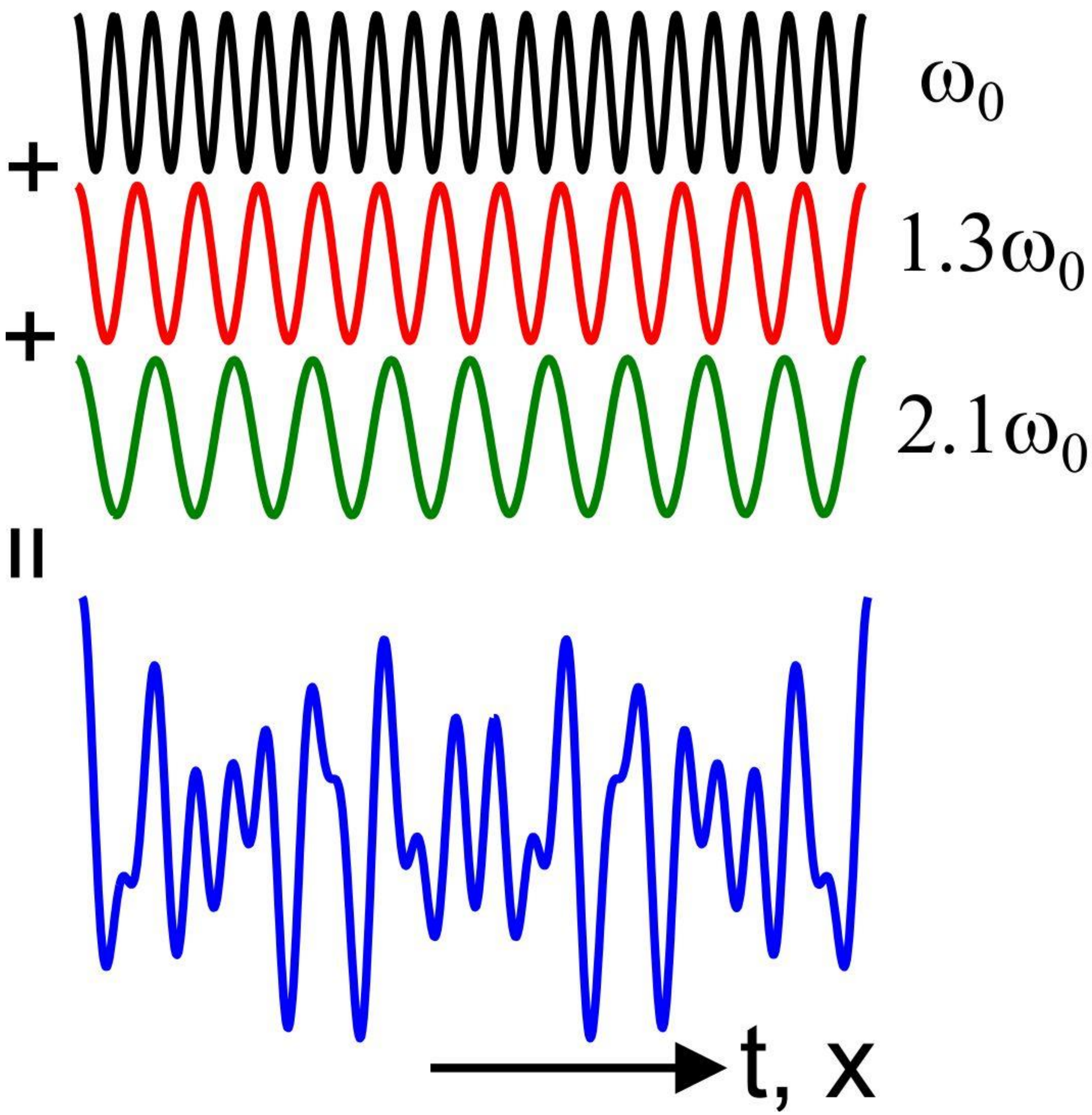
Wellenlänge: $\lambda = \dfrac{2\pi}{k}$

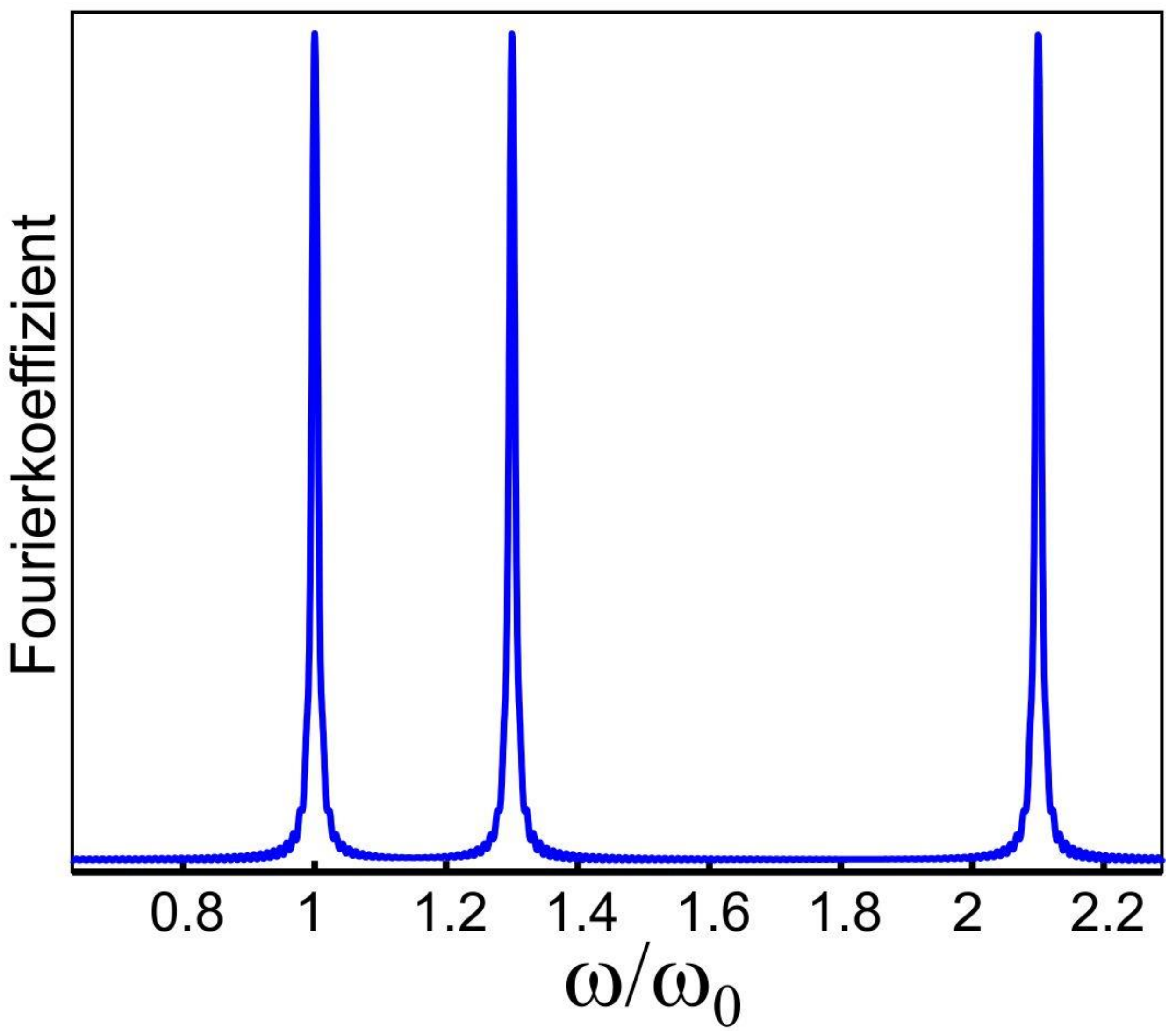
Periode: $T = \dfrac{2\pi}{\omega}$

Wellenformen



Anharmonische Wellen





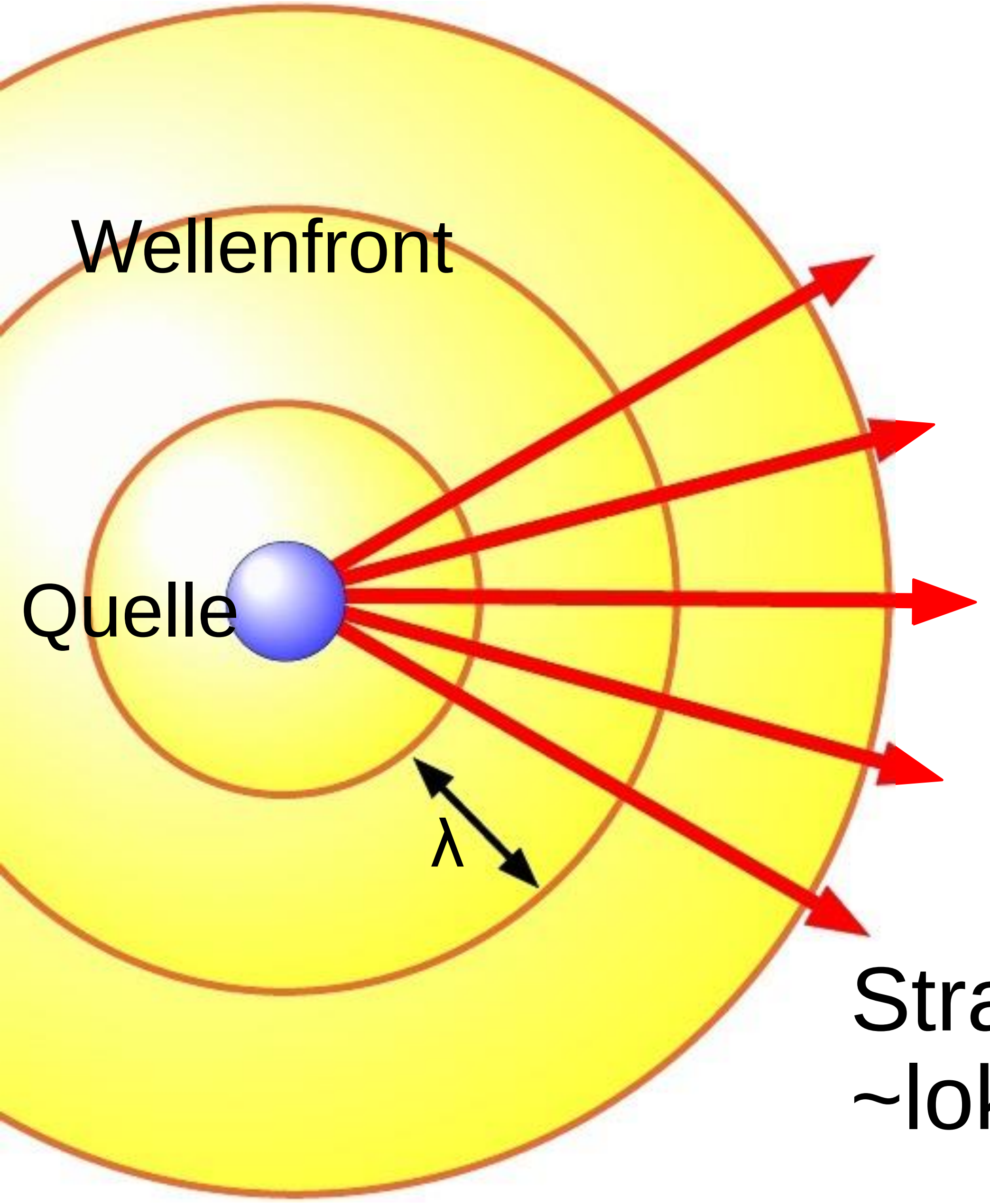
Foureirreihe

$$y(x,t) = \sum_{n=0}^{\infty} a_n \cos[n(\omega t - kx) + \varphi_n]$$

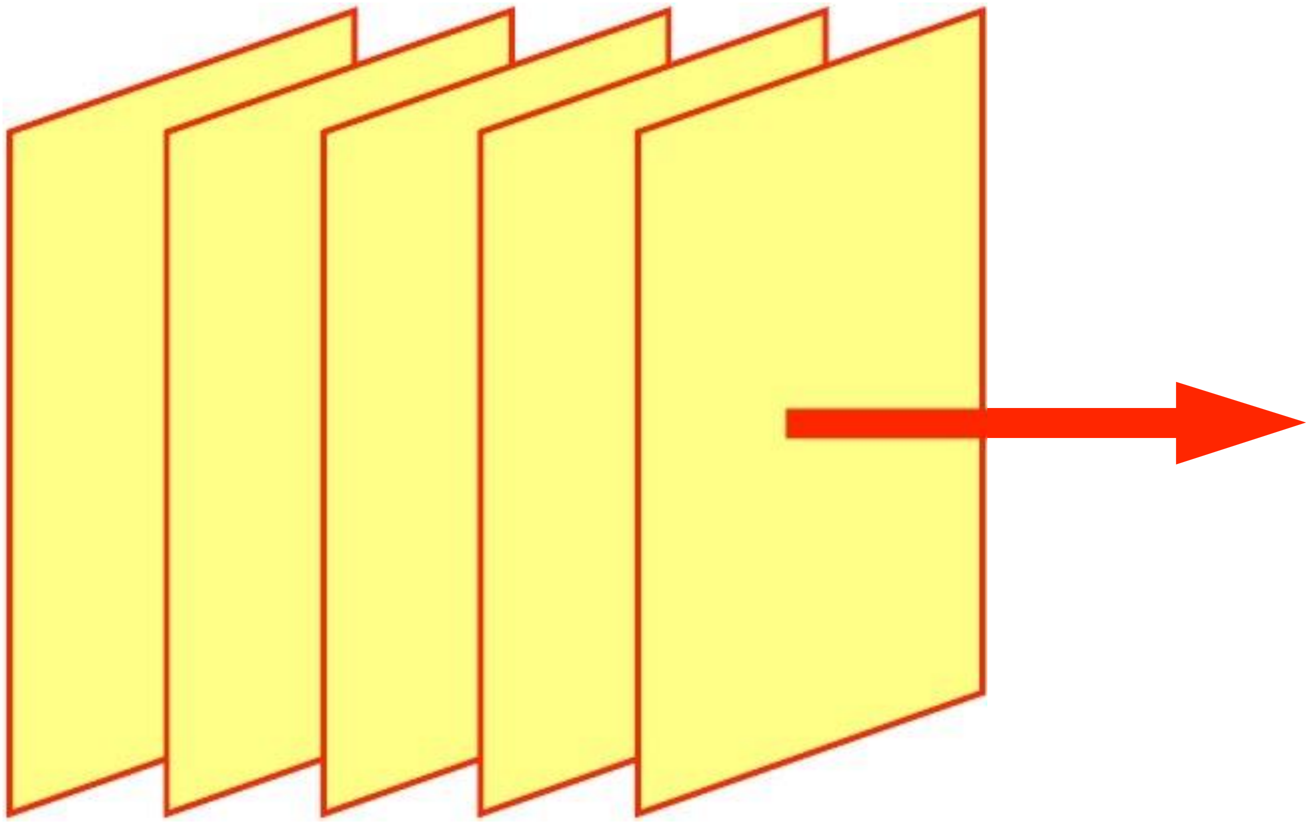
Ausbreitung von Wellen

Extremfälle

Kugelwelle

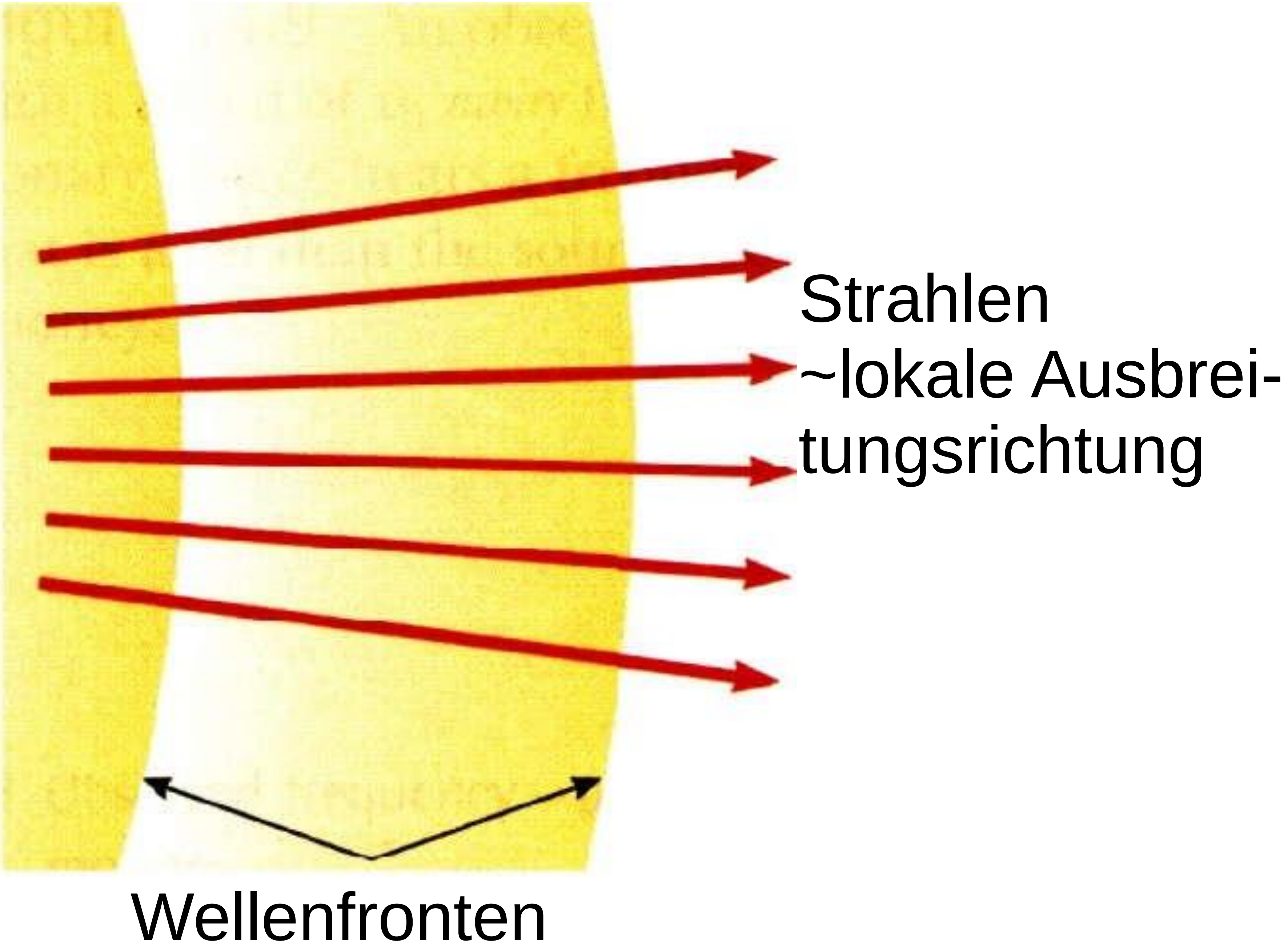


ebene Welle



Ausbreitung von Wellen

Kugelwellen und ebene Wellen sind Extremfälle

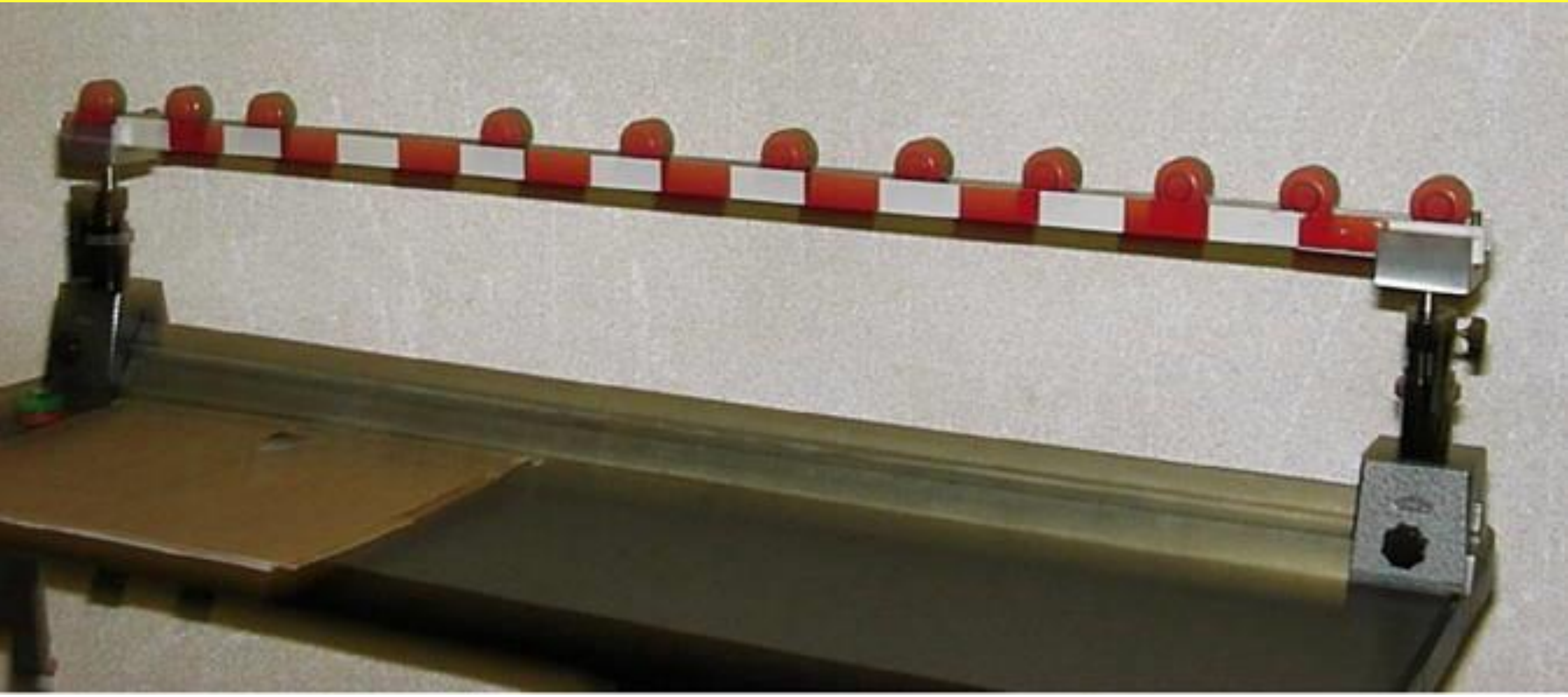


Arten von Wellen

Longitudinalwelle

Transversalwelle

Longitudinalwelle mit Magnetrollen



Transversalwelle

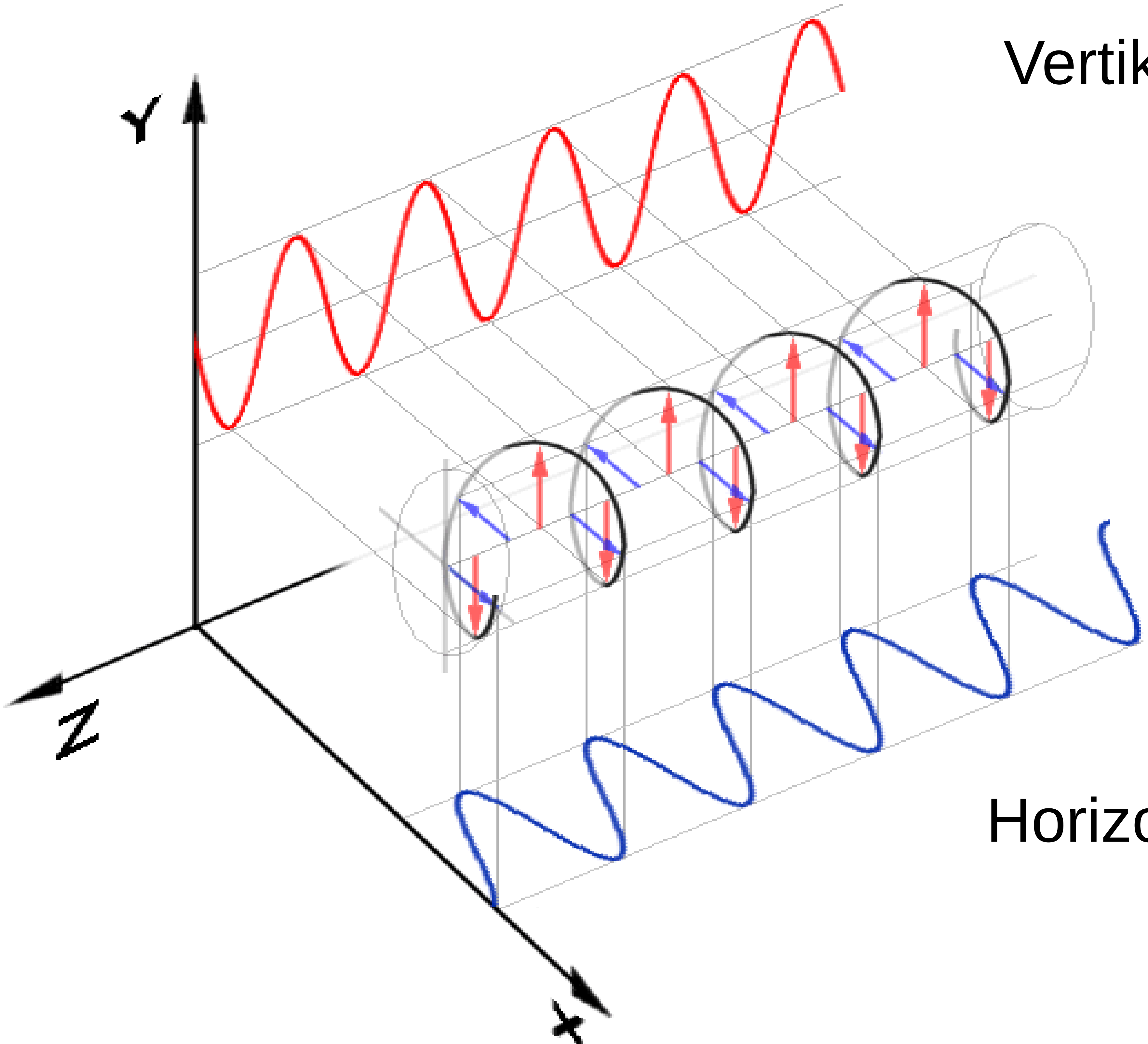
1/2-Welle

3/2-Welle

2/2-Welle

4/2-Welle

Transversalwelle

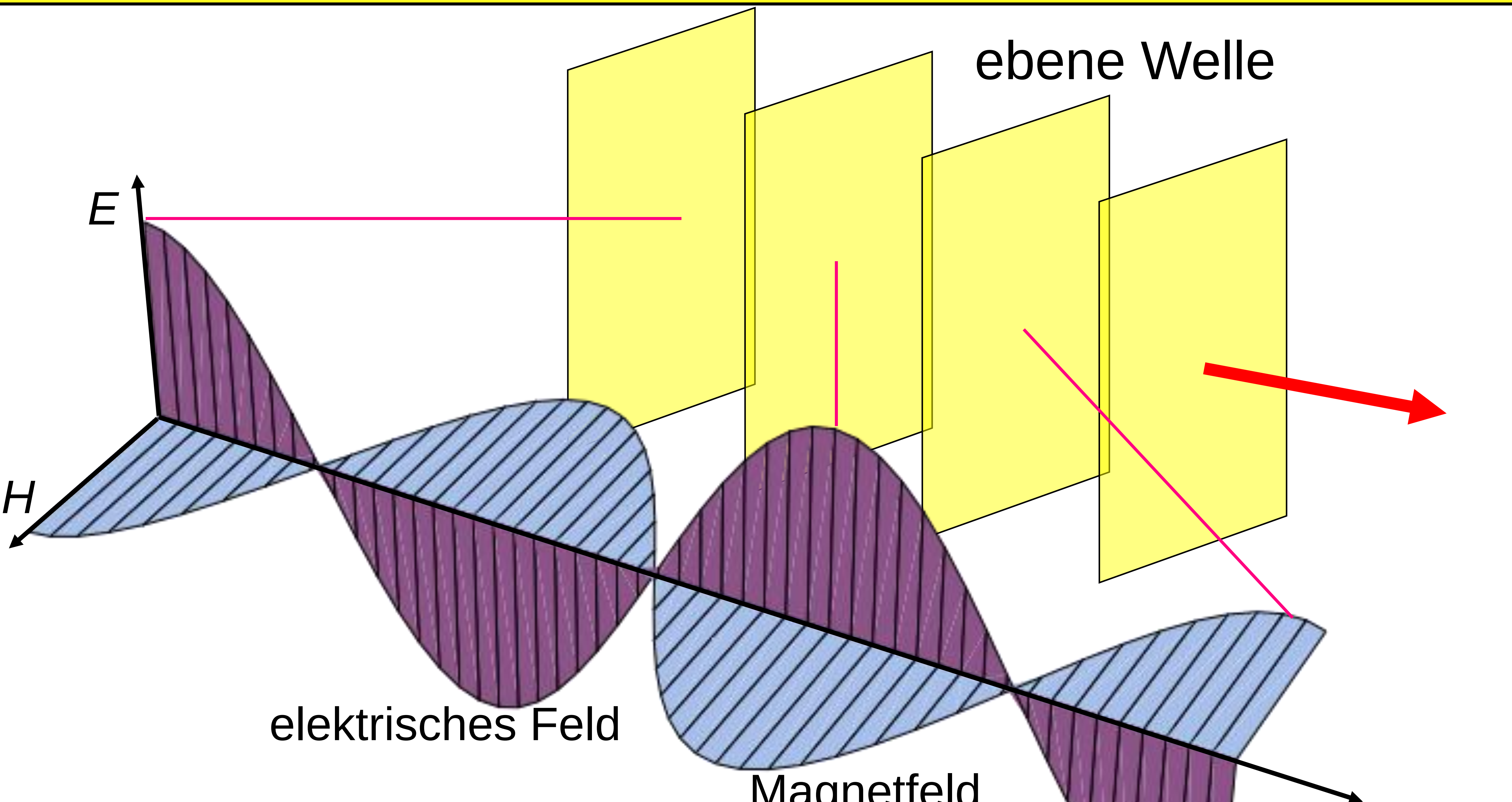


Vertikal polarisiert

Zirkular polarisiert

Horizontal polarisiert

Elektromagnetische Welle



Transversalwelle

Wellengleichung

Wellenfunktion: $\Psi(\vec{r},t)$

$$\Psi(x,t) = y(x,t) = y_0 \cdot \cos(\omega t - kx + \varphi)$$

$$\frac{d^2\Psi}{dt^2} = -\omega^2 \cdot y_0 \cdot \cos(\omega t - kx + \varphi)$$

$$\frac{d^2\Psi}{dx^2} = -k^2 \cdot y_0 \cdot \cos(\omega t - kx + \varphi)$$

$$\frac{d^2\Psi}{dt^2} = \frac{\omega^2}{k^2}\frac{d^2\Psi}{dx^2}$$

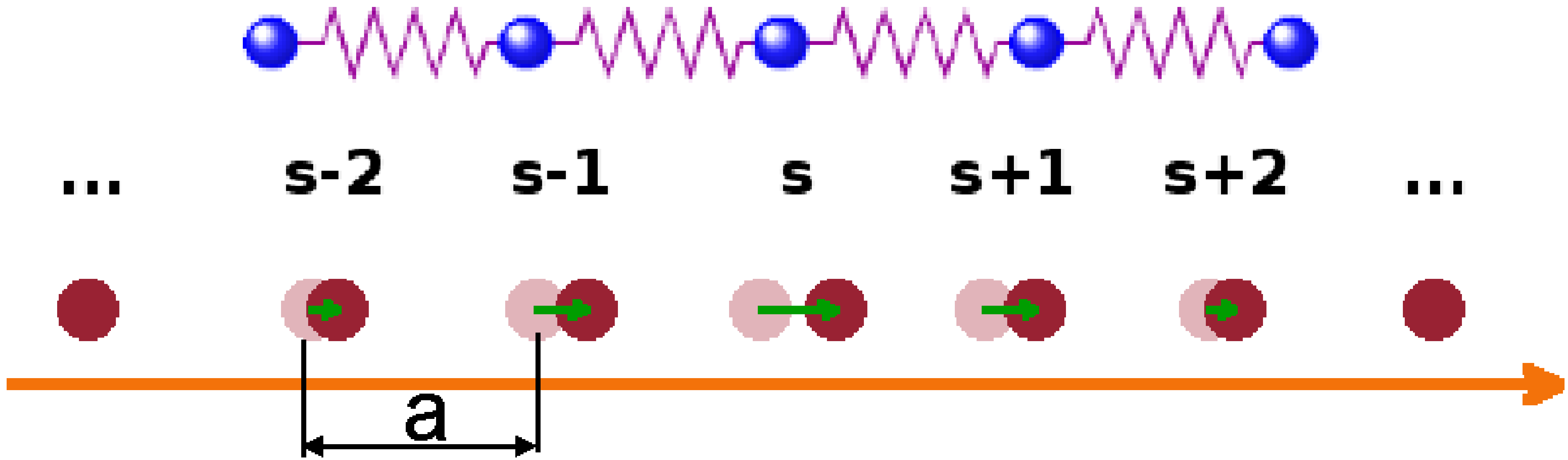
Wellengleichung

$$\frac{d^2\Psi(\vec{r},t)}{dt^2} = v_p^2 \cdot \Delta\Psi(\vec{r},t)$$

Laplace Operator $\Delta = \sum_{i=1}^{n} \frac{d^2}{dx_i^2}$

Phasengeschwindigkeit: $v_p = \frac{\omega}{k} = \frac{\lambda}{T}$

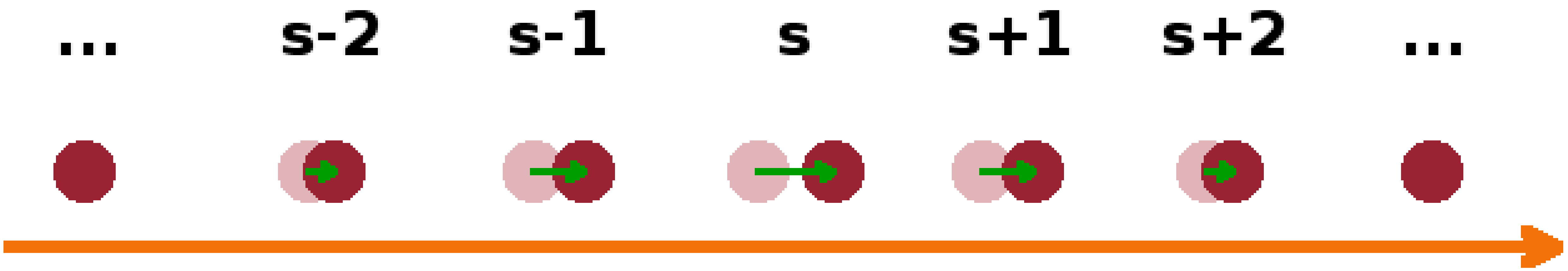
Lineare Kette



$$F_s = m \cdot \frac{d^2 y_s}{dt^2} = -C \cdot (y_s - y_{s-1}) - C \cdot (y_s - y_{s+1})$$

$$m \cdot \frac{d^2 y_s}{dt^2} = -C \cdot (2y_s - y_{s-1} - y_{s+1})$$

Lineare Kette



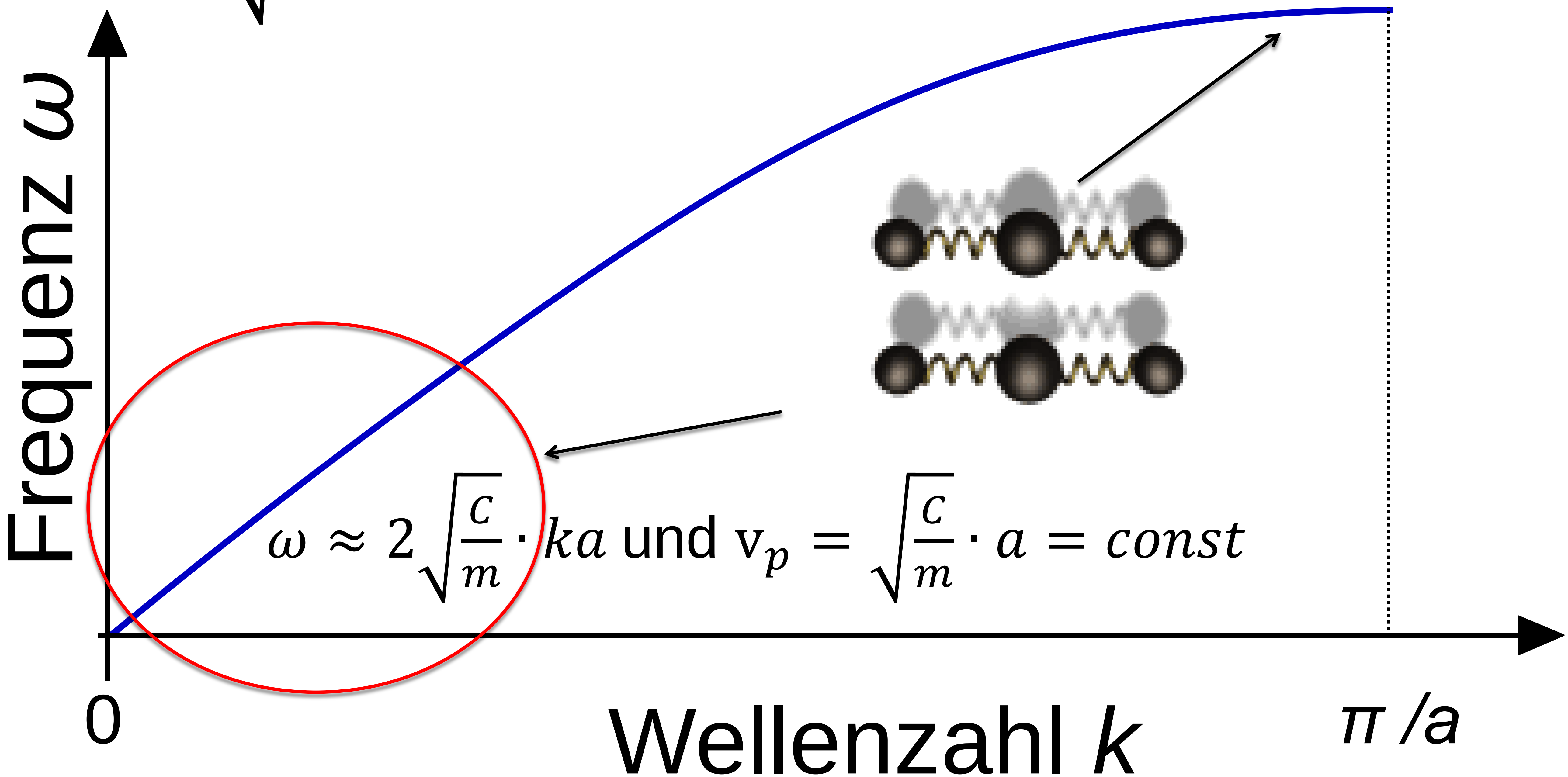
$$\frac{d^2 y_s}{dt^2} = -\omega^2 \cdot y_s = -2\frac{C}{m} \cdot (1 - \cos ka) \cdot y_s$$

$$\omega^2 = 2\frac{C}{m} \cdot (1 - \cos ka) = 4\frac{C}{m} \cdot sin^2 ka$$

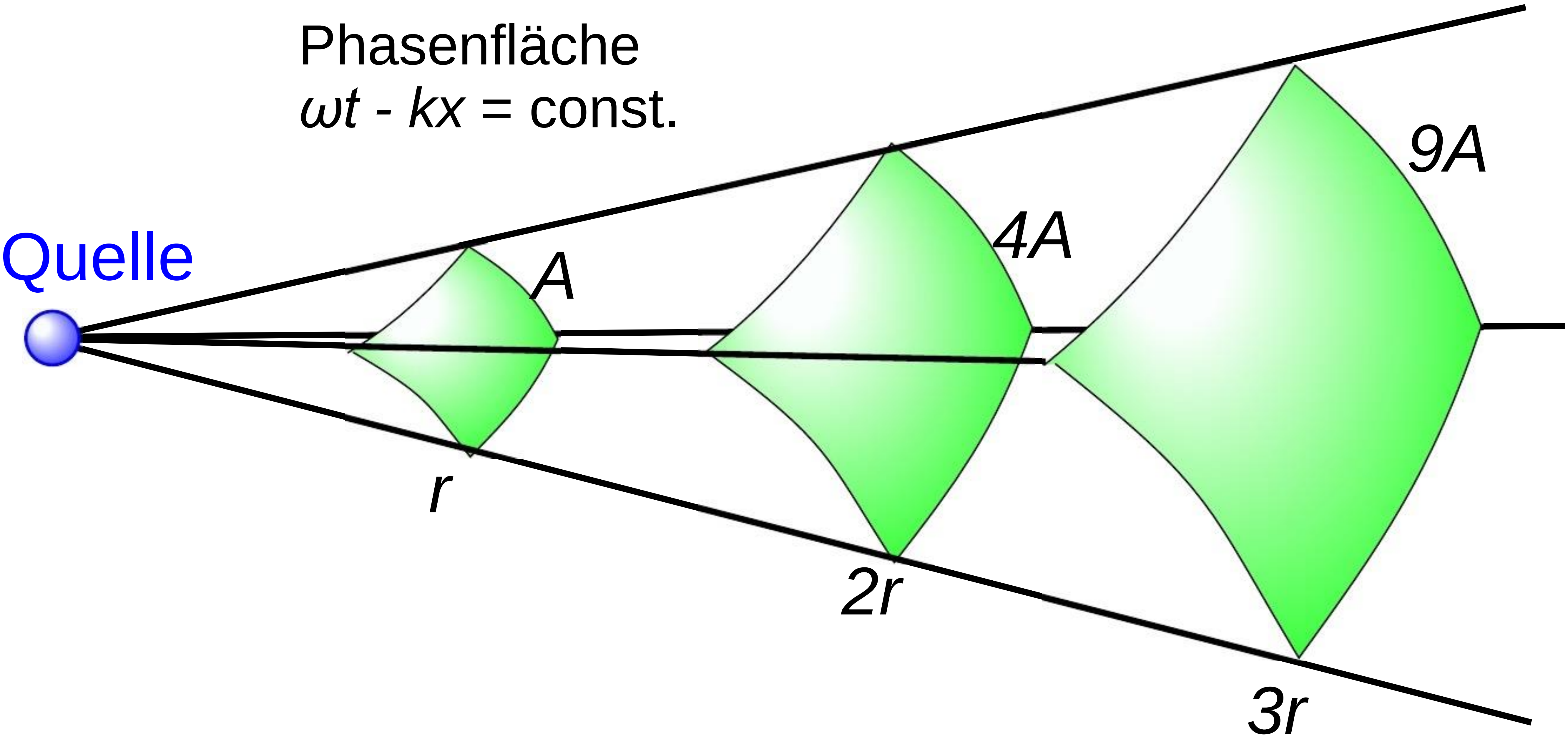
Dispersion auf der linearen Kette

$$\omega = 2\sqrt{\frac{C}{m}} \cdot |\sin(ka)|$$

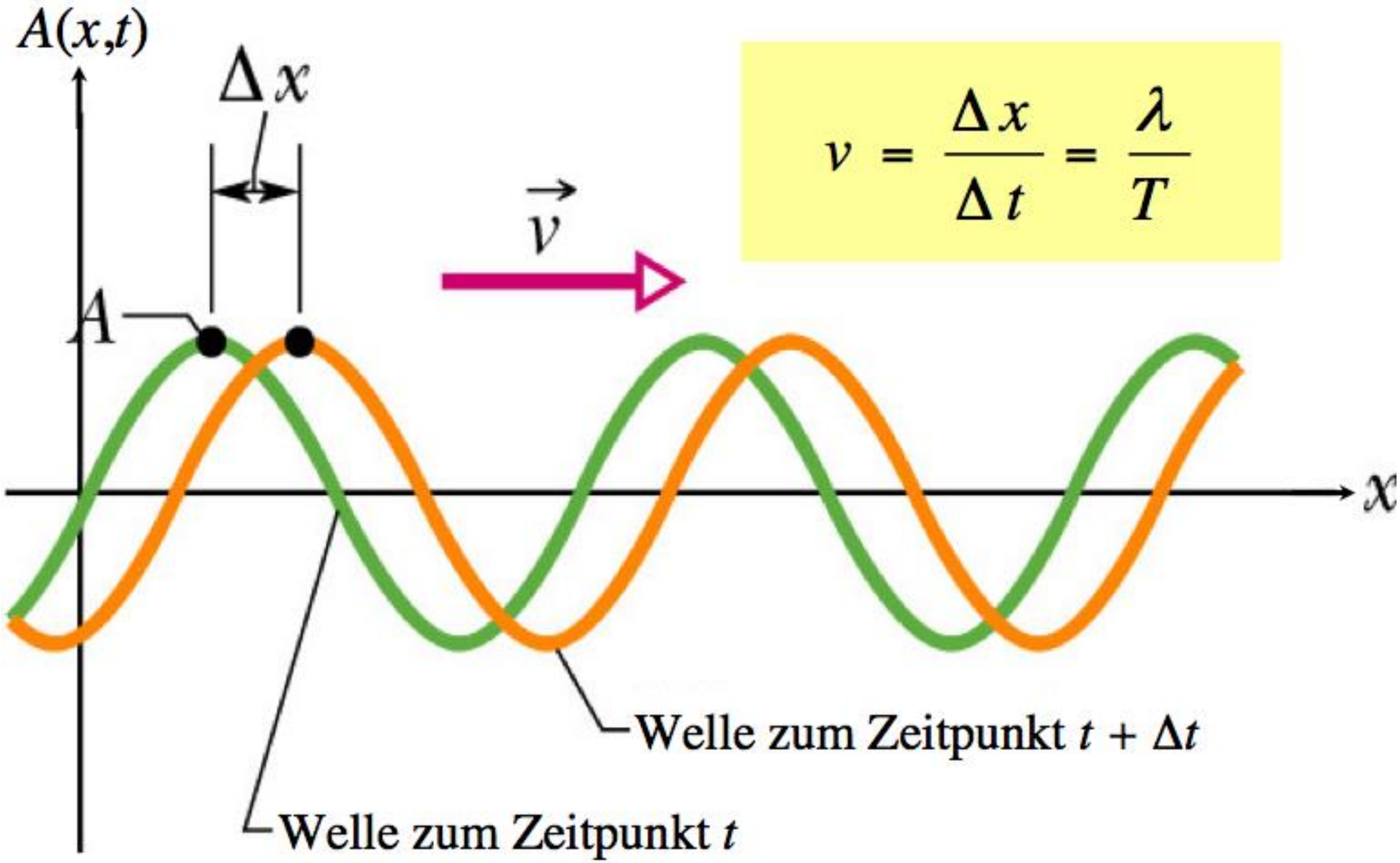
$$\omega = 2\sqrt{\frac{C}{m}};\ \mathrm{v}_p = \frac{\pi}{a}\sqrt{\frac{C}{m}}$$

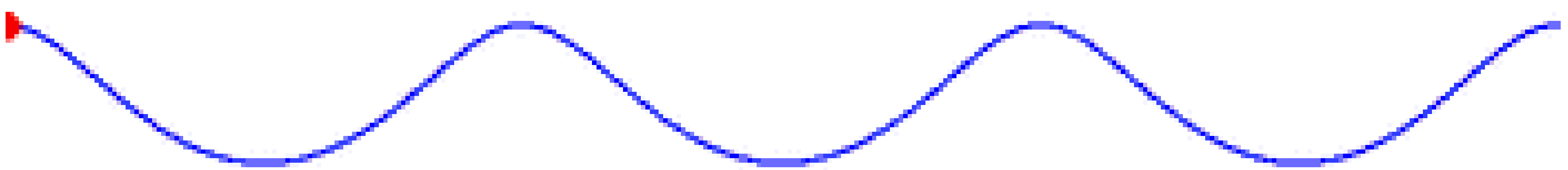


Phasengeschwindigkeit

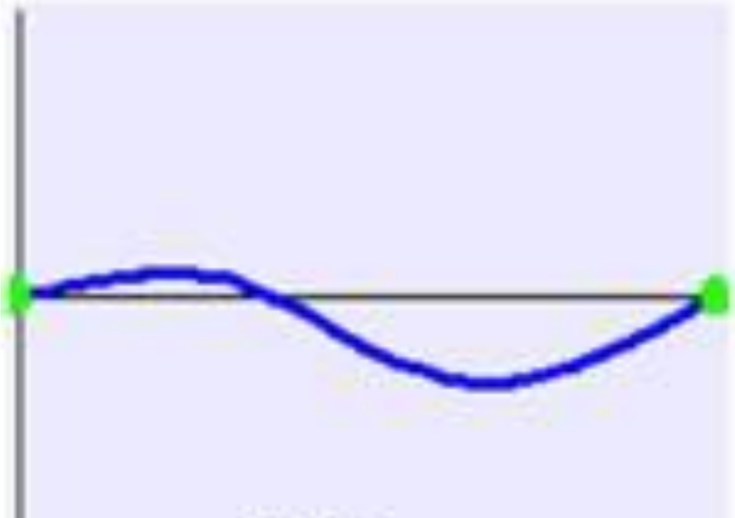


Phasengeschwindigkeit



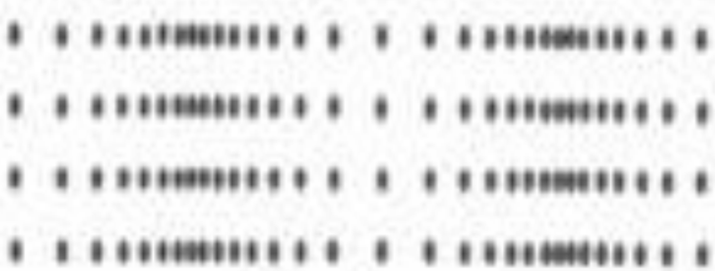


Phasengeschwindigkeit



Saite

$$c = \sqrt{\frac{F_0 / A}{\rho}}$$



Schall

$$c = \sqrt{\frac{K}{\rho}}$$

Festkörper

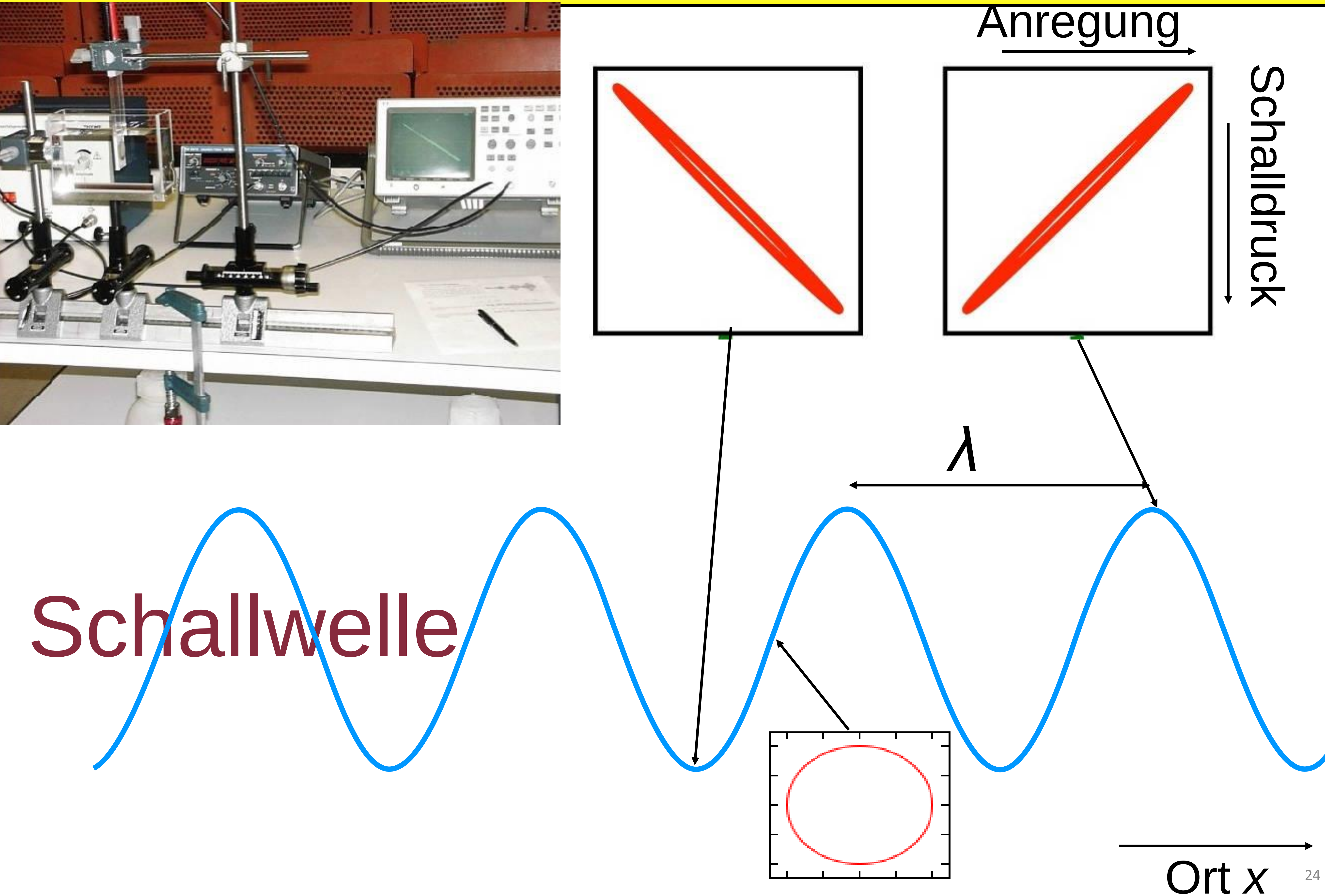
$$c = \sqrt{\frac{E}{\rho}}, \quad c = \sqrt{\frac{G}{\rho}}$$

c = Phasen-
geschwindigkeit
der Welle
=

$$\sqrt{\frac{\text{mechan. Eigenschaft}}{\text{Trägheit des Systems}}}$$

F_0 - Seilspannungskraft
K - Kompressionsmodul
E - Elastizitätsmodul
G - Torsionsmodul

Messung der Schallgeschwindigkeit

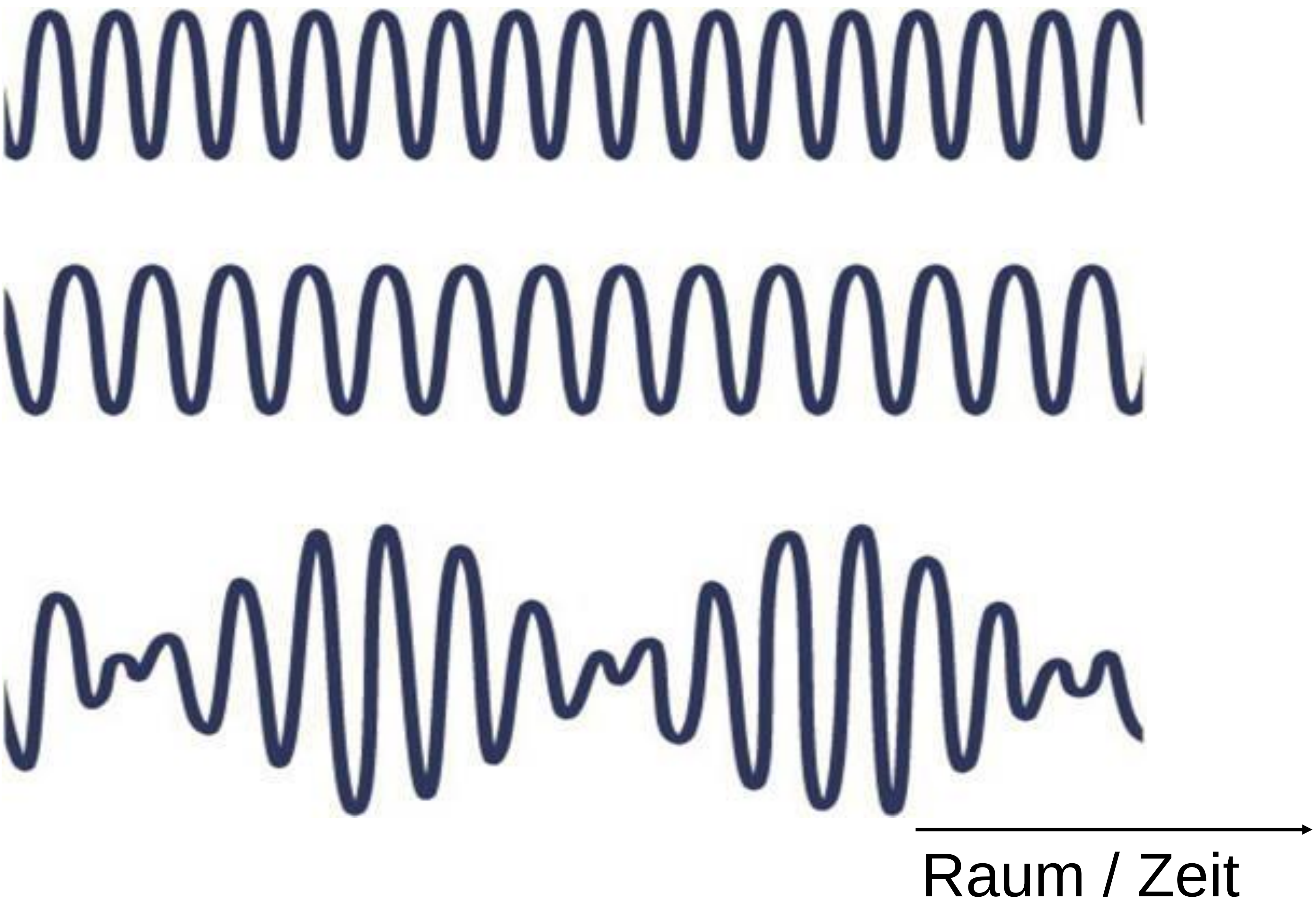


Dichte und Schallgeschwindigkeit

	Dichte ϱ in $\frac{kg}{m^3}$	Schallgeschwindigkeit c in $\frac{m}{s}$
Luft – 20 °C trocken	1,396	319
Luft 0 °C trocken	1,293	331
Luft 20 °C trocken	1,21	344
Luft 100 °C trocken	0,947	387
Wasserstoff 0 °C	0,090	1260
Wasserdampf 130 °C	0,54	450
Wasser 0 °C	1000	1400
20 °C	998	1480
Glyzerin	1260	1950
Eis	920	3200
Holz	600	4500
Glas	2500	5300
Beton	2100	4000
Stahl	7700	5050

Überlagerung

Überlagerung von Wellen



$$y(x,t) = y_0 \cdot \cos(\omega_1 t - k_1 x) + y_0 \cdot \cos(\omega_2 t - k_2 x)$$

Überlagerung von Wellen

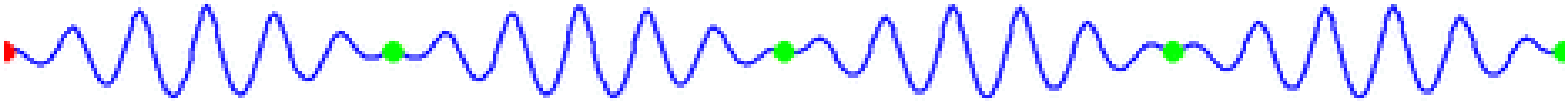
$$y = y_0 \cdot \cos(\omega t - kx) \cdot \cos(\Delta\omega t - \Delta\kappa x)$$

Grundwelle

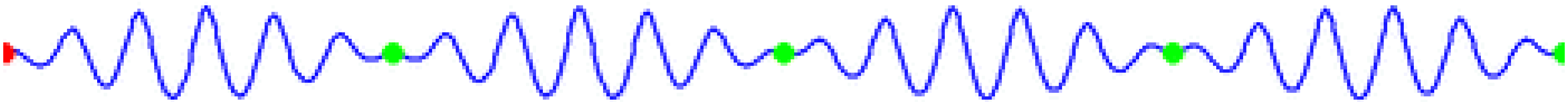
$$\omega = \frac{\omega_1 + \omega_2}{2} \qquad k = \frac{k_1 + k_2}{2}$$

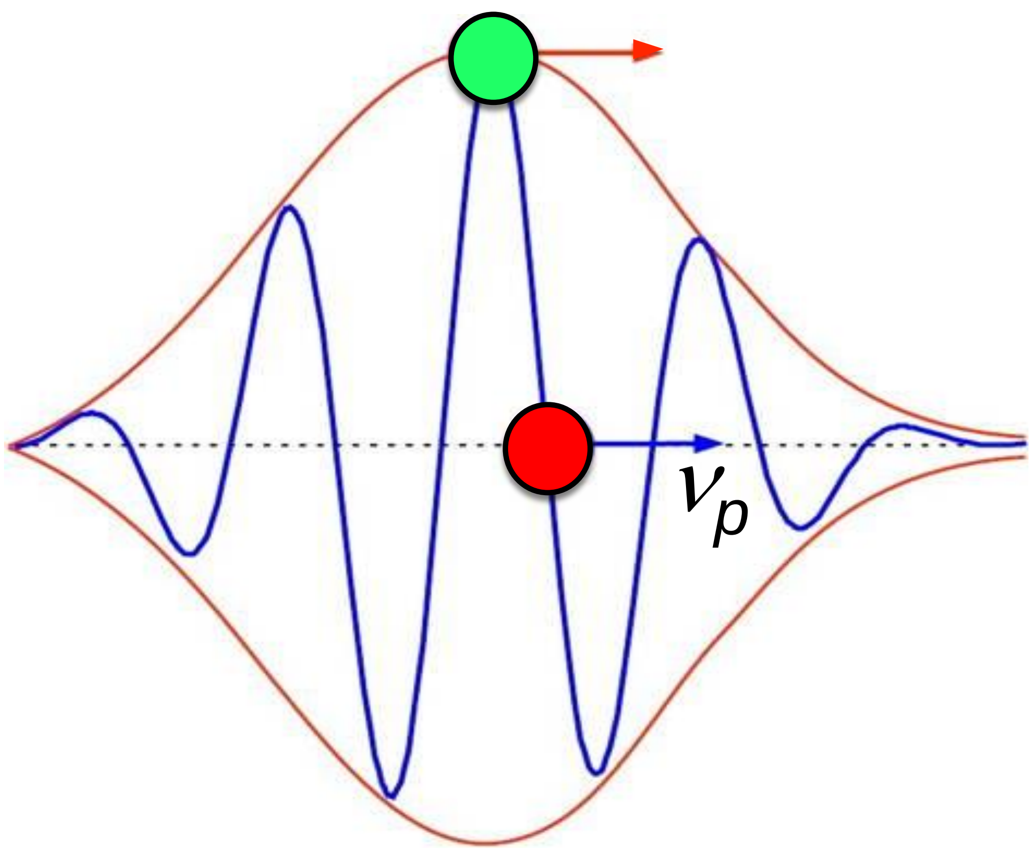
Modulation

$$\Delta\omega = \frac{\omega_1 - \omega_2}{2} \qquad \Delta k = \frac{k_1 - k_2}{2}$$



Phasen / Gruppengeschwindigkeit





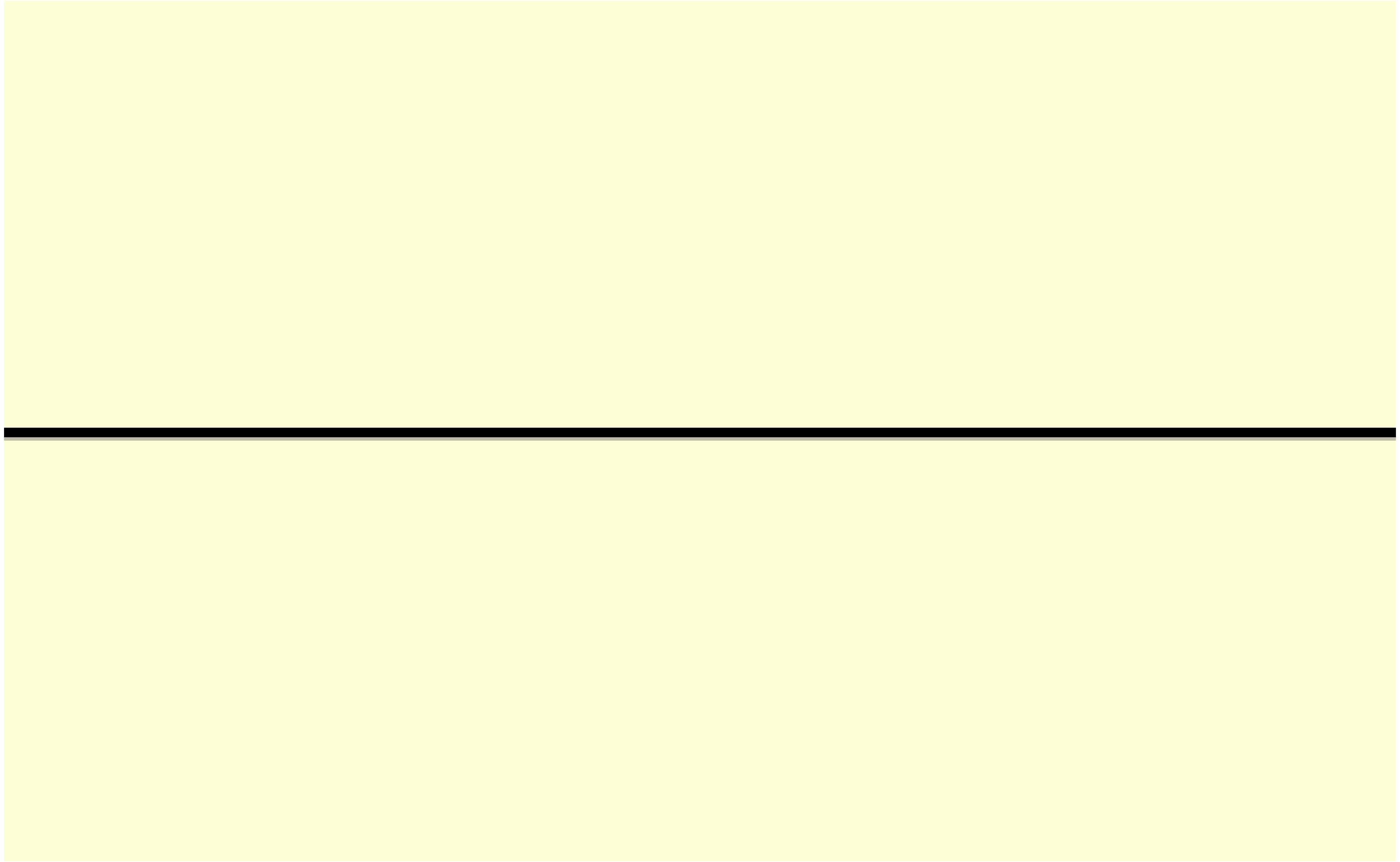
Gruppengeschwindigkeit:

$$v_g = \frac{\Delta\omega}{\Delta k}$$

Phasengeschwindigkeit:

$$v_p = \frac{\omega}{k}$$

Welle und Wellengruppe

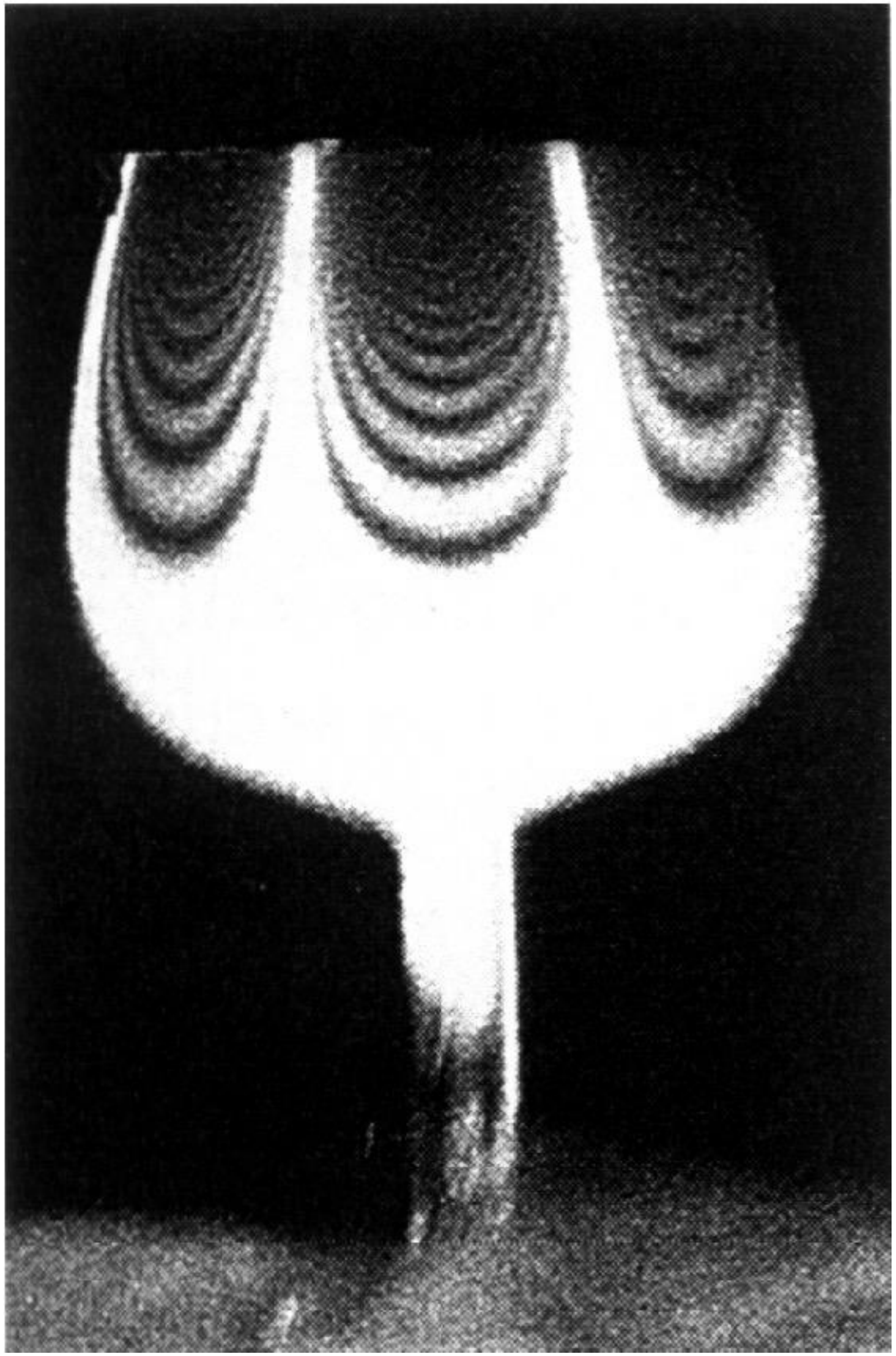


Gruppengeschwindigkeit = Signalgeschwindigkeit

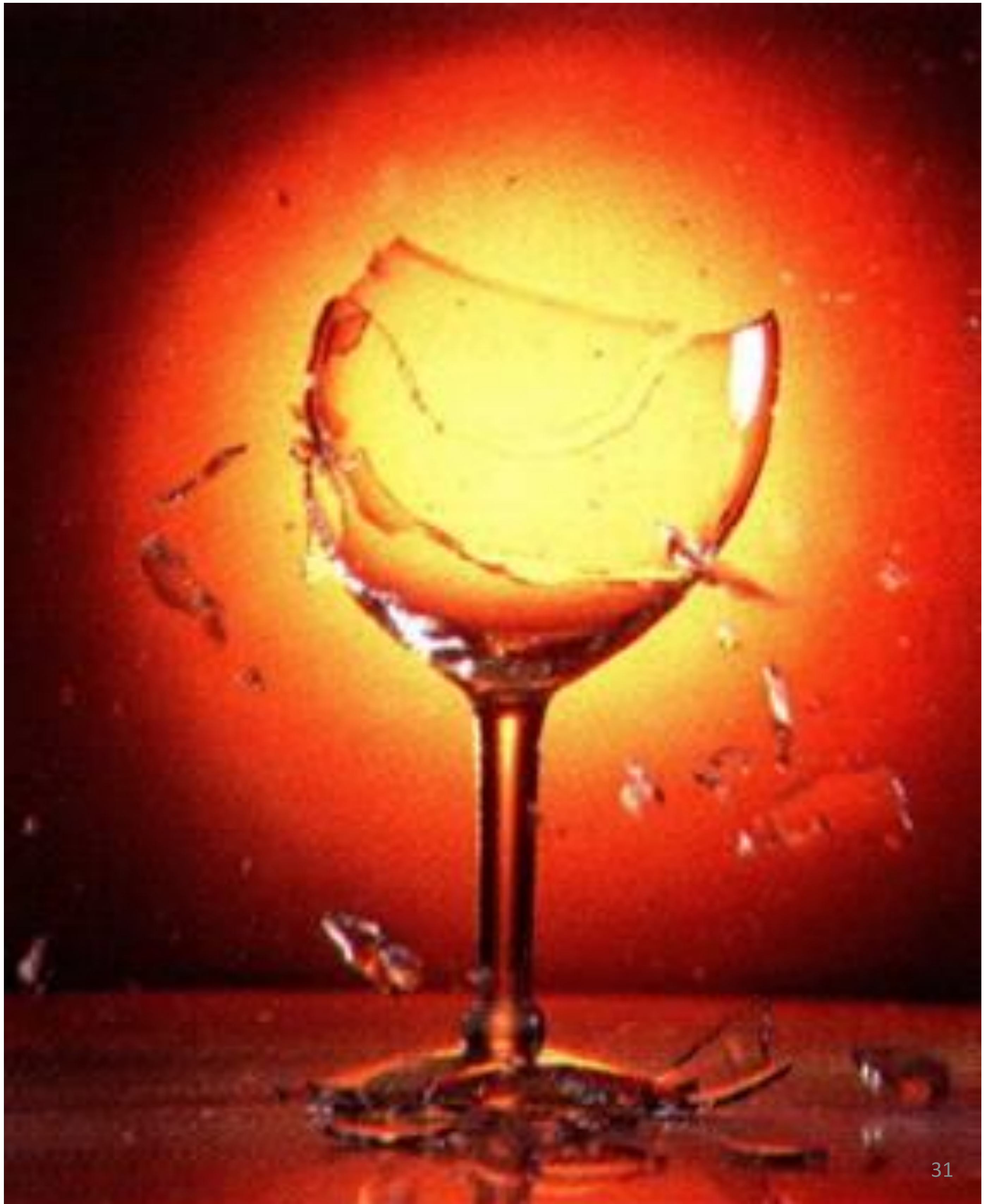
Informationsübertragung!

Energietransport

Schwingung enthält Energie



Welle transportiert Energie

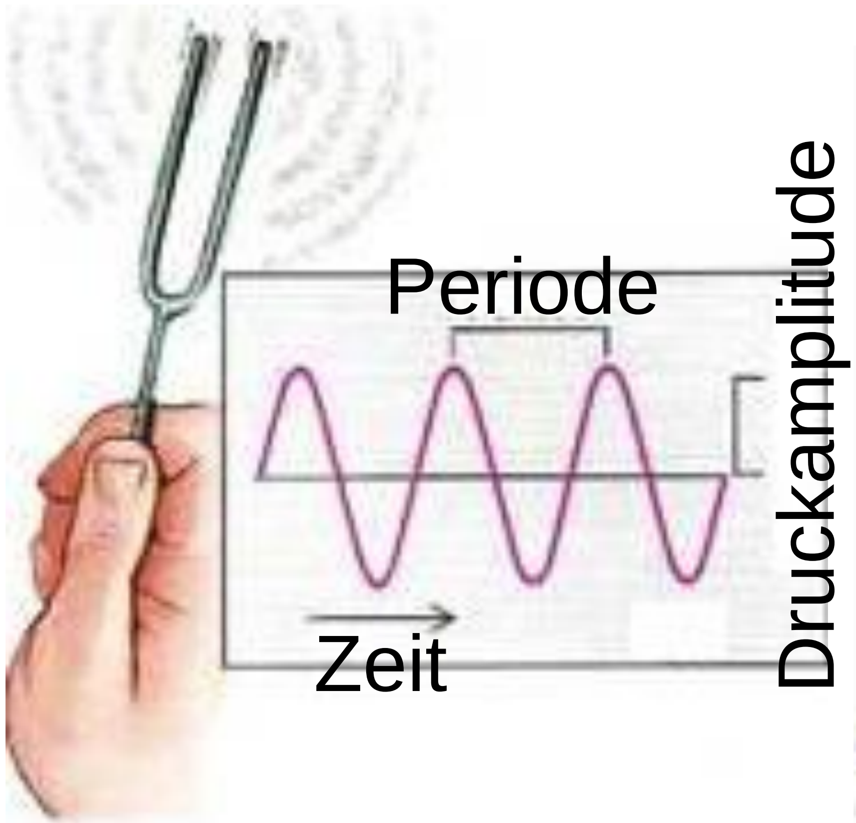


Akustische Wellen

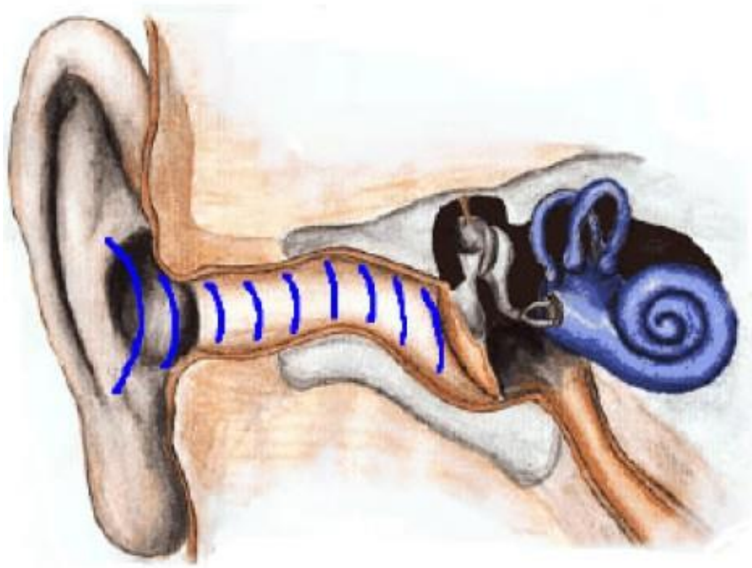
Ton

Klang

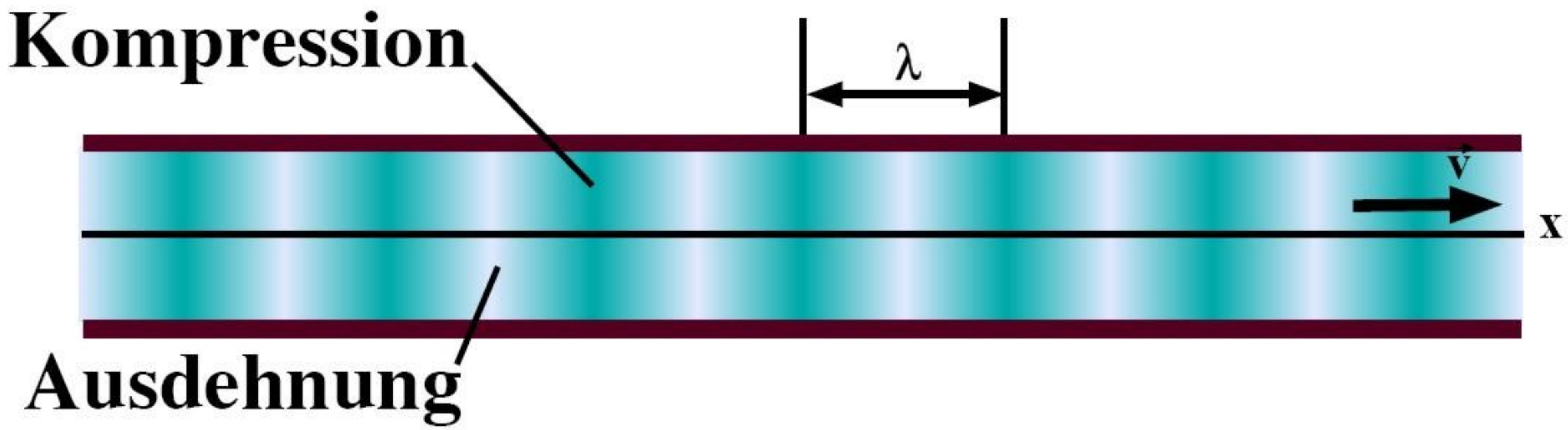
Geräusch



Akustik



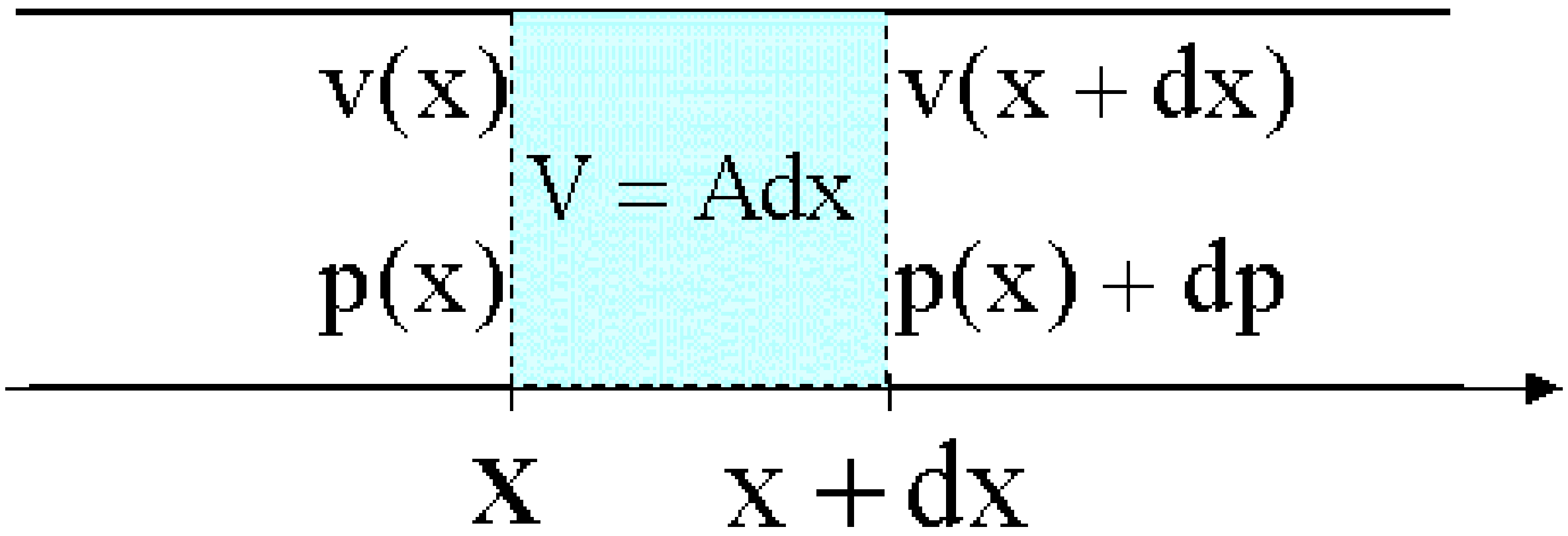
Akustik: Lehre vom Schall und seiner Ausbreitung in einem Medium, aber auch sämtliche damit zusammenhängenden Gesichtspunkte, wie Entstehung und Erzeugung, Beeinflussung und Analyse von Schall.



Schall: Akustische Welle etwa in Luft / Longitudinalwelle

Kein Schall ohne Medium

Longitudinale Druckwelle

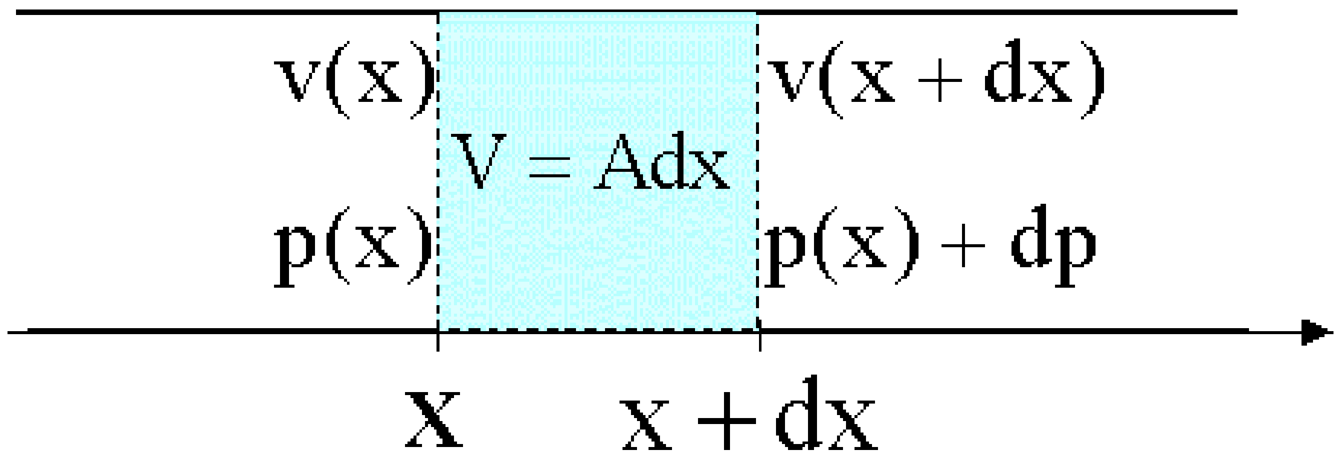


Räumliche Änderung des Druckes

$$dF = A \cdot [p(x) - p(x - dx)] = -A \cdot dx \cdot \frac{dp}{dx} = -V \cdot \frac{dp}{dx}$$

$$\frac{d\mathrm{v}}{dt} = \frac{dF}{dm} = -\frac{1}{\rho_0} \cdot \frac{dp}{dx}$$

Longitudinale Druckwelle

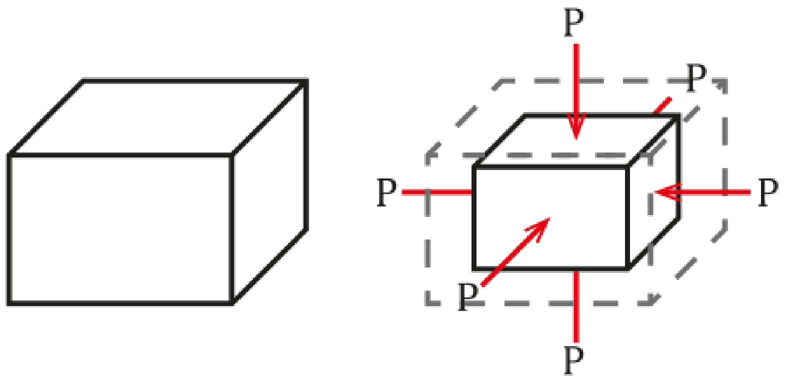


Zeitliche Änderung des Druckes

$$\frac{dV}{dt} = A \cdot \frac{d}{dt} dx = A \cdot d\mathrm{v} \Rightarrow \frac{dV}{V} = \frac{d\mathrm{v}}{dx} \cdot dt$$

Kompressionsmodul:

$$dp = -K \cdot \frac{dV}{V}$$



$$\frac{dp}{dt} = -K \cdot \frac{d\mathrm{v}}{dx}$$

Longitudinale Druckwelle

Druck

$$\frac{dp}{dt} = -K \cdot \frac{d\mathrm{v}}{dx} \qquad \frac{dp}{dx} = -\rho_0 \cdot \frac{d\mathrm{v}}{dt}$$

Wellengleichung:

$$\frac{d^2p}{dt^2} = \frac{K}{\rho_0} \cdot \frac{d^2p}{dx^2}$$

Schallgeschwindigkeit: $\mathrm{v}_s = \sqrt{\frac{K}{\rho_0}}$

Longitudinale Druckwelle

Lokale Dichte

$$\frac{d^2p}{dt^2} = \mathrm{v}_s^2 \cdot \frac{d^2p}{dx^2}$$

$$\left.\frac{dp}{d\rho}\right|_{\rho_0} = \frac{K}{\rho_0}$$

Wellengleichung:

$$\frac{d^2\rho}{dt^2} = \mathrm{v}_s^2 \cdot \frac{d^2p}{dx^2}$$

Longitudinale Druckwelle

Lokale Geschwindigkeit (Schallschnelle)

$$\frac{dp}{dt} = -K \cdot \frac{d\mathrm{v}}{dx} \qquad \frac{dp}{dx} = -\rho_0 \cdot \frac{d\mathrm{v}}{dt}$$

Wellengleichung:

$$\frac{d^2\mathrm{v}}{dt^2} = \mathrm{v}_s^2 \cdot \frac{d^2\mathrm{v}}{dx^2}$$

Longitudinale Druckwelle

Lösung

Druck $$p(x,t) = p_0 + \Delta p_0 \cdot \cos(\omega t - kx)$$

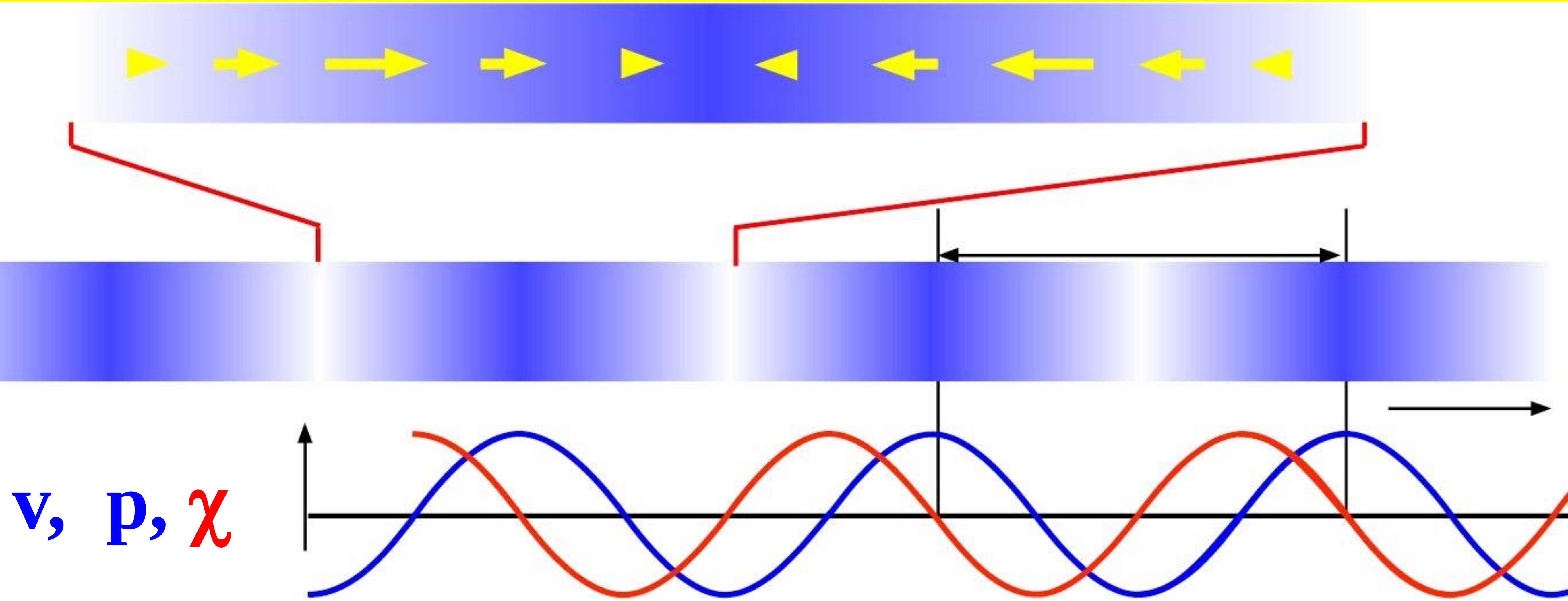
Dichte $$\rho(x,t) = \rho_0 + \Delta \rho_0 \cdot \cos(\omega t - kx)$$

Lokale geschw. $$\mathrm{v}(x,t) = \mathrm{v}_0 \cdot \cos(\omega t - kx)$$

Lokale Verschiebung $$\chi(x,t) = \chi_0 \cdot sin(\omega t - kx)$$

$$\mathrm{v}_0 = \omega \cdot \chi_0 \qquad \Delta p_0 = \rho_0 \cdot \mathrm{v}_0 \cdot \mathrm{v}_s = \mathrm{v}_0 \cdot \sqrt{K \rho_0}$$

Druck und Geschwindigkeit

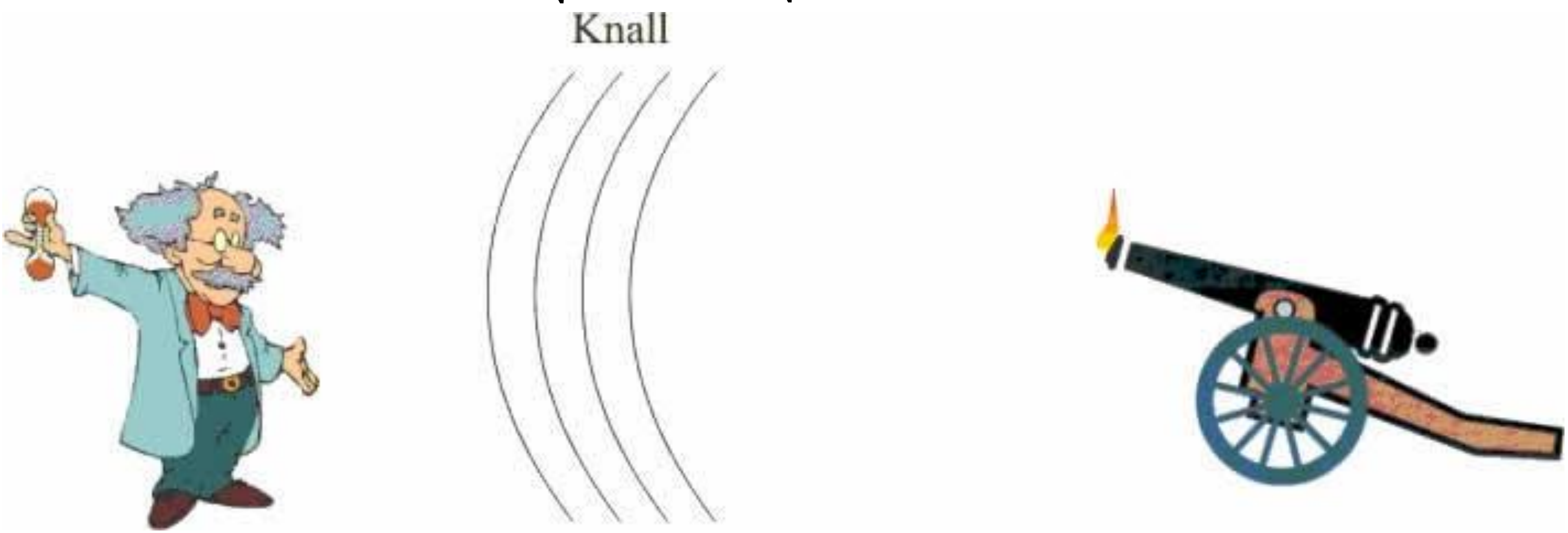


Verschiebung $\chi(x,t) = \chi_0 \cdot sin(\omega t - kx)$

Druck $p(x,t) = p_0 + \Delta p_0 \cdot \cos(\omega t - kx)$

Schallgeschwindigkeit

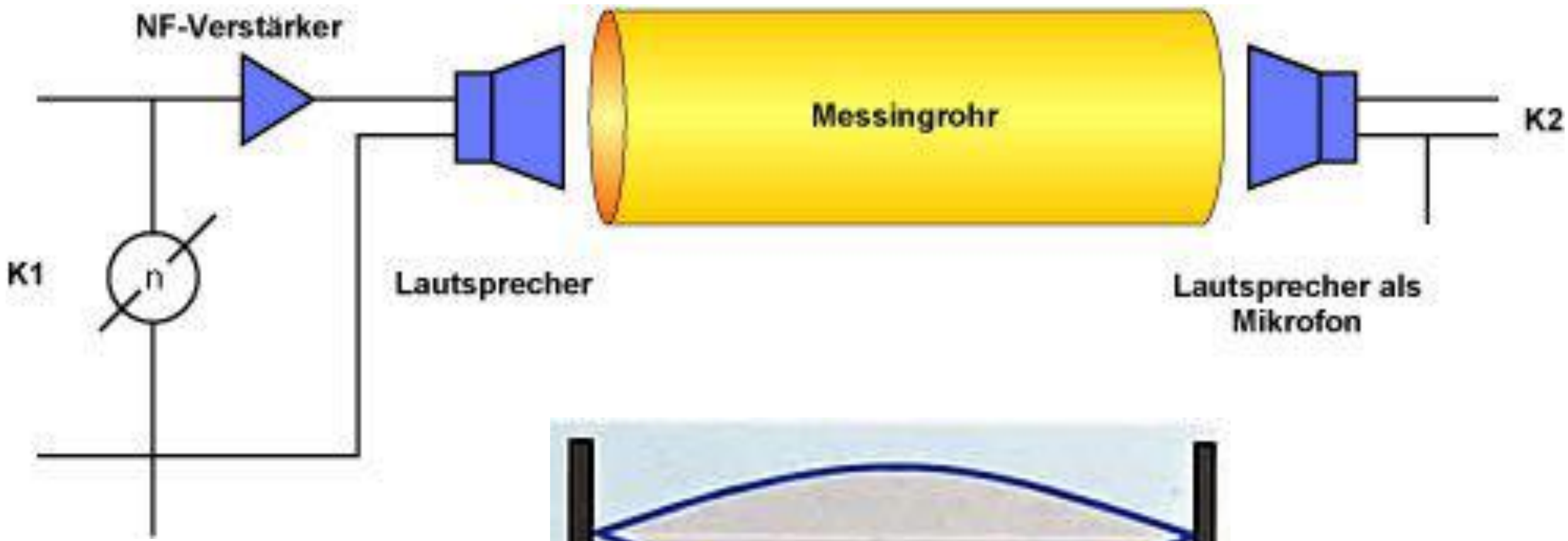
Lüft: $$\mathrm{v}_s = \sqrt{\frac{K}{\rho_0}} = \sqrt{\frac{140kPa}{1.2\ kg/m^3}} = 342m/s$$

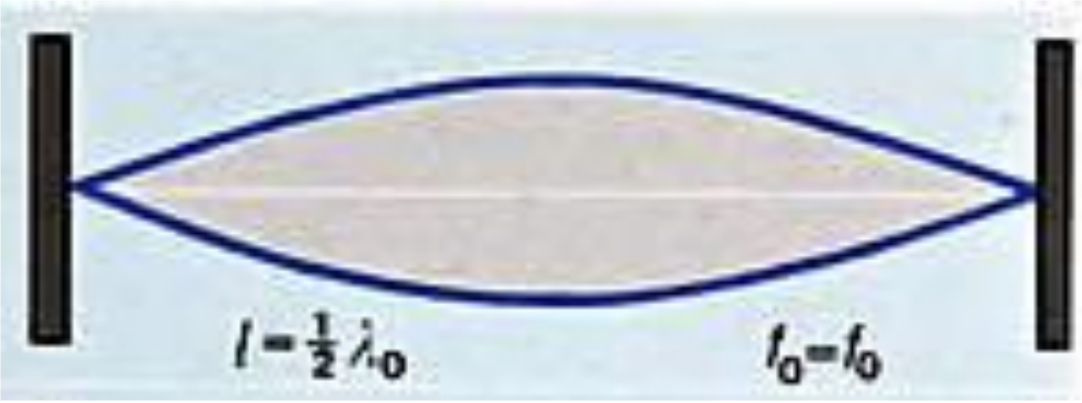


$$t_{Knall} - t_{Licht} = \mathrm{v}_s \cdot L$$

$$\mathrm{v}_s \approx 300m/s$$

Schallgeschwindigkeit





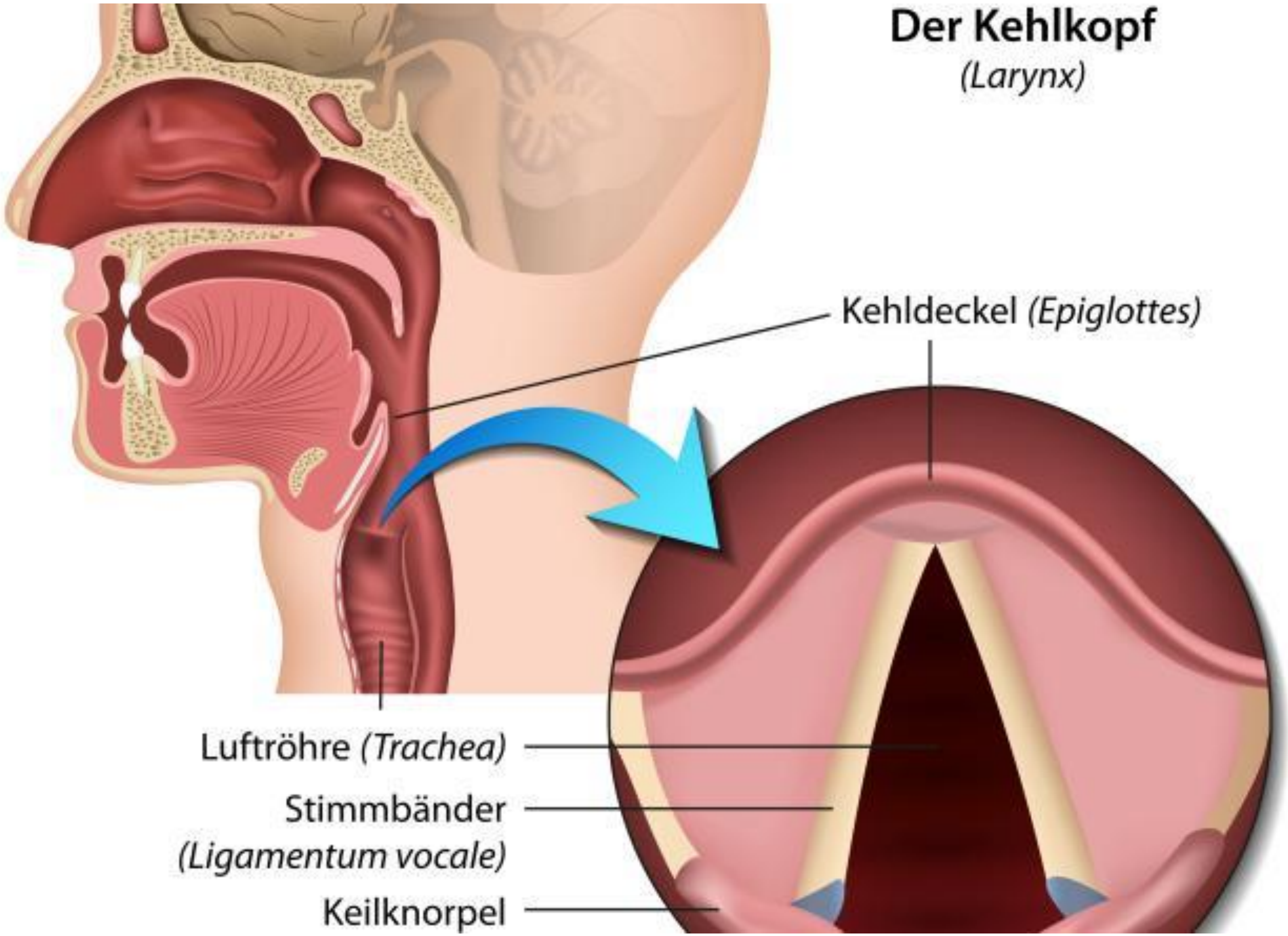
$$v_s = 2 \cdot \nu_0 \cdot l$$

Schallgeschwindigkeit

Schallgeschwindigkeiten bei 20° C

Material	c in [m s^{-1}]
Luft	330
Helium	1007
Azeton	1190
Wasser	1485
Blei	1300
Eisen	5100
Kronglas	5300

Menschliche Stimme



$$\nu(He)/\nu(Lüft) \approx 3$$

Schallimpedanz

Druck $$p(x,t) = p_0 + \Delta p_0 \cdot \cos(\omega t - kx)$$

Lokale geschw. $$\mathrm{v}(x,t) = \mathrm{v}_0 \cdot \cos(\omega t - kx)$$

$$\frac{dp}{dx} = -\rho_0 \cdot \frac{d\mathrm{v}}{dt}$$

$$\Delta p_0 \cdot k \cdot \sin(\omega t - kx) = \mathrm{v}_0 \cdot \rho_0 \cdot \omega \cdot \sin(\omega t - kx)$$

$$\Delta p_0 = \rho_0 \cdot \mathrm{v}_0 \cdot \mathrm{v_s}$$

Schallimpedanz

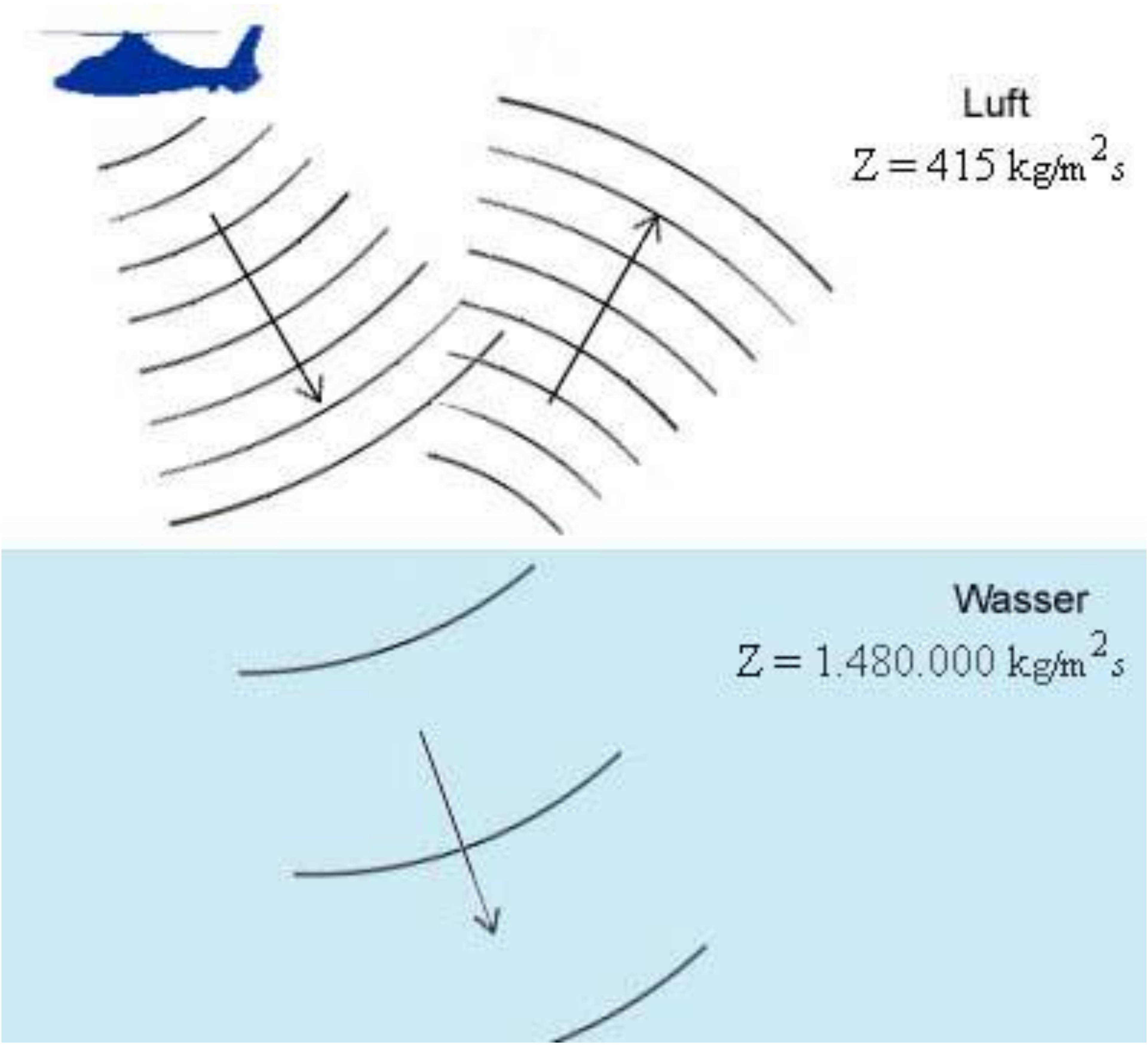
$$Z = \frac{\Delta p_0}{\mathrm{v}_0} = \rho_0 \cdot \mathrm{v_s} = \sqrt{\rho_0 K} = const$$

„ohmsches Gesetz als akustische Äquivalenz“ $$R = \frac{U}{I} = const$$

Schallimpedanz

	Dichte ϱ in $\frac{kg}{m^3}$	Schallgeschwindigkeit c in $\frac{m}{s}$	Schallkennimpedanz Z_0 in $\frac{kg}{m^2 s}$
Luft – 20 °C trocken	1,396	319	445
Luft 0 °C trocken	1,293	331	427
Luft 20 °C trocken	1,21	344	416
Luft 100 °C trocken	0,947	387	366
Wasserstoff 0 °C	0,090	1260	113
Wasserdampf 130 °C	0,54	450	243
Wasser 0 °C	1000	1400	$1{,}40 \cdot 10^6$
20 °C	998	1480	$1{,}48 \cdot 10^6$
Glyzerin	1260	1950	$2{,}46 \cdot 10^6$
Eis	920	3200	$2{,}94 \cdot 10^6$
Holz	600	4500	$2{,}70 \cdot 10^6$
Glas	2500	5300	$13{,}0 \cdot 10^6$
Beton	2100	4000	$8{,}4 \cdot 10^6$
Stahl	7700	5050	$39 \cdot 10^6$

Schallimpedanz



Energie

Energiedichte

$$w = w_{kin} + w_{pot}$$

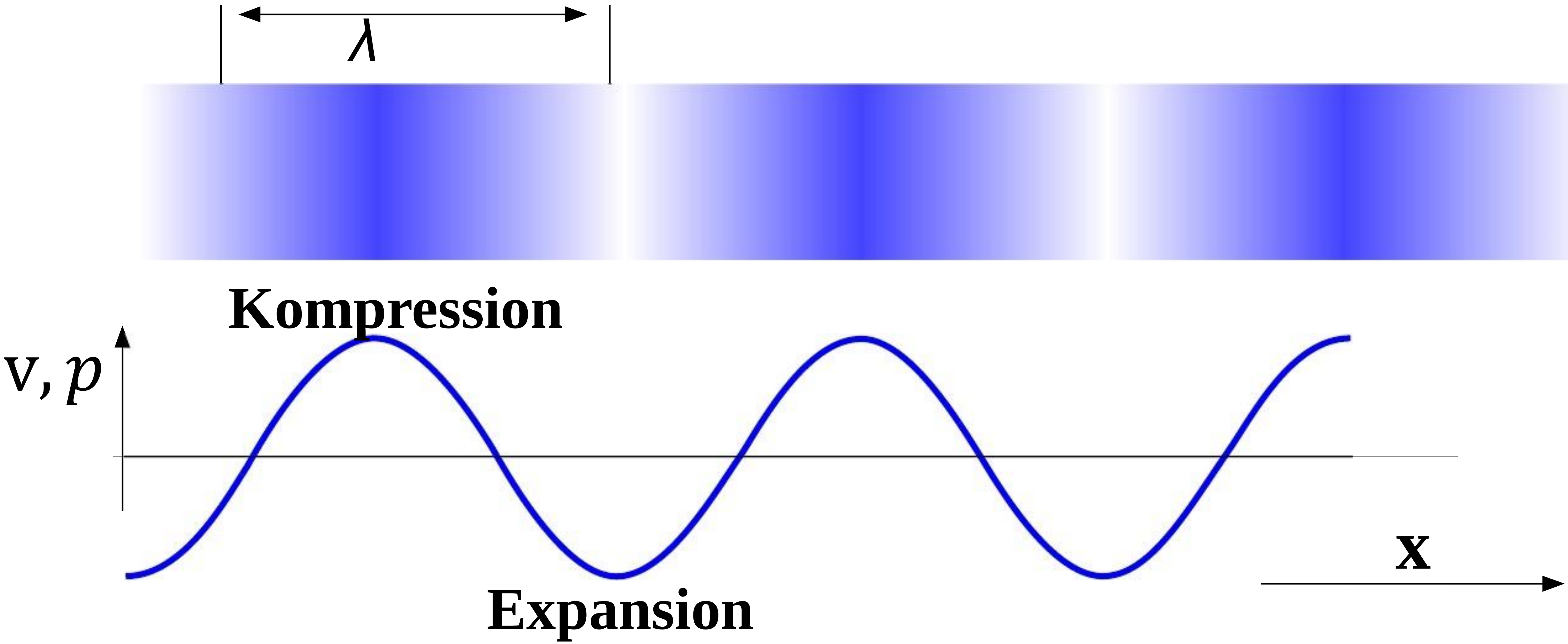
$$w_{kin} = \frac{E_{kin}}{\Delta V} = \frac{1}{2} \cdot \frac{m}{\Delta V} \cdot \mathrm{v}^2 = \frac{1}{2} \cdot \rho_0 \cdot \chi_0^2 \cdot \omega^2 \cdot sin^2(\omega t - kx)$$

$$w_{pot} = \frac{-\int p \cdot dV}{\Delta V} = \frac{1}{K} \int p \cdot dp = \frac{1}{2K} \cdot p^2 = \frac{1}{2} \cdot \rho_0 \cdot \chi_0^2 \cdot \omega^2 \cdot sin^2(\omega t - kx)$$

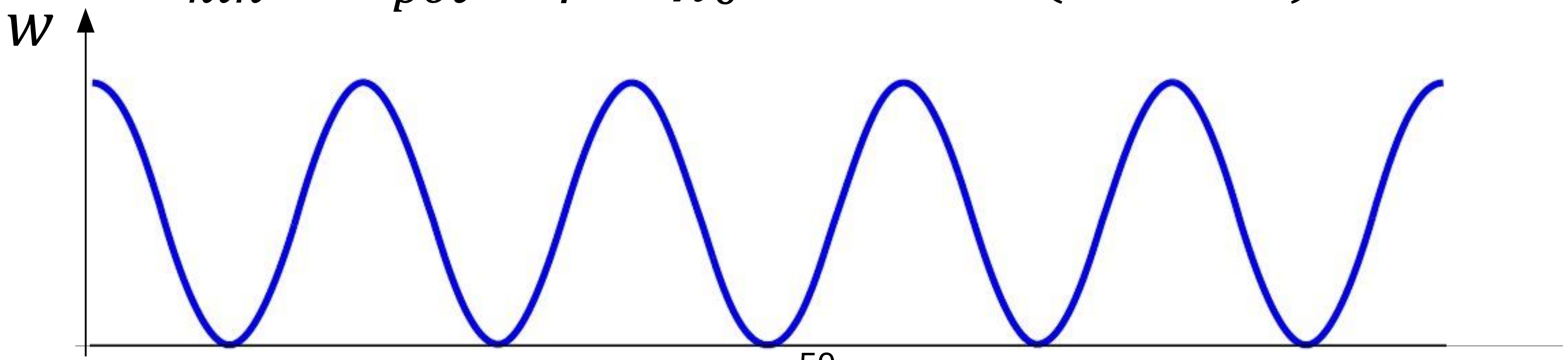
$$dp = -K \cdot \frac{dV}{V}$$

$$w = \rho_0^2 \cdot \chi_0^2 \cdot \omega^2 \cdot sin^2(\omega t - kx)$$

Energiedichte



$$w_{kin} + w_{pot} = \rho^2 \cdot \chi_0^2 \cdot \omega^2 \cdot sin^2(\omega t - kx)$$



Intensität

Intensität $I = \frac{Energie}{Fläche \cdot Zeit}$

$$I = \frac{dE}{A \cdot dt} = \frac{dE}{A \cdot d\chi} \cdot \frac{d\chi}{dt} = w \cdot \mathrm{v}_s = \frac{1}{2} \cdot \frac{\Delta p_0^2}{\rho_0 \cdot \mathrm{v}_s} = \frac{1}{2} \cdot \frac{\Delta p_0^2}{Z}$$

Lautstärkeskala

Schallpegel

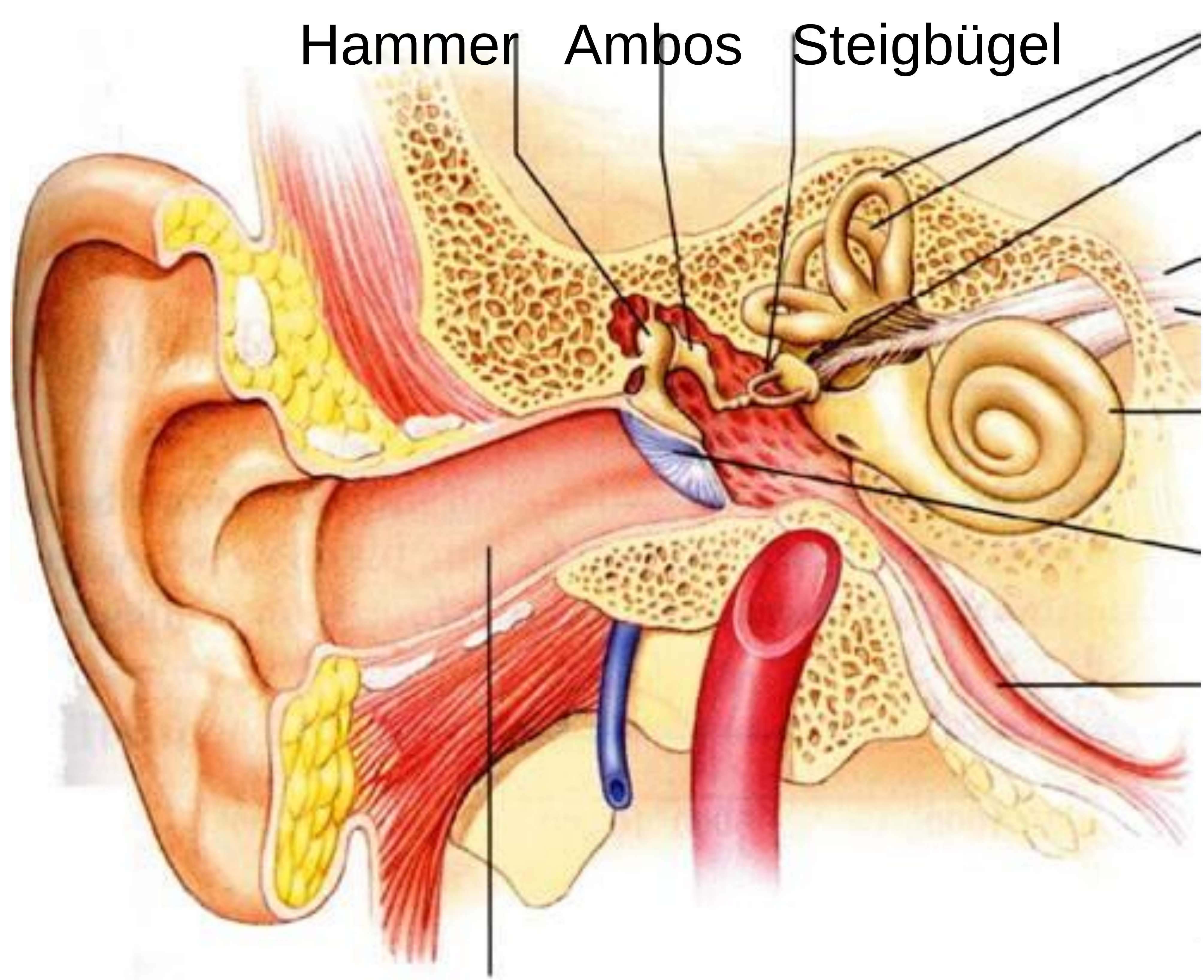
$$L = 20 \cdot \log\left(\frac{\Delta p}{\Delta p_0}\right) = 20 \cdot \log\left(\frac{I}{I_0}\right) [dBA]$$

Menschliche Hörschwelle

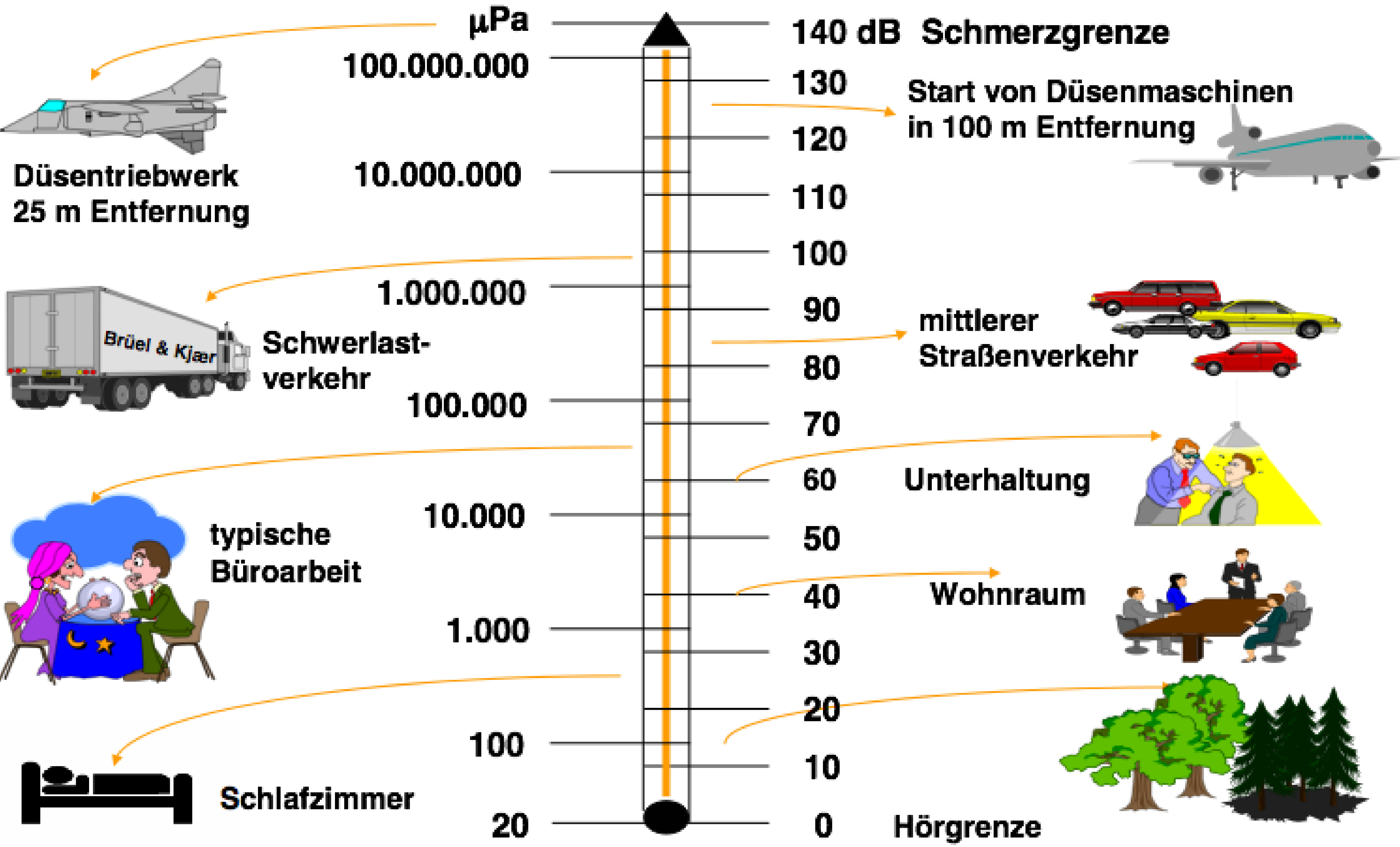
$$\Delta p_0 = 20\ \mu Pa$$

$$I_0 = 10^{-12} W/m^2$$

Das Ohr als Schallwandler



Lautstärke


µPa
100.000.000
10.000.000
1.000.000
100.000
10.000
1.000
100
20
140 dB Schmerzgrenze
130
120
110
100
90
80
70
60
50
40
30
20
10
0
Düsentriebwerk
25 m Entfernung
Start von Düsenmaschinen
in 100 m Entfernung
Brüel & Kjær
Schwerlast-
verkehr
mittlerer
Straßenverkehr
Unterhaltung
typische
Büroarbeit
Wohnraum
Schlafzimmer
Hörgrenze

Mathematische Ergänzung

10:00 Raum: Chemie HS3

Thema:

Wellengleiching => Schröginger-Gleichung

Elektronen - Atome - Orbitale