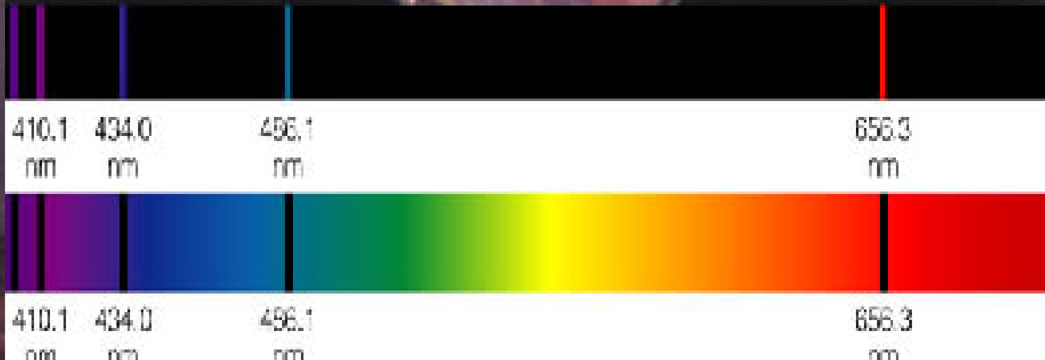
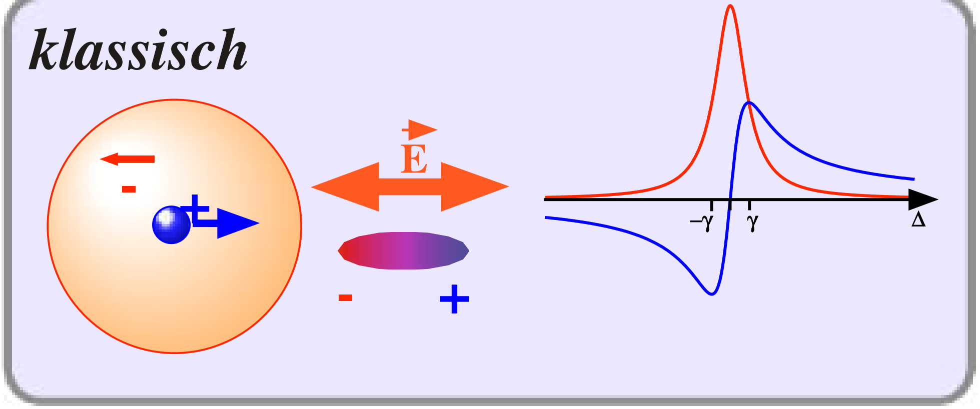
# 3) Licht-Materie Wechselwirkung






# Inhalt

3.1 Klassische Dispersionstheorie

3.2 Das Jaynes-Cummings Modell

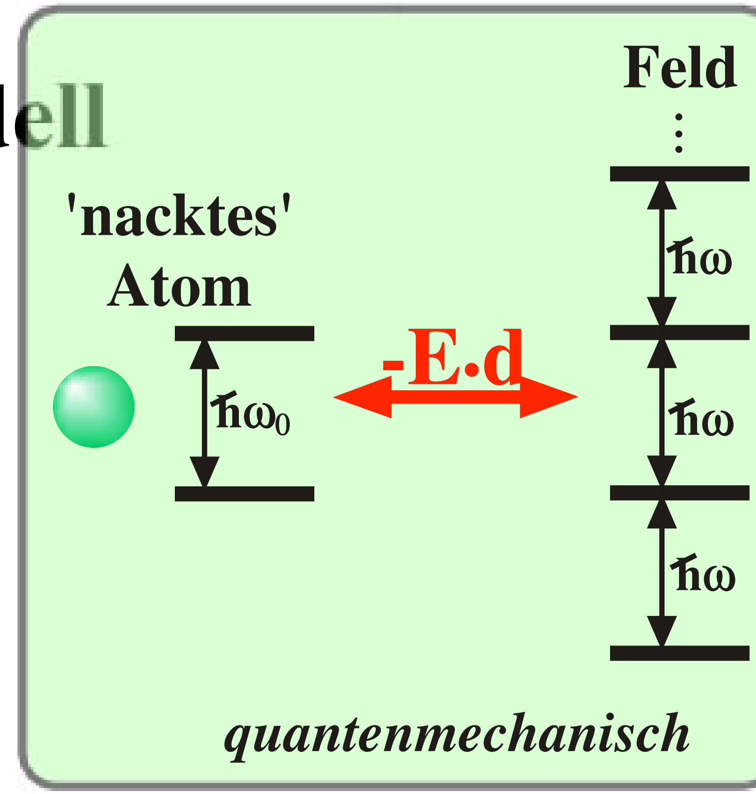
3.3 Das Zweiniveaumodell

3.4 Der Dichteoperator

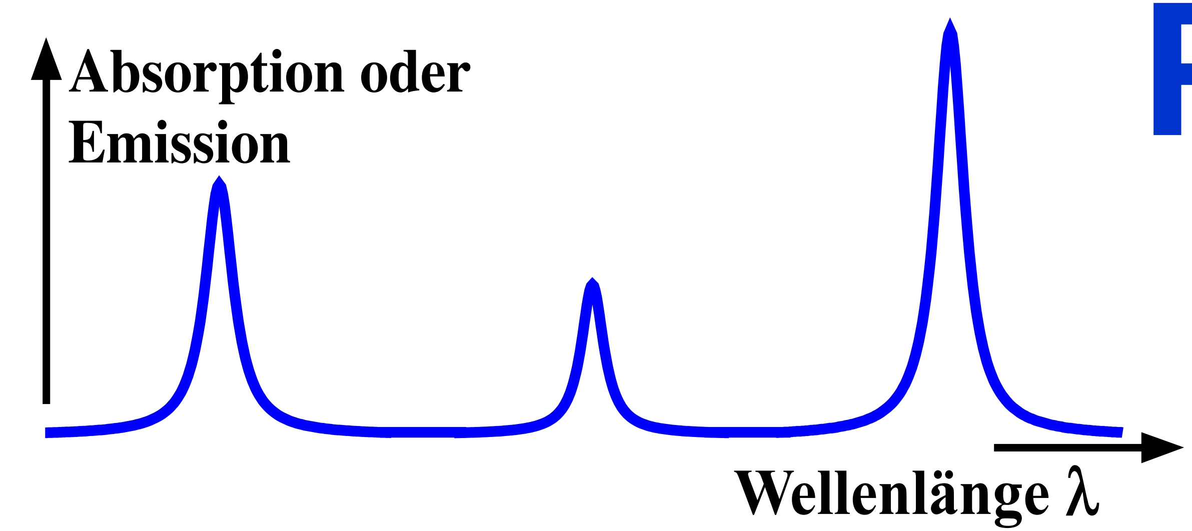
3.5 Optische Blochgleichung

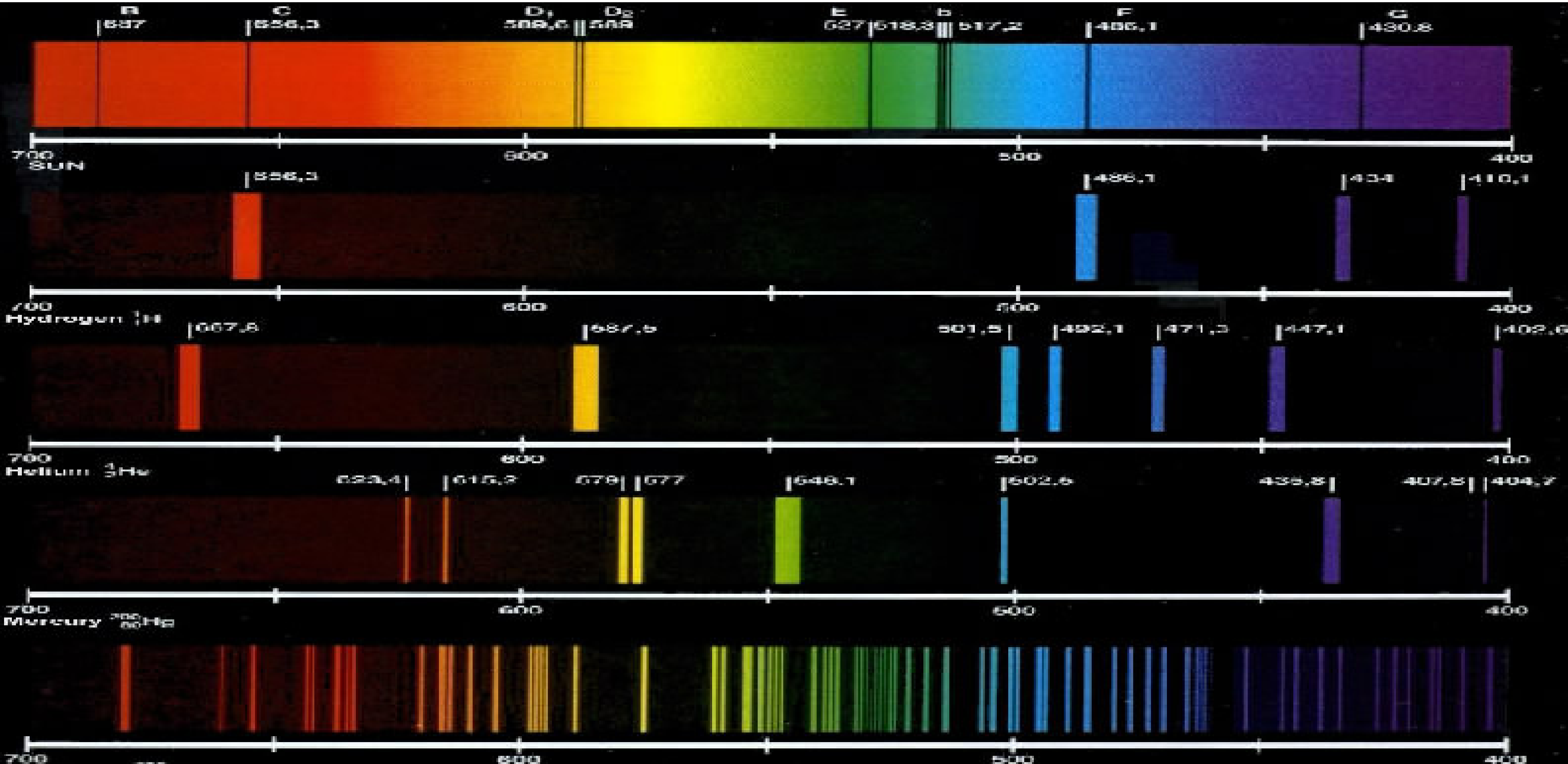
3.6 Laserpulse

3.7 Stationäre Lösung



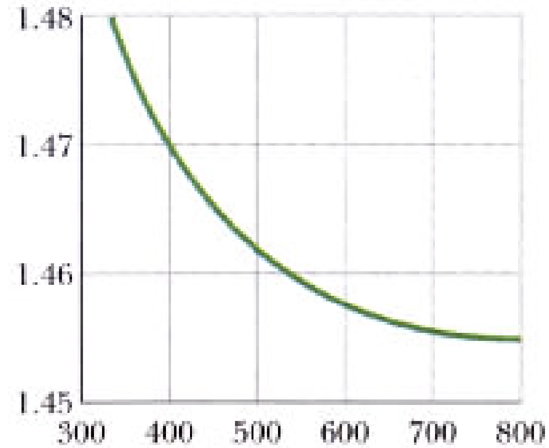
# Resonanz


Absorption oder
Emission
Wellenlänge λ


700
600
500
400
SUN
Hydrogen
Helium
Mercury

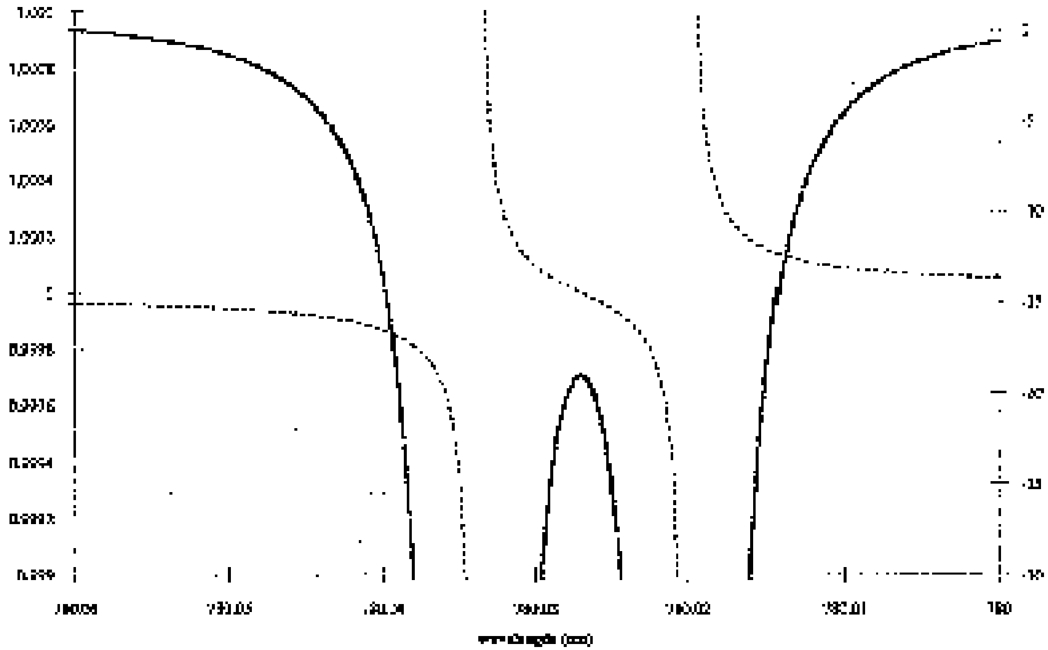
# Resonante Dispersion

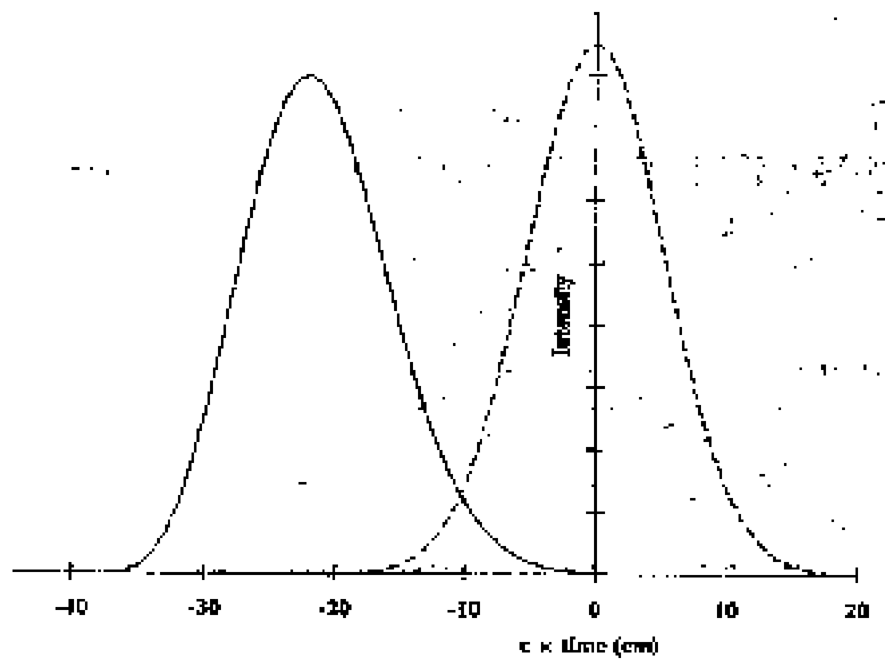
**Brechungsindex von Quartz**




**Wellenlänge / nm**

**Brechungsindex in Rb**

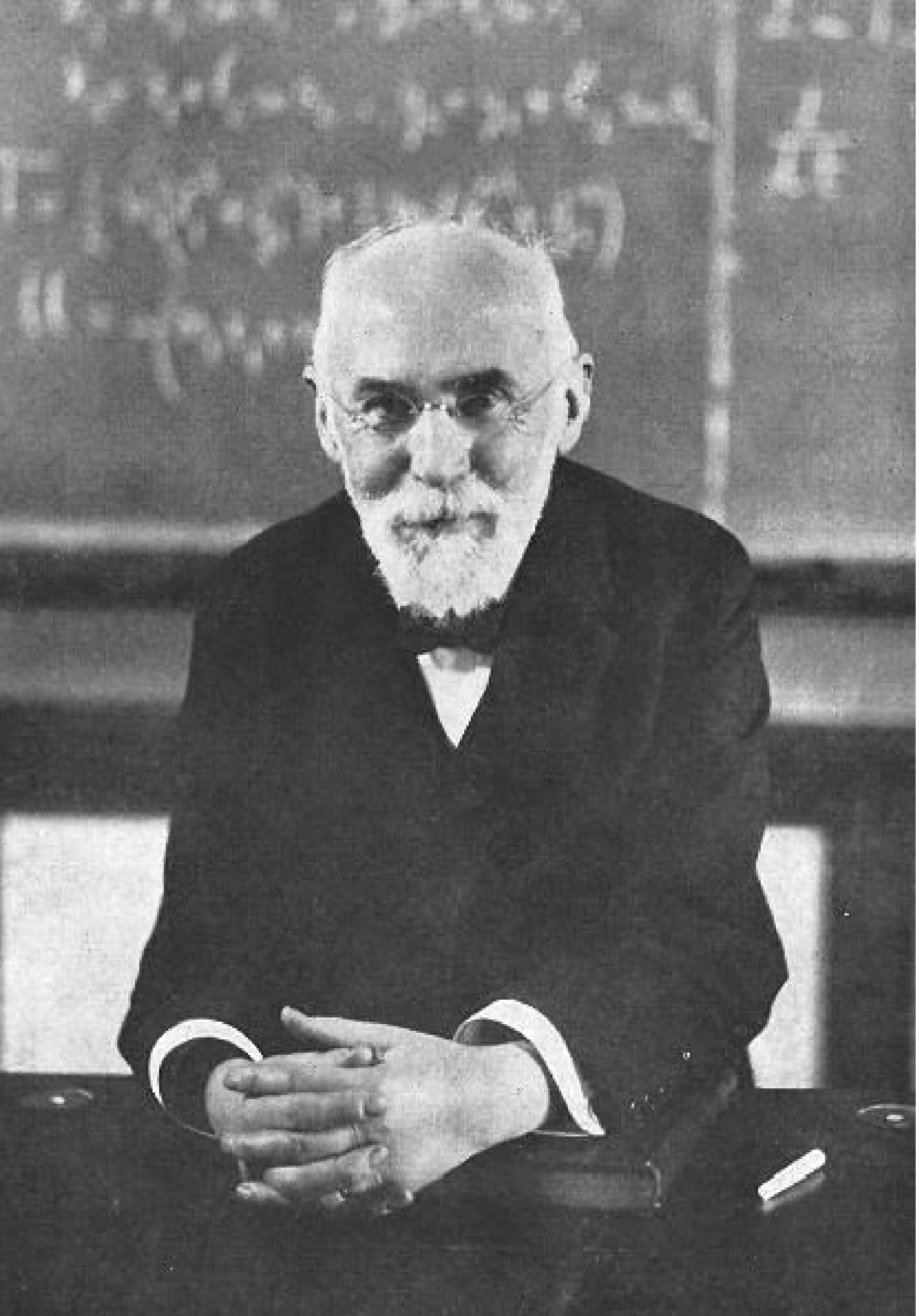





**Wellenlänge / nm**

**c · Zeit / cm**

# Lorentz



**Hendrik Antoon Lorentz**

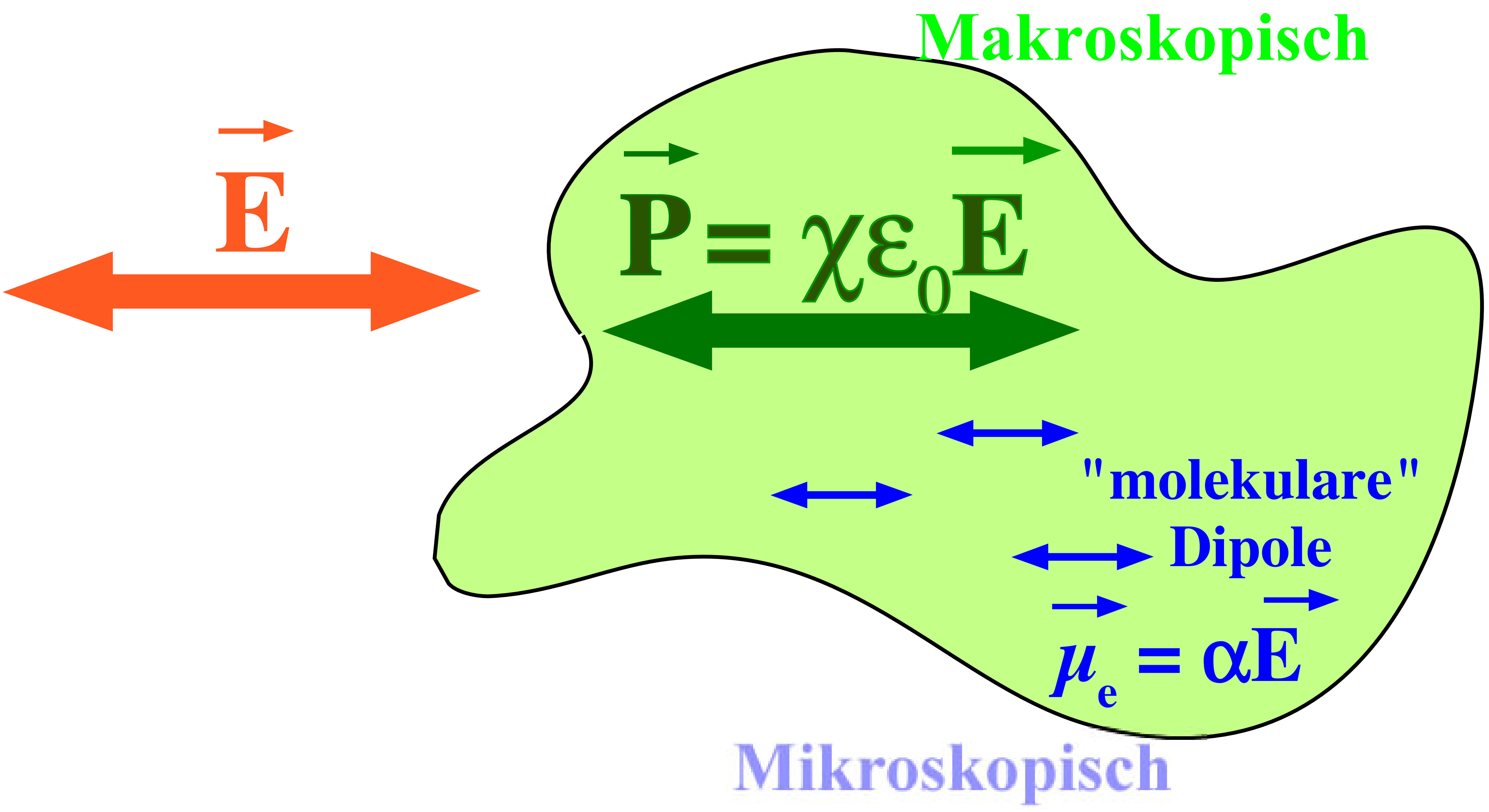
**1853–1928**

**1902 Nobelpreis für Physik**

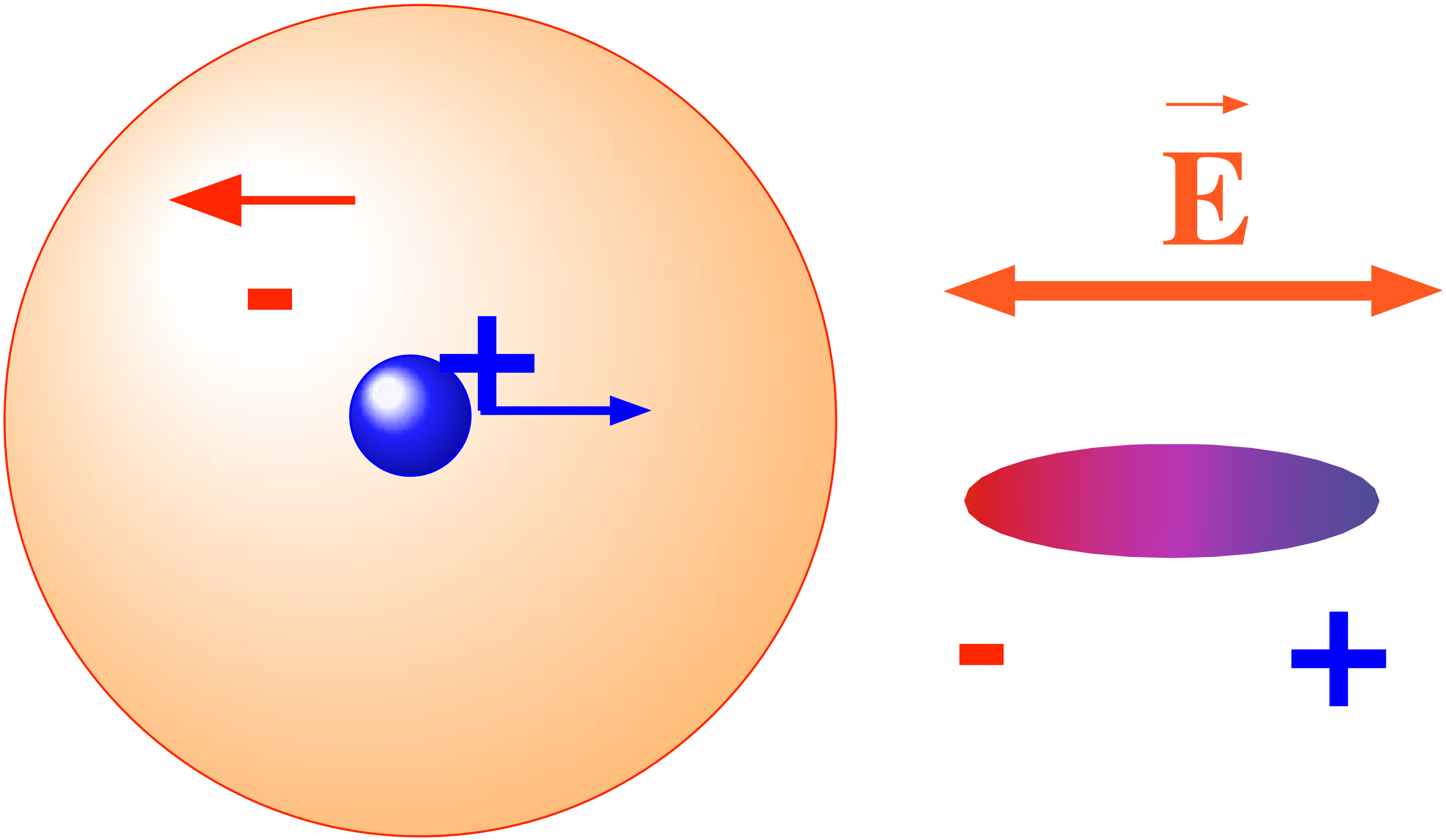
**Postulierte das Elektron**

**Lorentz-Transformation**

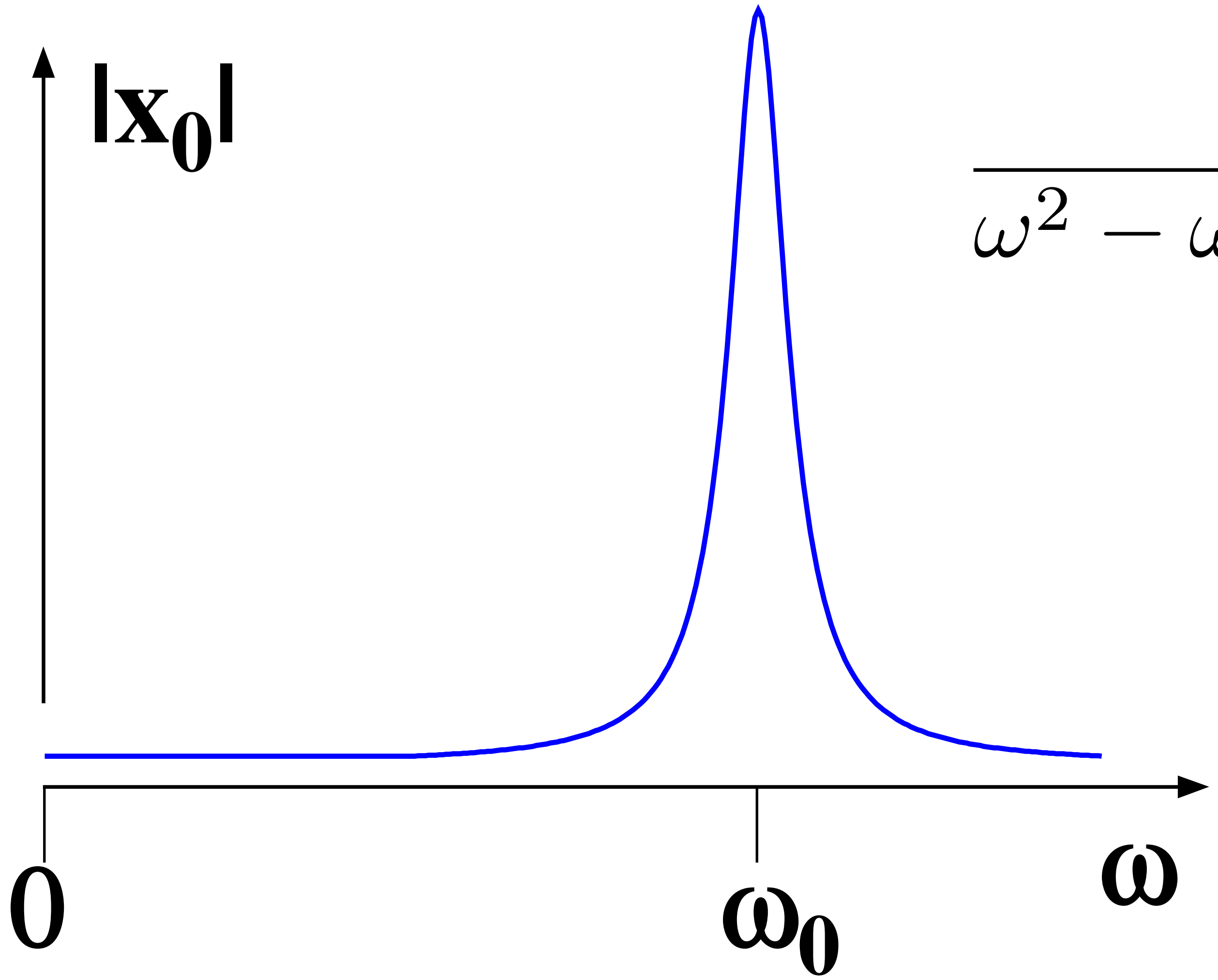
# Polarisation



# Dipol



# Resonanz



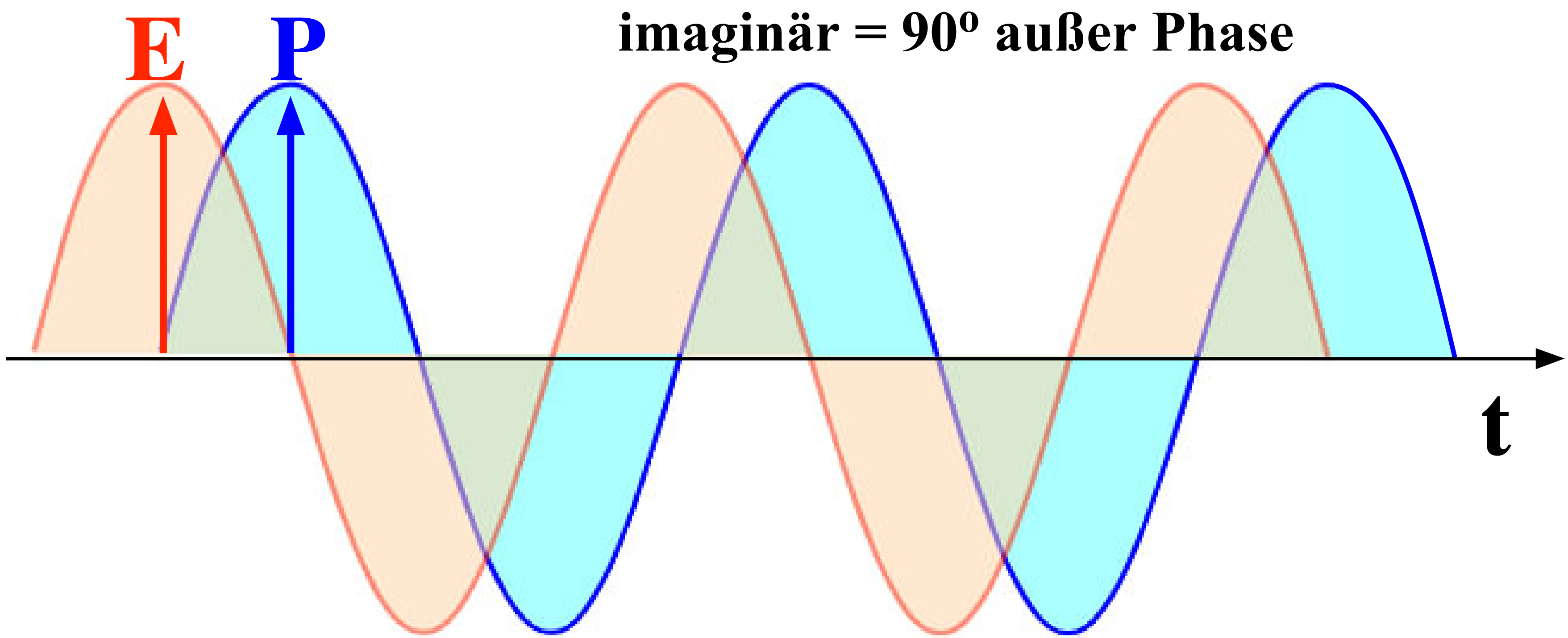

$$\frac{1}{\omega^2 - \omega_0^2 - 2i\gamma\omega}$$

# Zeitabhängige Polarisation

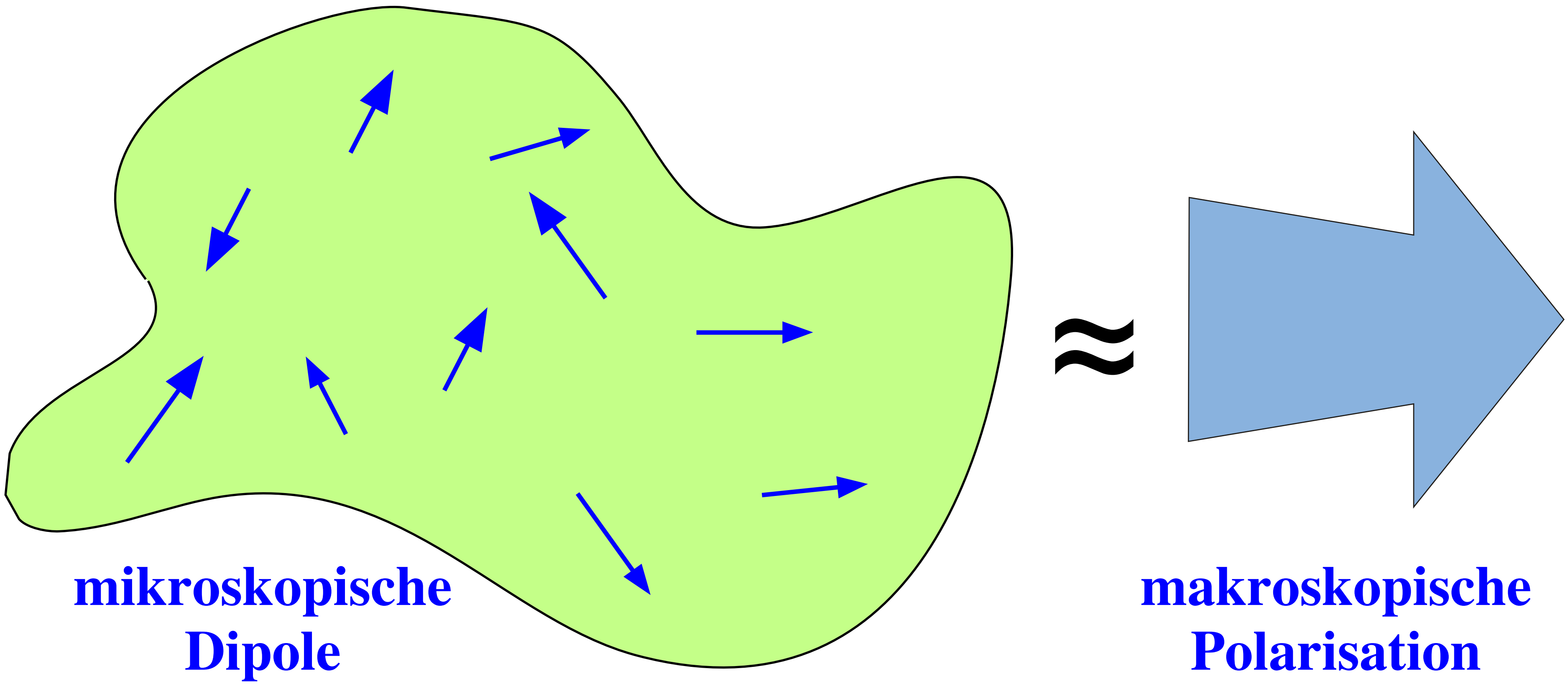
**Amplitude:** $$x_0 = \frac{eE_0}{2m\omega_0} \frac{\Delta + i\gamma}{\Delta^2 + \gamma^2}$$

**resonante Amplitude :** $$x_0(\Delta = 0) = i \, \frac{eE_0}{2m\gamma\omega_0}$$

**imaginär = 90° außer Phase**

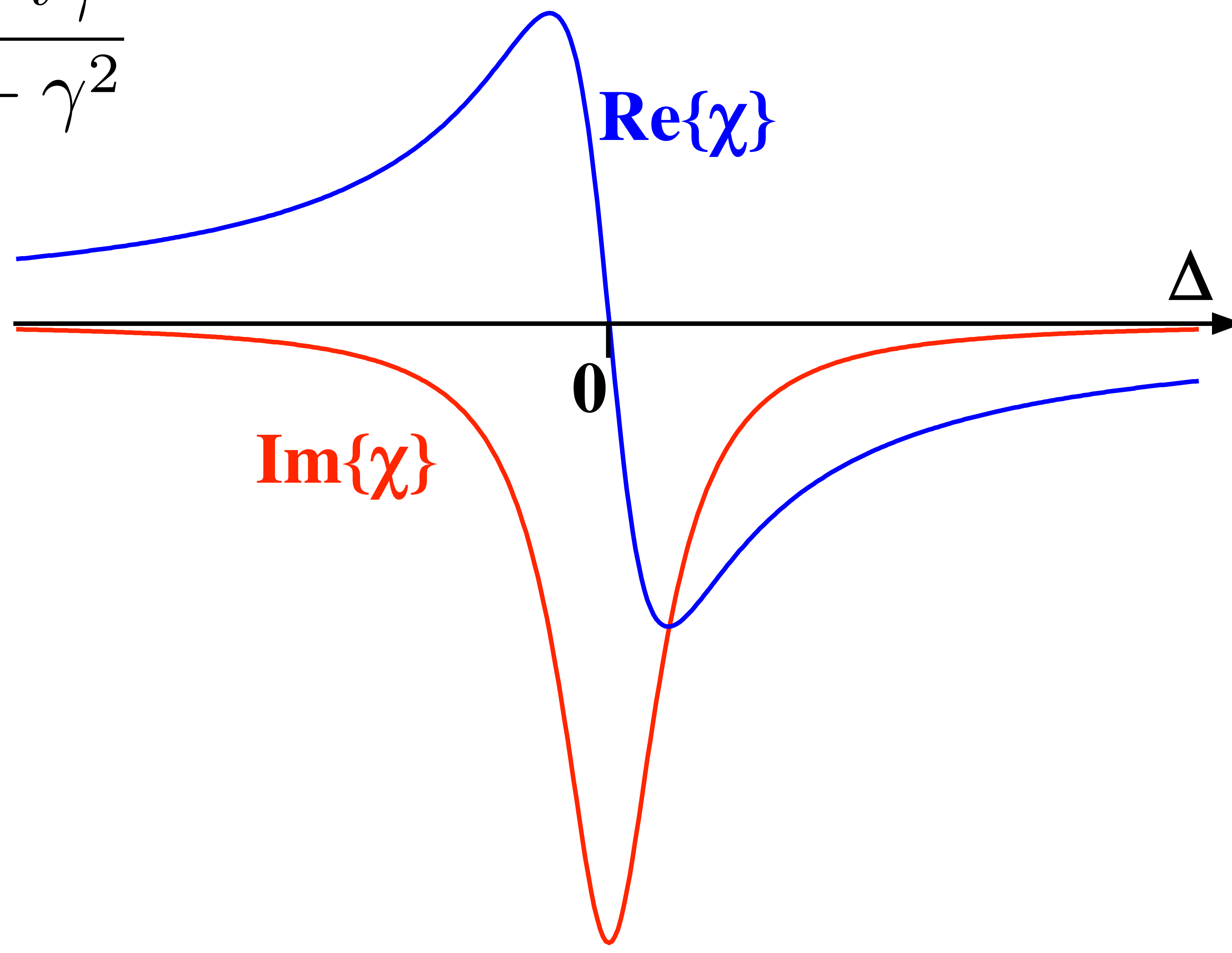


# Dipole und Polarisation

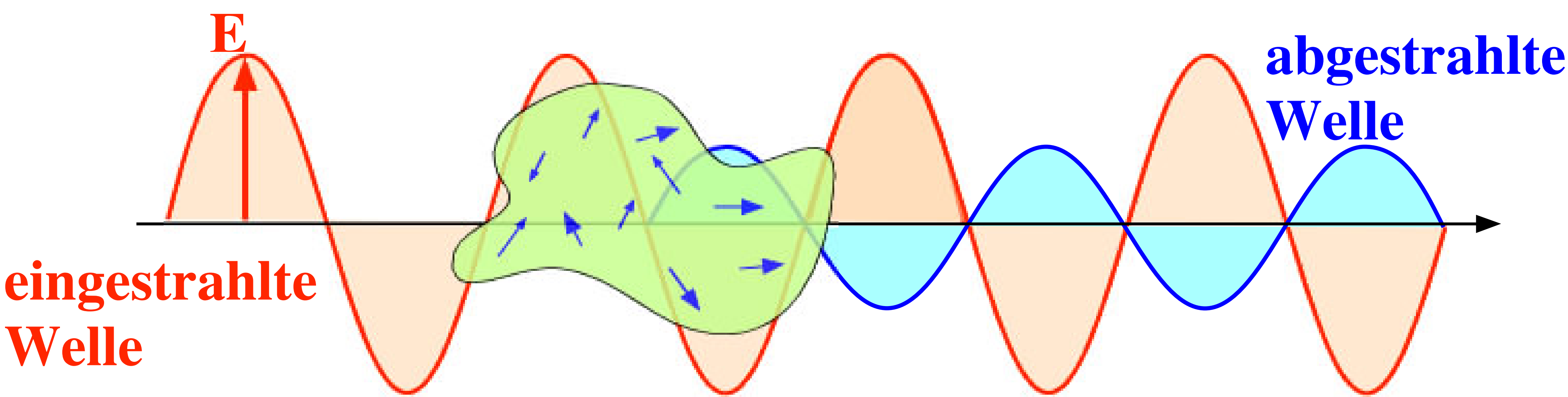


# Komplexe Suszeptibilität

$$\chi \propto -\frac{\Delta + i\gamma}{\Delta^2 + \gamma^2}$$



# Lichtausbreitung




**Maxwellgleichungen**

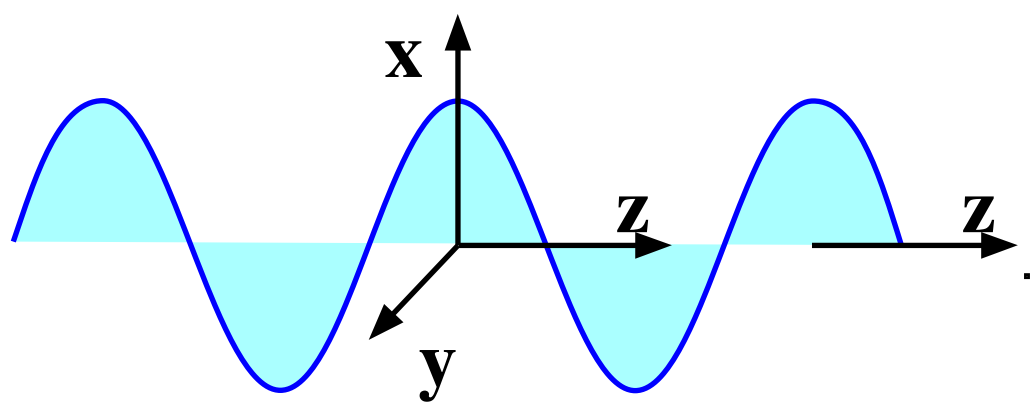
$$\vec{\nabla} \times \vec{H} = \frac{\partial}{\partial t}\vec{D}$$

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_e$$

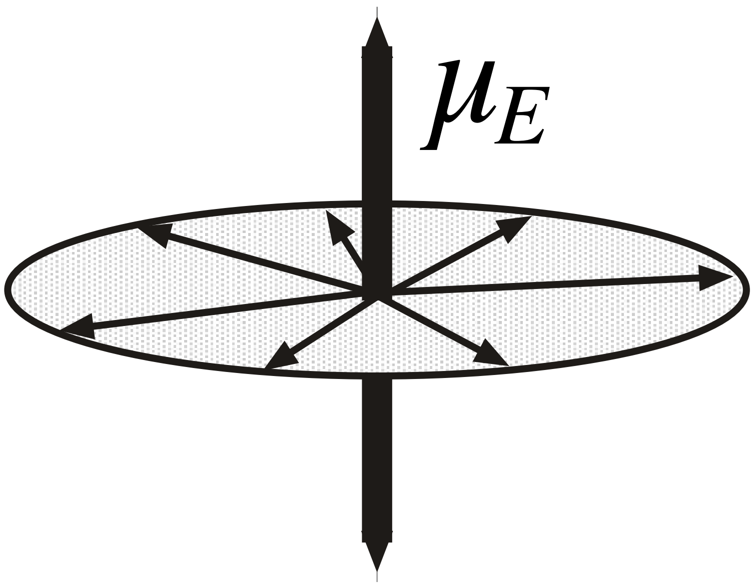
$$\vec{\nabla} \cdot \vec{B} = 0$$

**Lösungsansatz : ebene Welle || z**

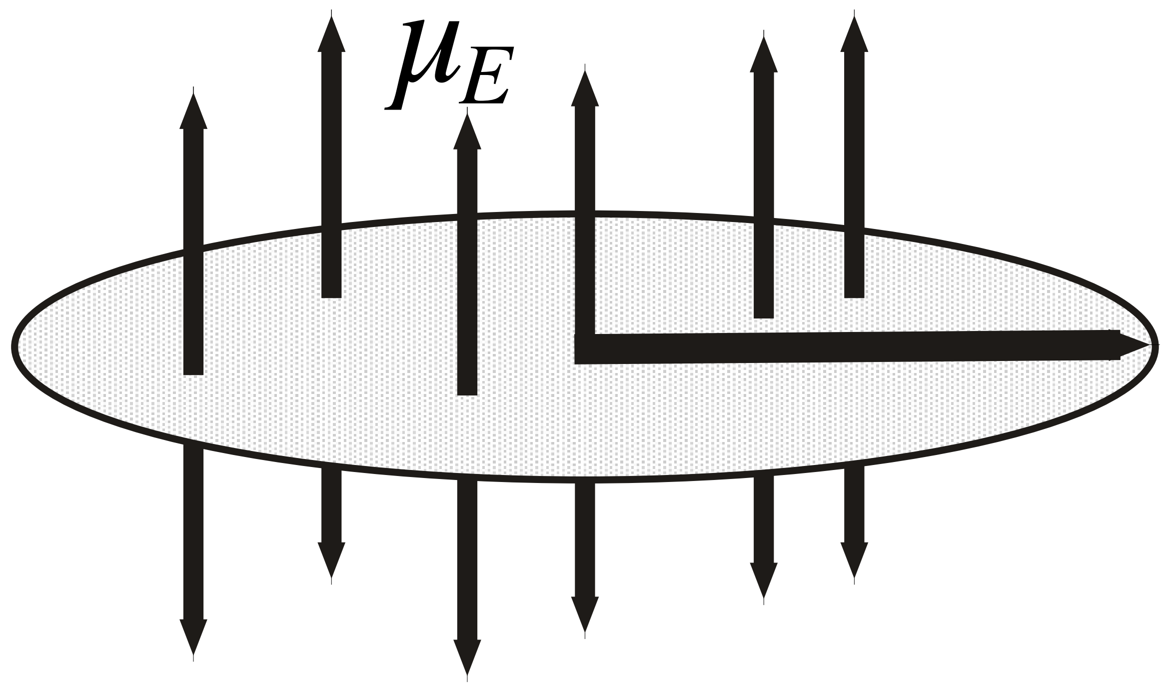


# Ausbreitung

**Einzelnes Atom**




**Ensemble**



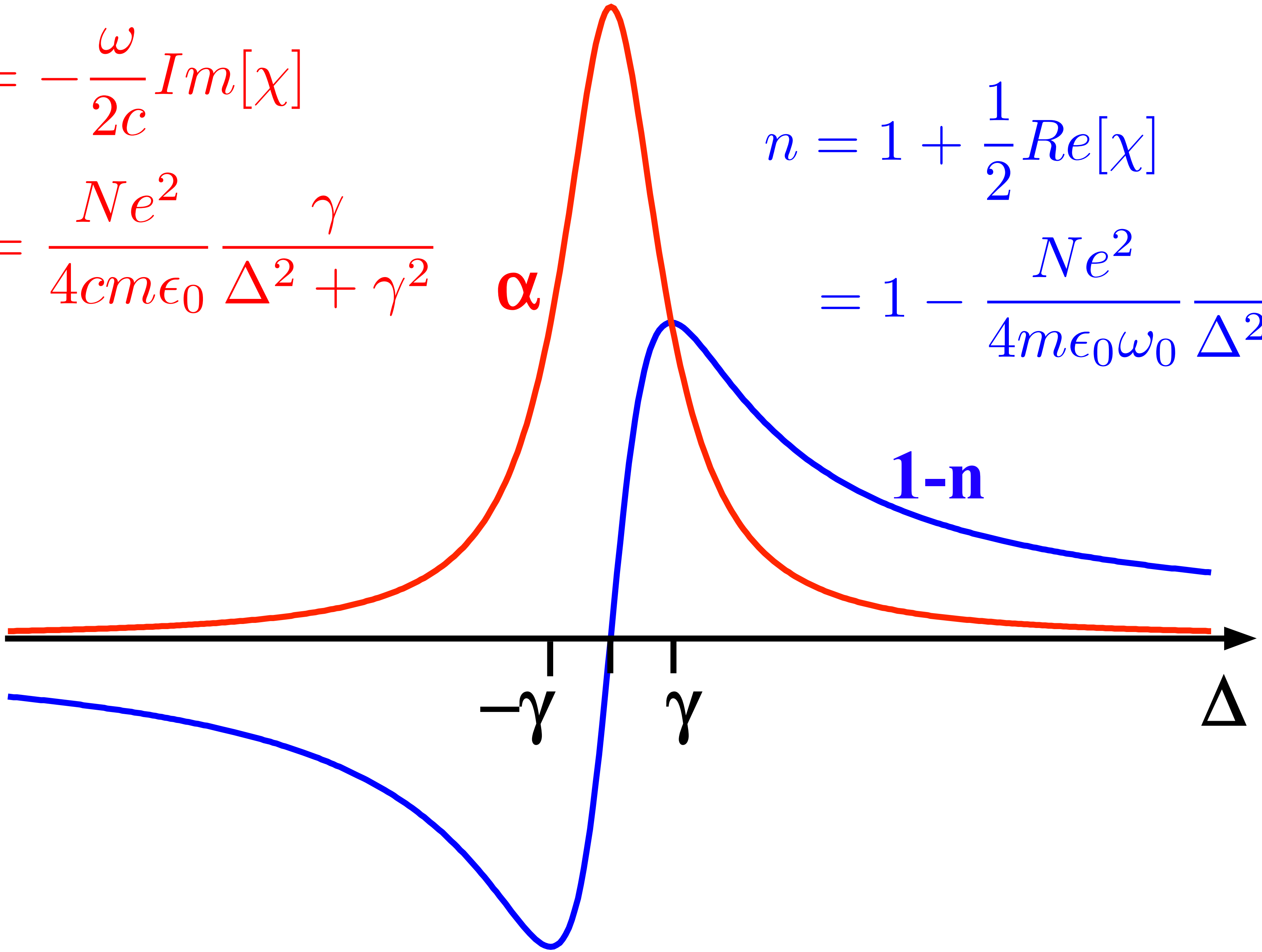
# Absorption und Dispersion

$$\alpha = -\frac{\omega}{2c} Im[\chi]$$

$$= \frac{Ne^2}{4cm\epsilon_0} \frac{\gamma}{\Delta^2 + \gamma^2}$$

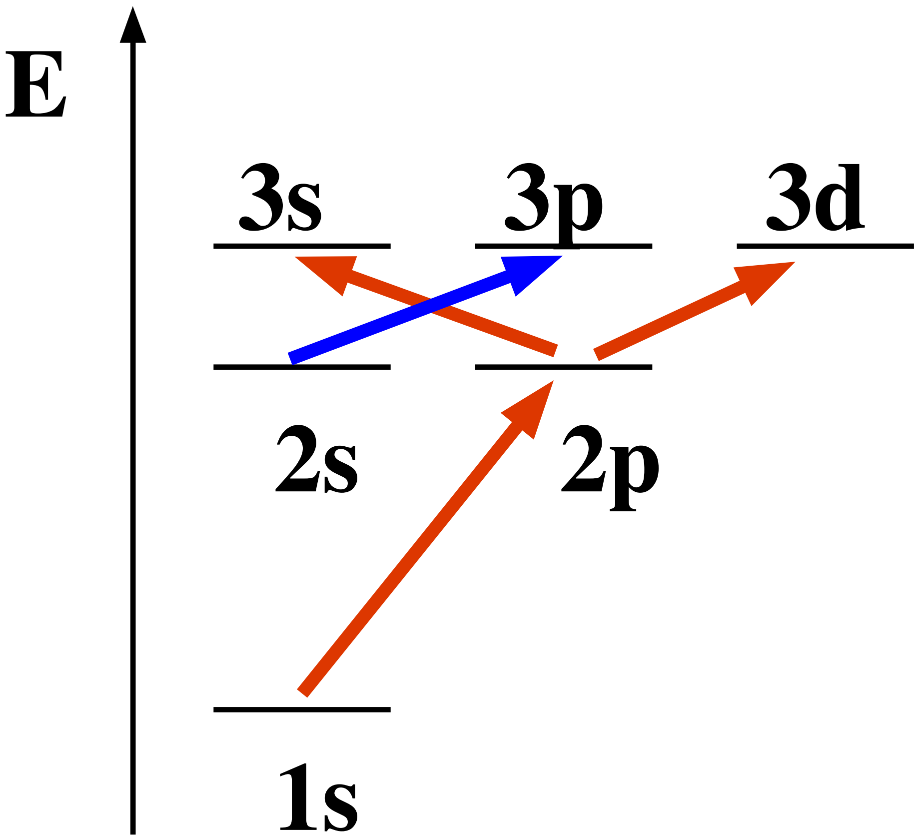
$$n = 1 + \frac{1}{2} Re[\chi]$$

$$= 1 - \frac{Ne^2}{4m\epsilon_0\omega_0} \frac{\Delta}{\Delta^2 + \gamma^2}$$

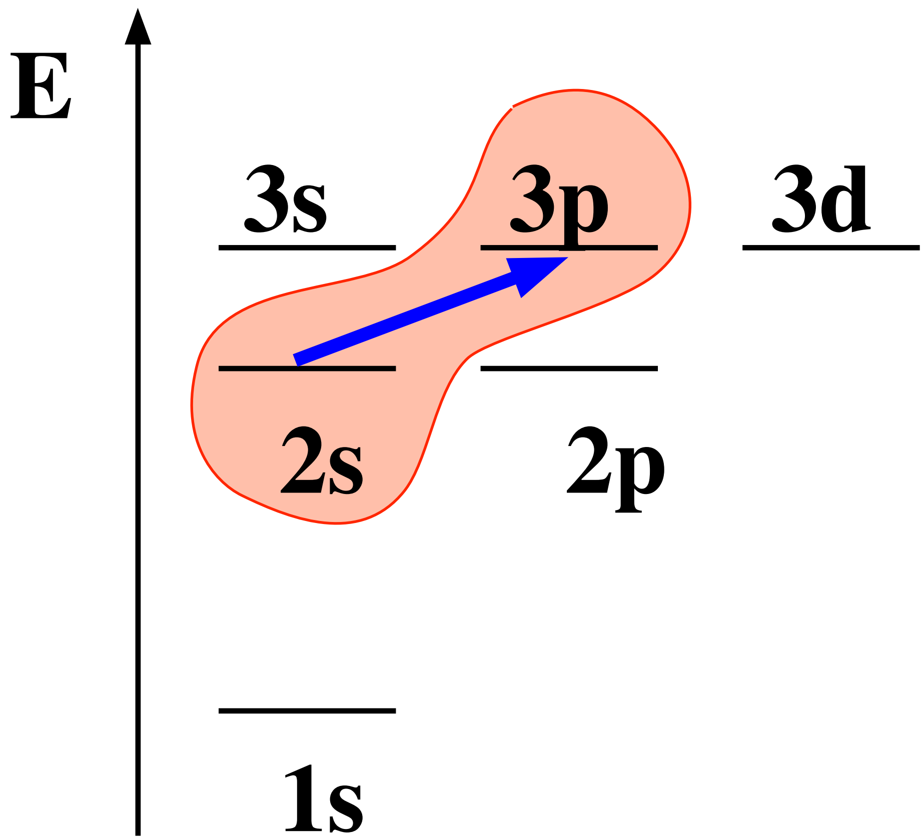


# 2-Niveauatome

**typ. atomare Niveaustruktur**

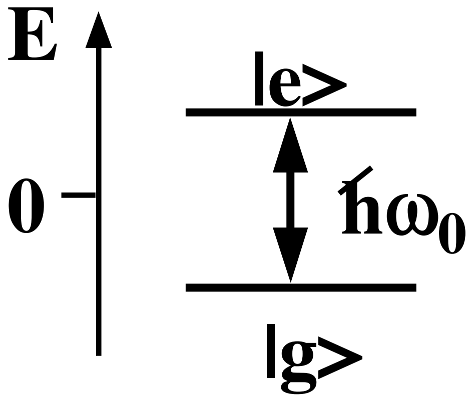



**WW mit 2 Zuständen**

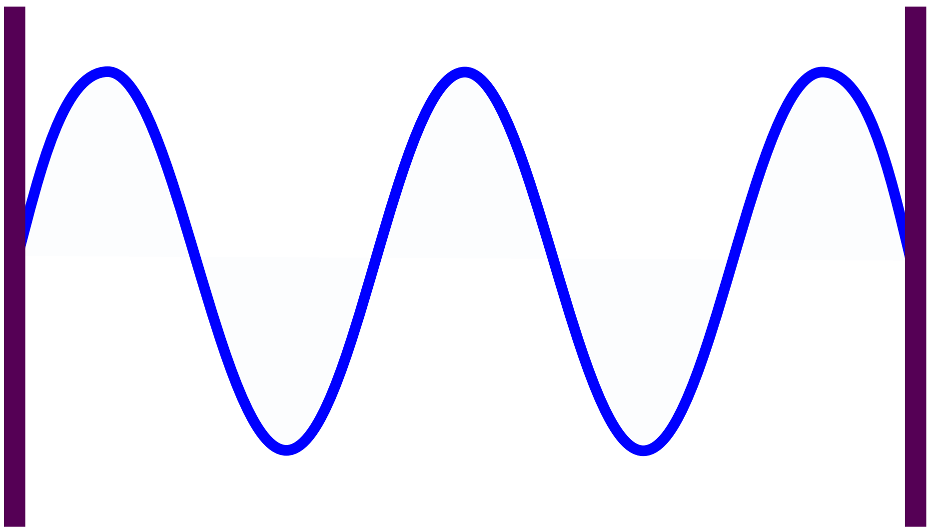


# Wechselwirkung

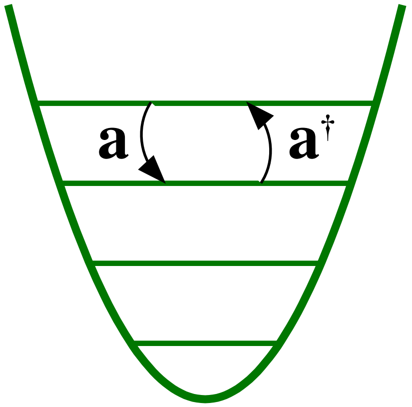
System:



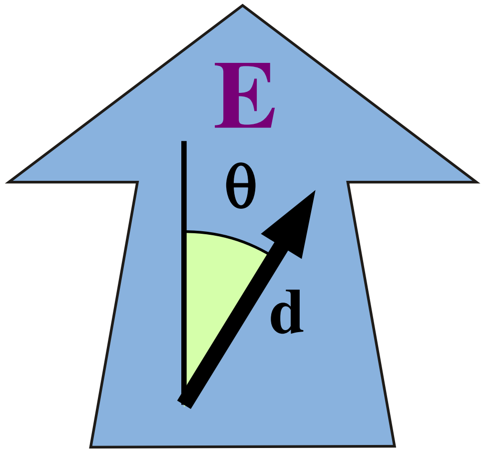

Resonatormode



harmonischer Oszillator

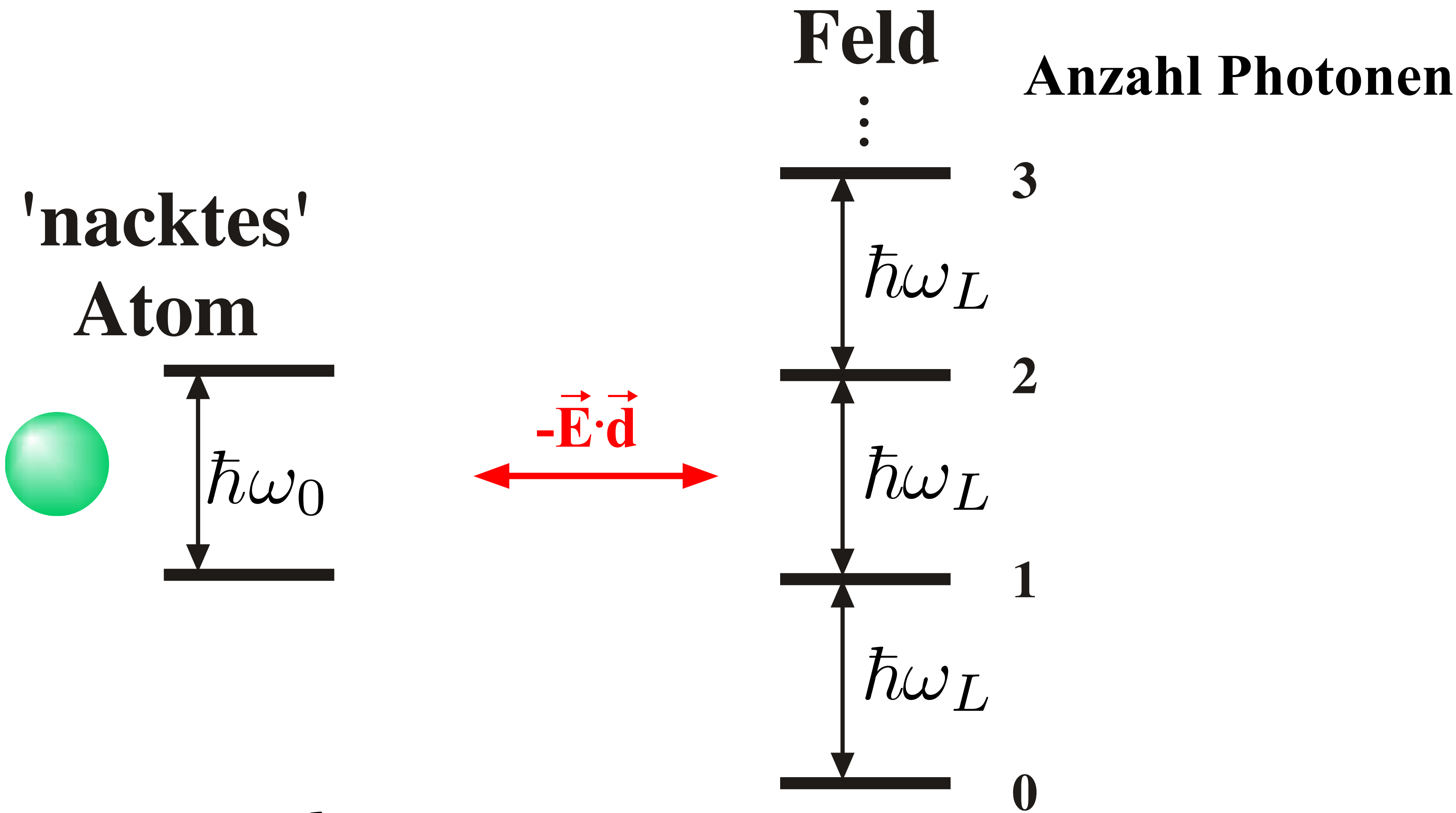



klass. Wechselwirkung



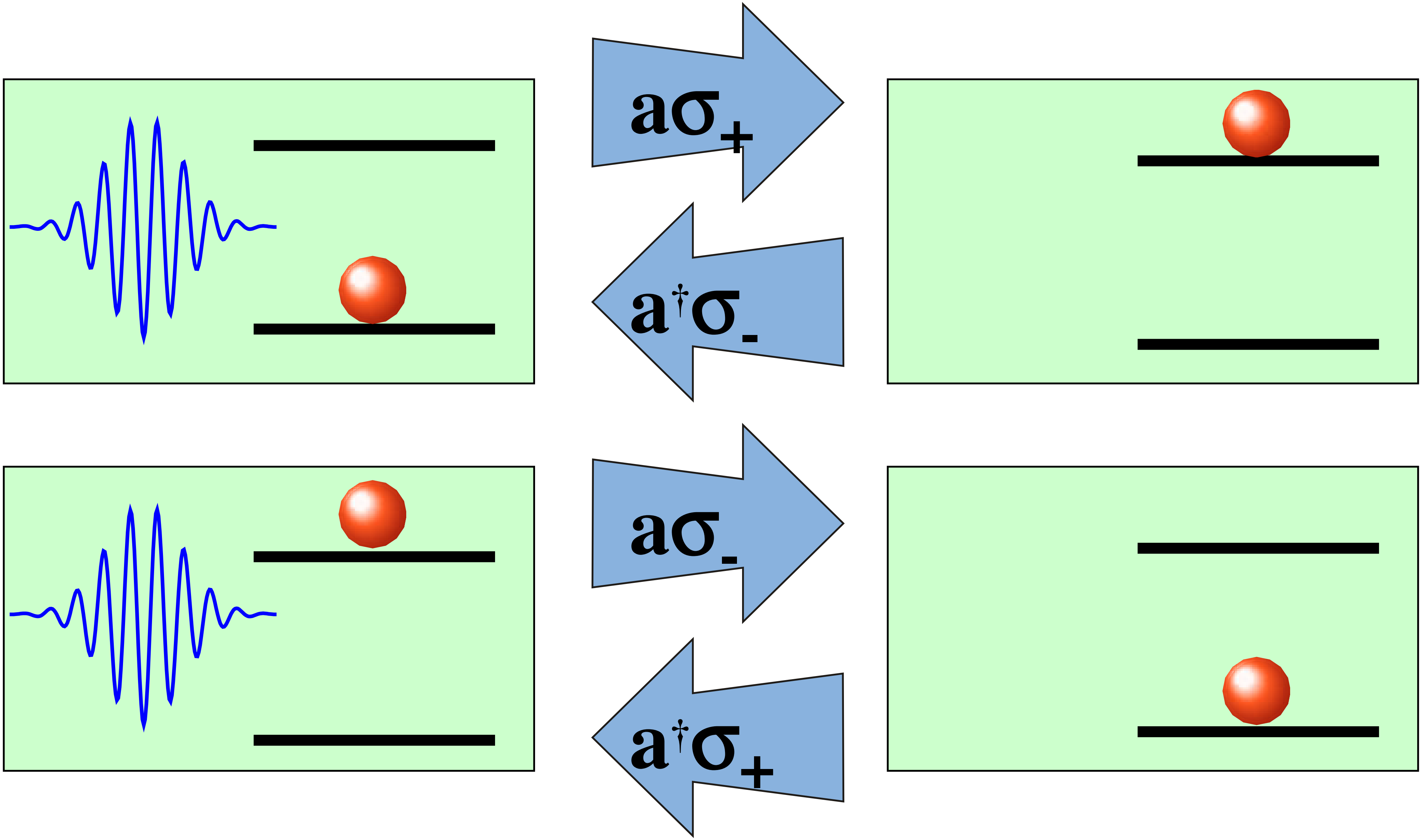
# Jaynes-Cumming Modell

*E.T. Jaynes and F.W. Cummings, Proc. IEEE 51, 89 (1963)*

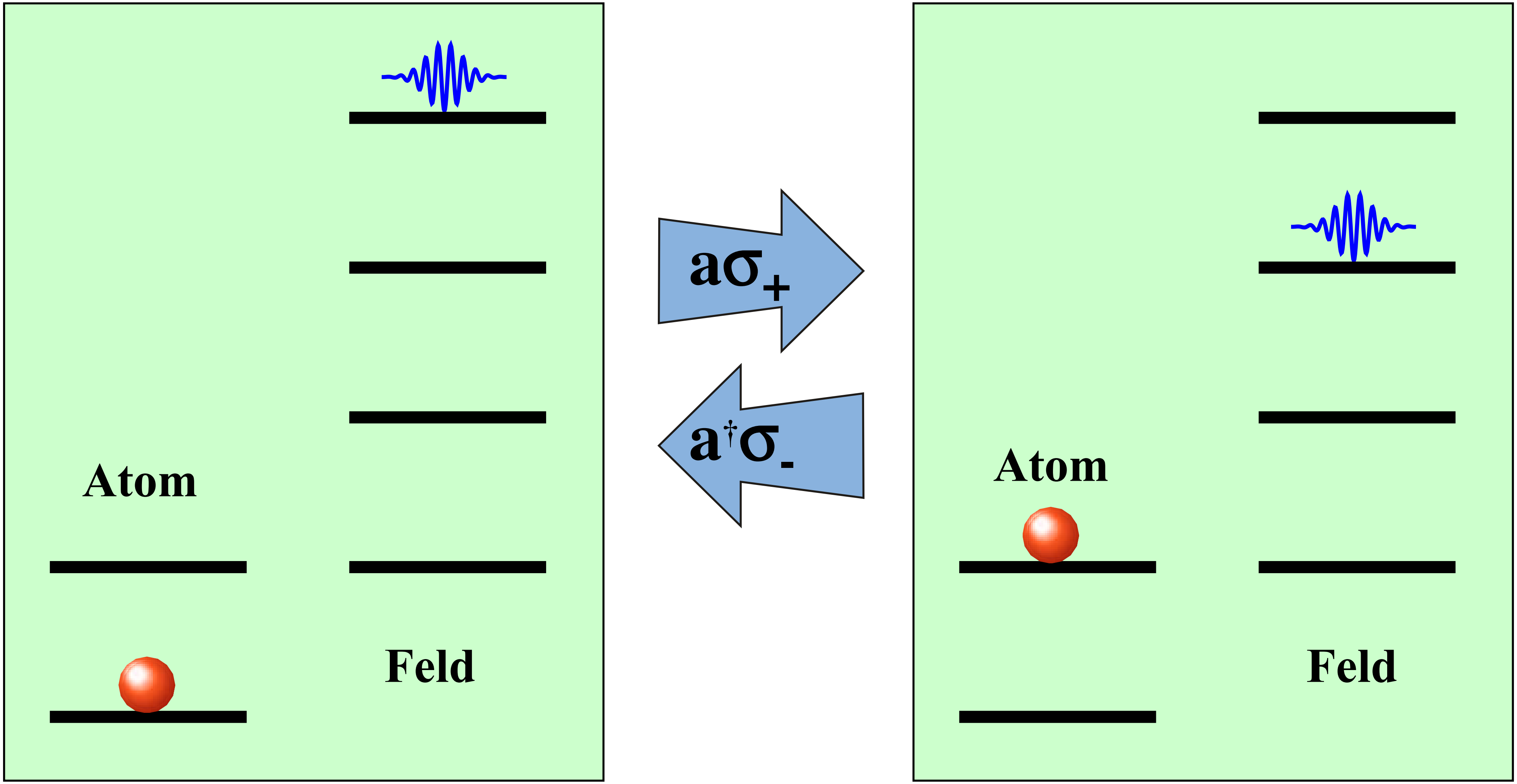



$$\mathcal{H}_{atom} = -\frac{\hbar}{2}\omega_0\sigma_z \qquad \mathcal{H}_{Feld} = \hbar\omega_L(a^\dagger a + \frac{1}{2})$$

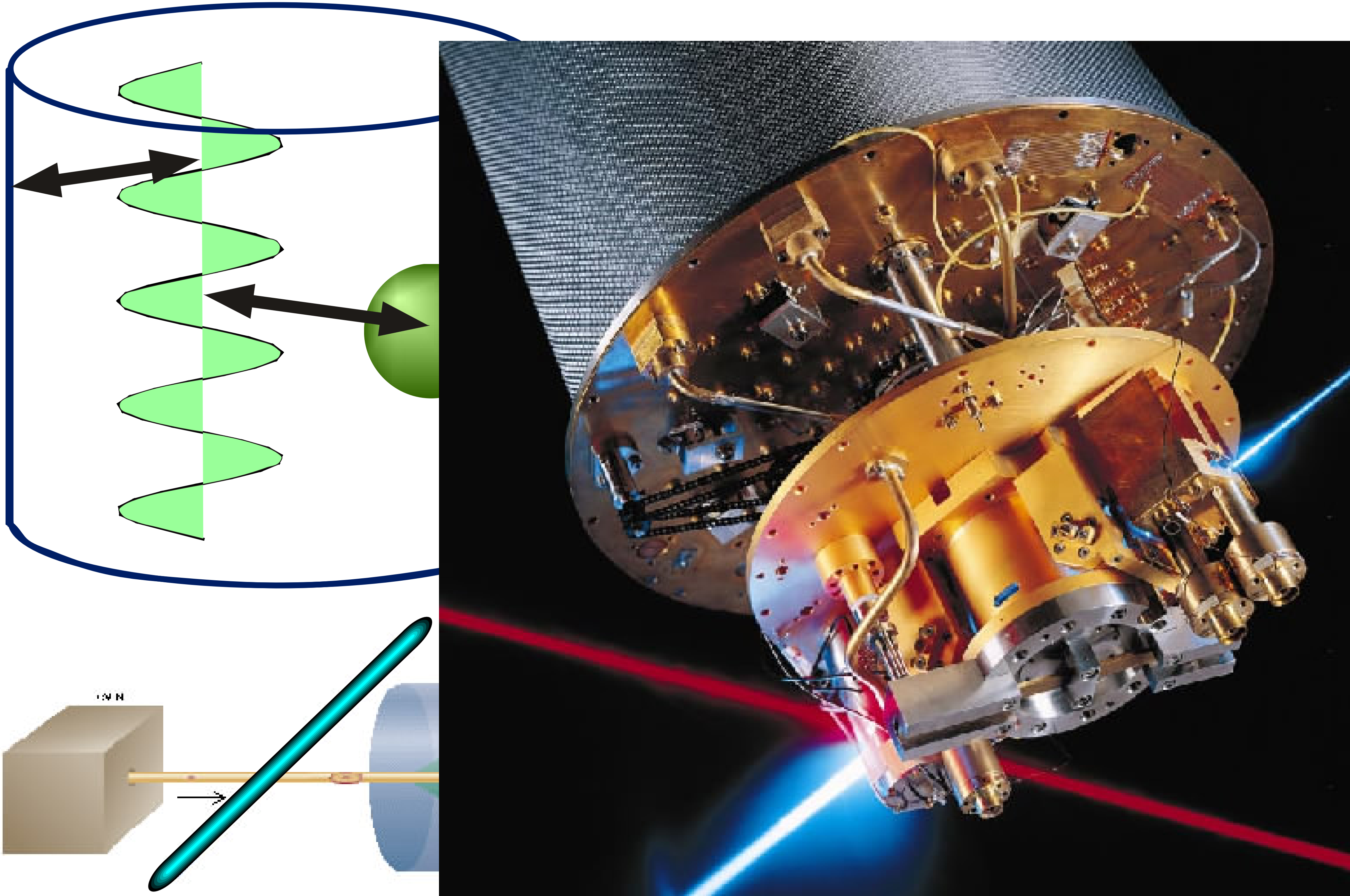
# WW-Prozesse



# WW-Prozess



# Experimentelle Realisierung



# Literatur


**P. Goy, J.M. Raimond, M. Gross, and S. Haroche, 'Observation of cavity-enhanced single-atom spontaneous emission', Phys. Rev. Lett. 50, 1903-1906 (1983).**

**H. Walther, 'The single atom maser and the quantum electrodynamics in a cavity', Physica Scripta T23, 165-169 (1988).**

**S. Haroche and D. Kleppner, 'Cavity quantum electrodynamics', Physics Today January 1989, 24-30 (1989).**

**E.A. Hinds, 'Cavity Quantum Electrodynamics', in Adv. atomic, mol. opt. phys. 28, Editor: D. Bates, Academic Press, Boston (1991).**

**S. Haroche, 'Cavity Quantum Electrodynamics', in Fundamental Systems in Quantum Optics; Proceedings of the Les Houches summer scool, Editor: J. Dalibard, J.M. Raimond, and J. Zinn-Justin, North-Holland, Amsterdam (1992).**

**H. Walther, 'Experiments on cavity quantum electrodynamics', Phys. Rep. 219, 263-281 (1992).**

# Hamiltonoperator

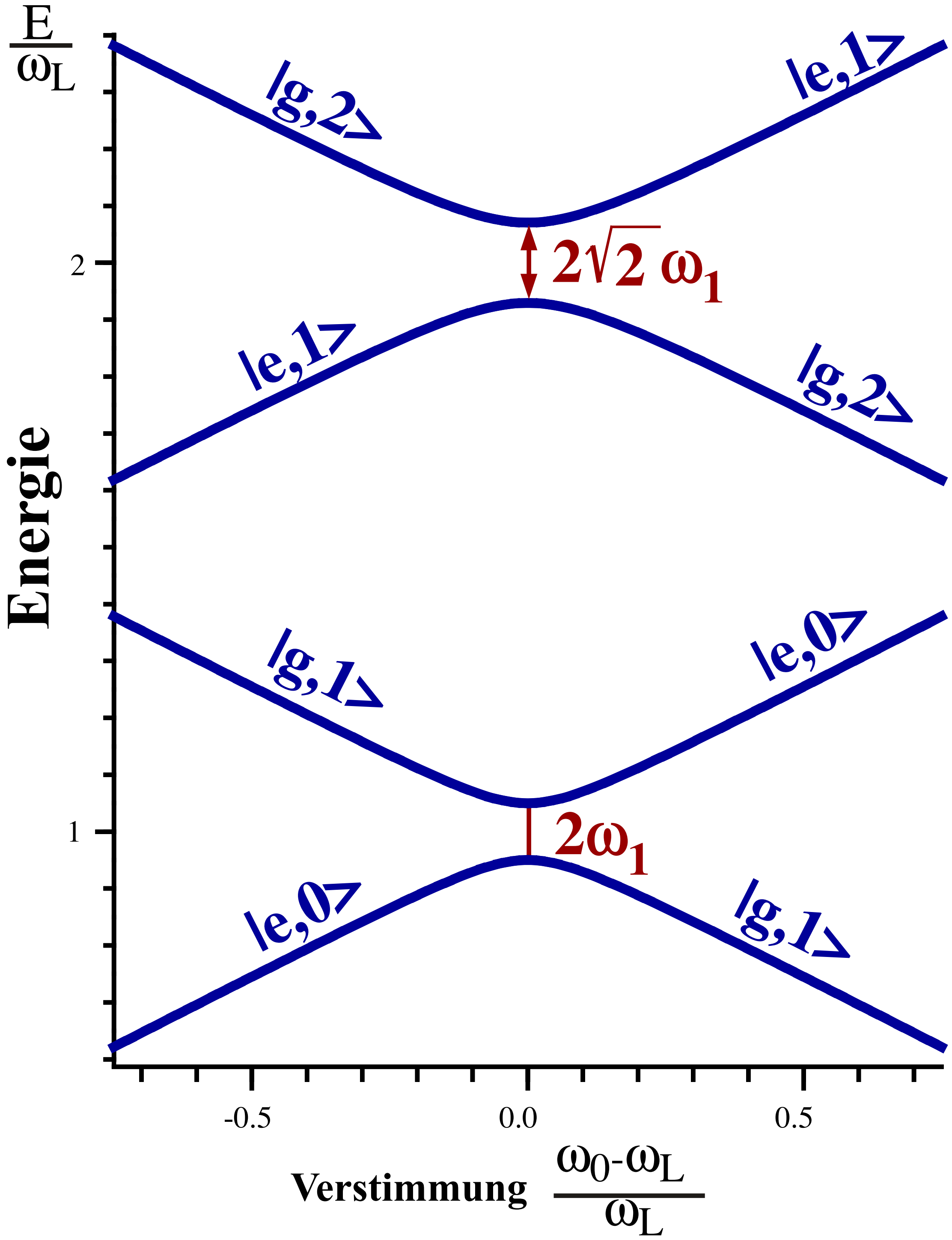
$$\mathcal{H}_n = \hbar \begin{pmatrix} (n+\frac{1}{2})\omega_L - \frac{\omega_0}{2} & \omega_1\sqrt{n} \\ \omega_1\sqrt{n} & (n-\frac{1}{2})\omega_L + \frac{\omega_0}{2} \end{pmatrix}$$

$$\begin{pmatrix} \boxed{\mathcal{H}_\mathbf{n}} & 0 & 0 \\ 0 & \boxed{\mathbf{H}_{\mathbf{n+1}}} & 0 \\ 0 & 0 & \boxed{\mathbf{H}_{\mathbf{n+2}}} \end{pmatrix}$$

| **\|Ψ>** | $\mathbf{E_0}$ |
|---|---|
| \|n,g> | |
| \|n-1,e> | |
| \|n+1,g> | $(n+\frac{3}{2})\,\omega_L - \frac{\omega_0}{2}$ |
| \|n,e> | $(n+\frac{1}{2})\,\omega_L + \frac{\omega_0}{2}$ |
| \|n+2,g> | |
| \|n+1,e> | |

**n = 1, 2, 3, ...**

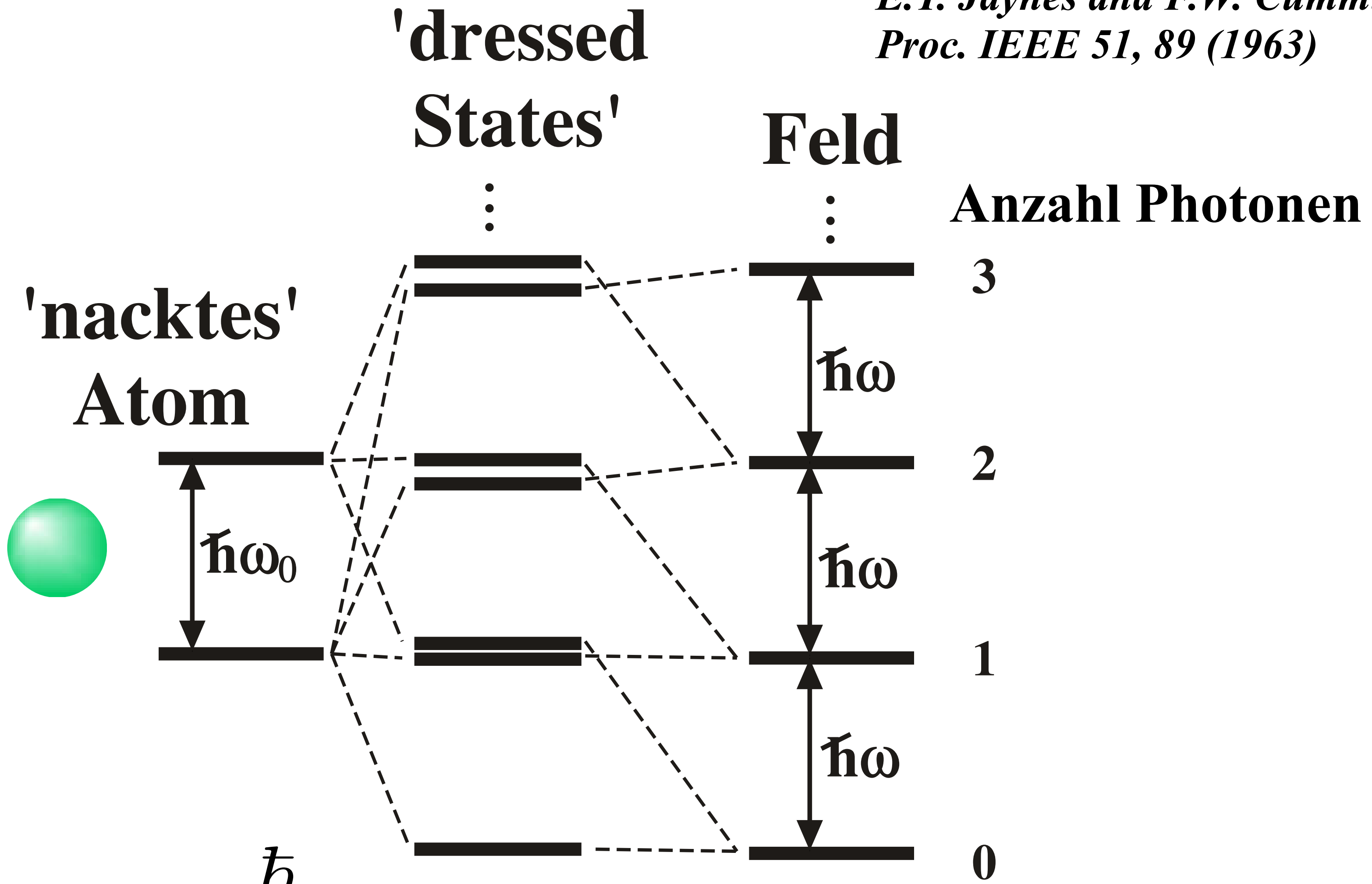
# Energien



$\frac{E}{\omega_L}$
Energie
2
1
|g,2>
|e,1>
|e,1>
|g,2>
$2\sqrt{2}\,\omega_1$
|g,1>
|e,0>
|e,0>
|g,1>
$2\omega_1$
-0.5
0.0
0.5
Verstimmung $\frac{\omega_0-\omega_L}{\omega_L}$

# Jaynes-Cumming Modell

*E.T. Jaynes and F.W. Cummings, Proc. IEEE 51, 89 (1963)*




$$\mathcal{H}_{atom} = -\frac{\hbar}{2}\omega_0\sigma_z \qquad \mathcal{H}_{Feld} = \hbar\omega_L(a^\dagger a + \frac{1}{2})$$

$$\mathcal{H}_{ww} \approx \hbar\omega_1(a\sigma_+ + a^\dagger\sigma_-)$$

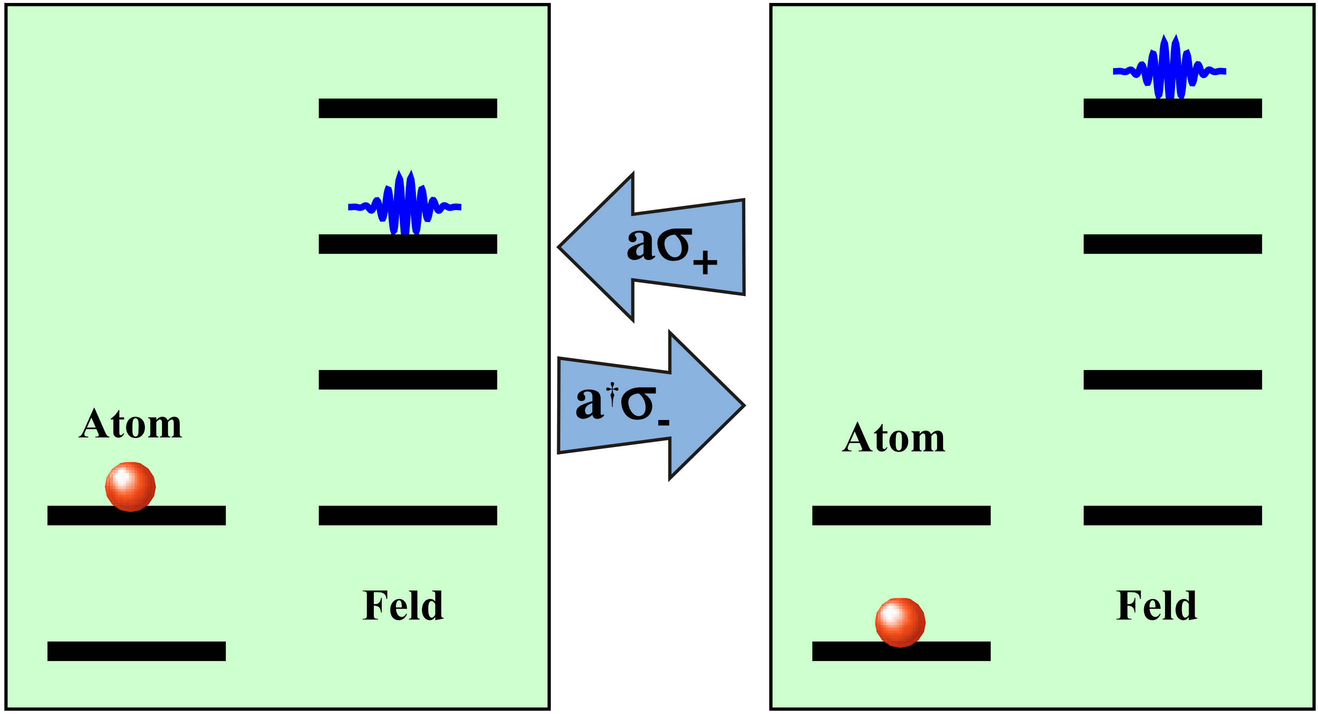
Übung

# Dynamik des JC Systems

$$\omega_L = \omega_0 \quad \Rightarrow \quad \mathcal{H}_n = \hbar \begin{pmatrix} n\omega_0 & \omega_1\sqrt{n} \\ \omega_1\sqrt{n} & n\omega_0 \end{pmatrix}$$

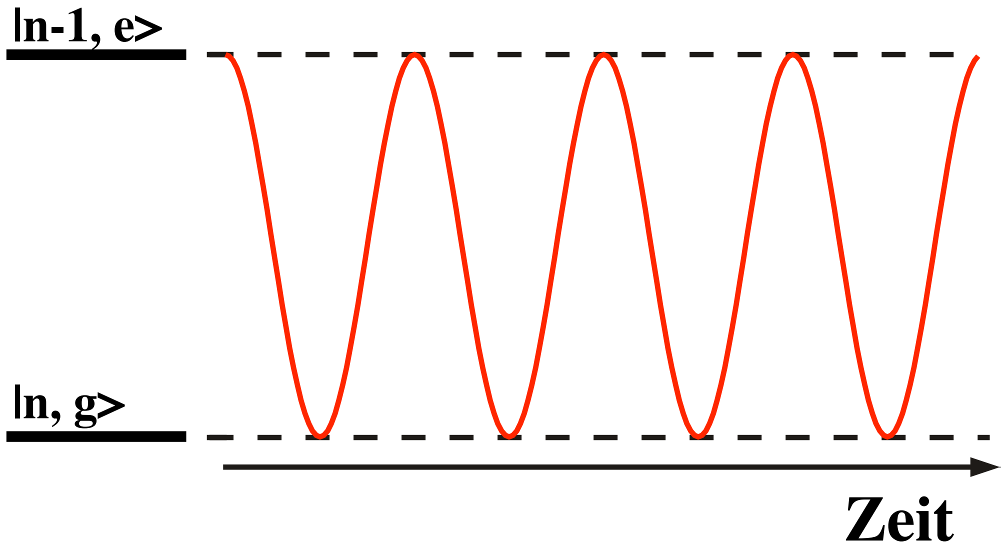
$$\psi(0) = |n, e\rangle$$

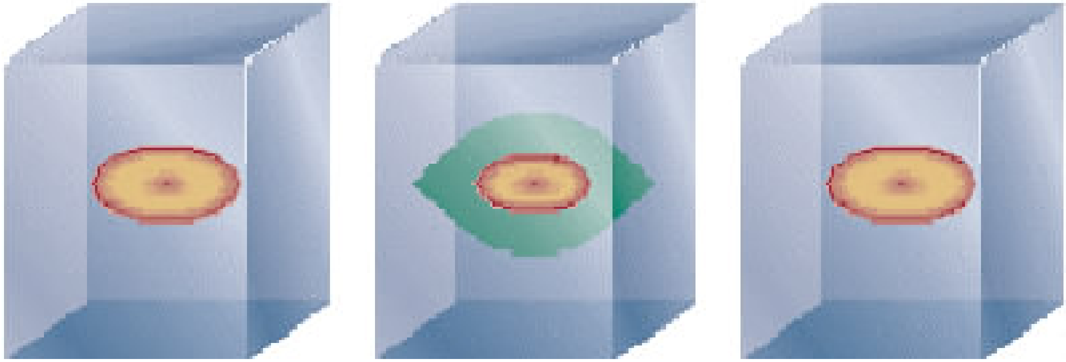
$$\psi(t) = ?$$



# Evolution

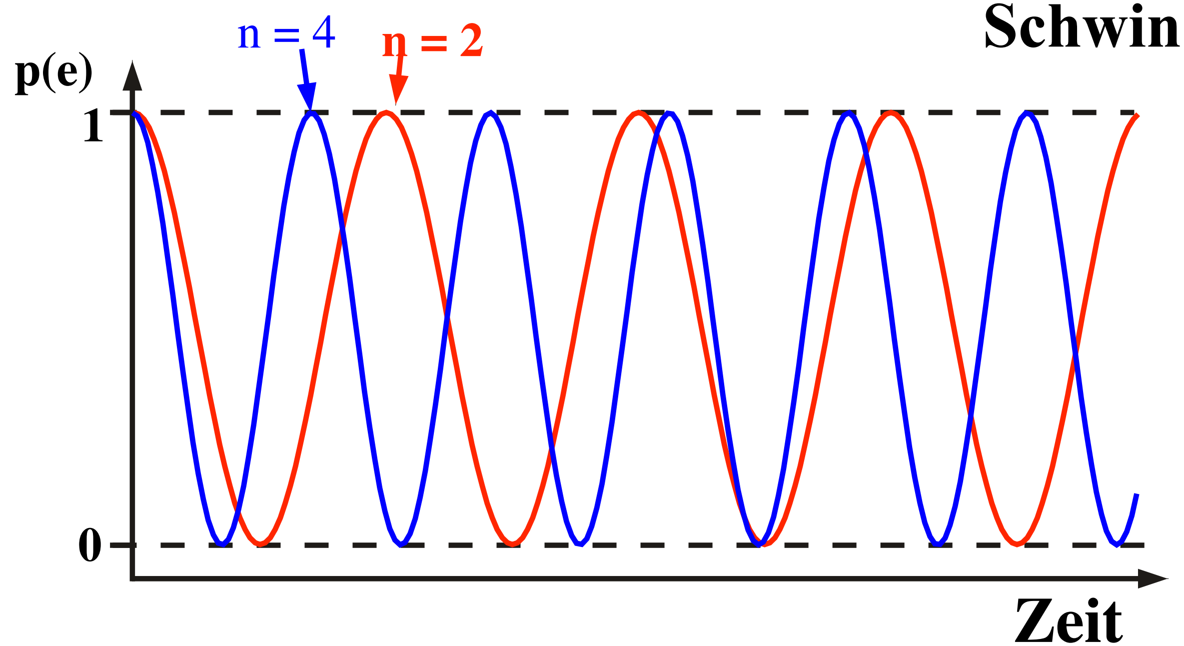
$$|\langle\psi(t)|e,n-1\rangle|^2 = \frac{1}{2}(1+cos(2\omega_1\sqrt{n}t))$$

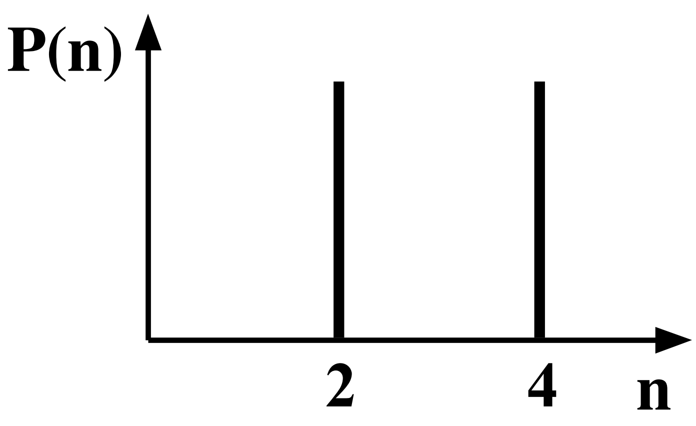


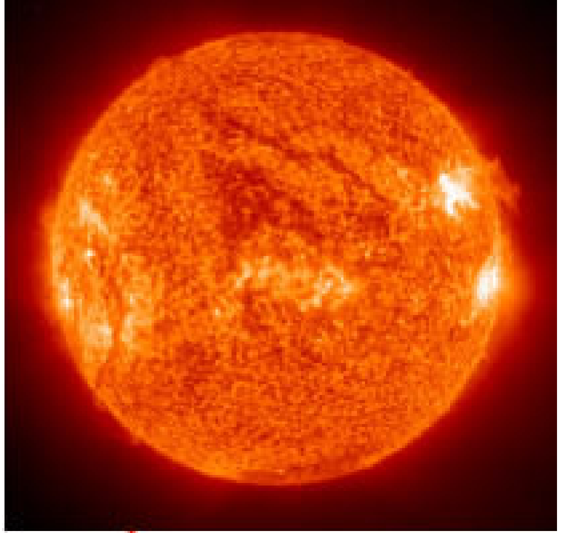
Schwingungsfrequenz $\omega \propto \omega_1\sqrt{n}$

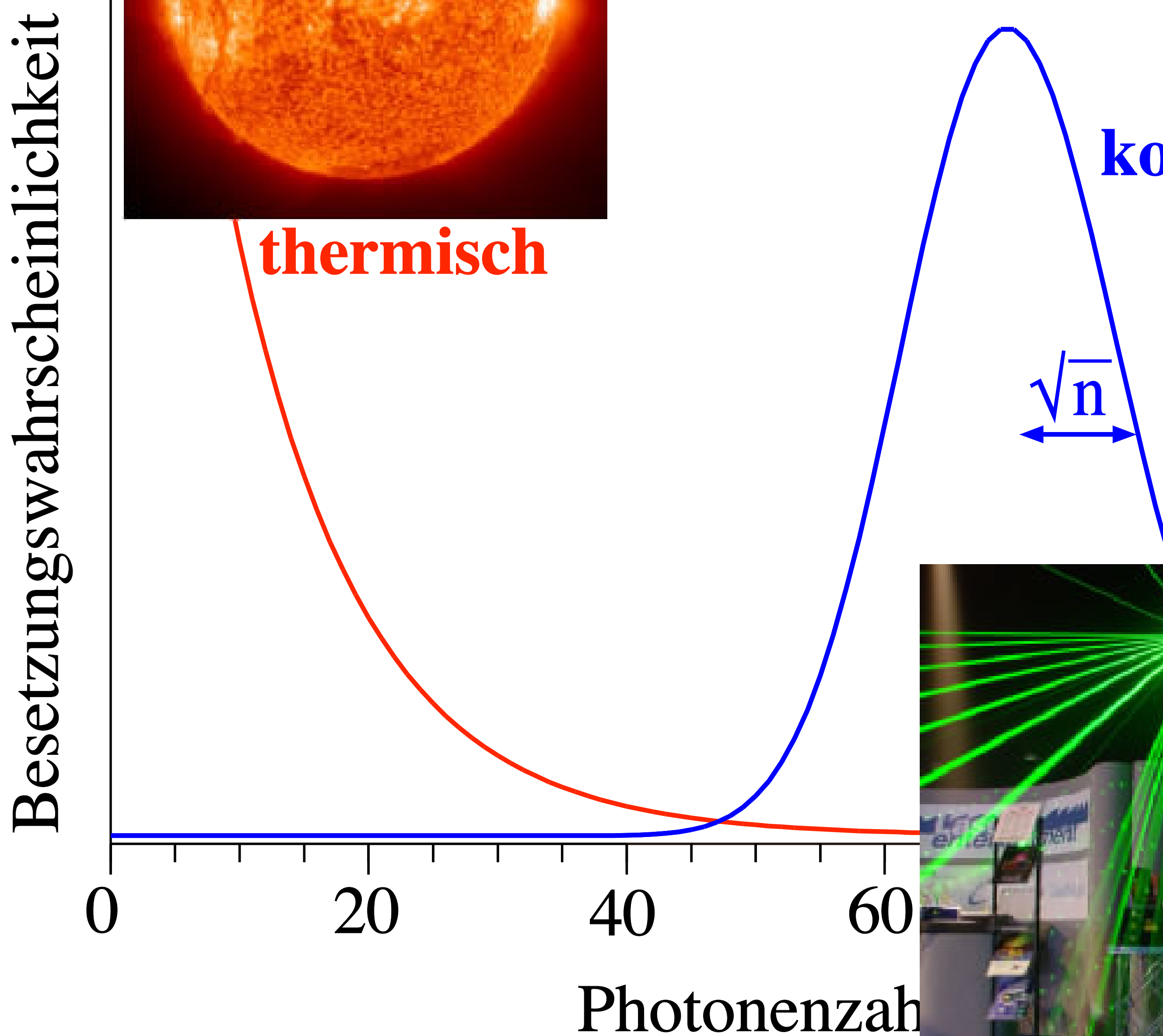
Schwingungsamplitude $A \propto P(n)$



# Zustände

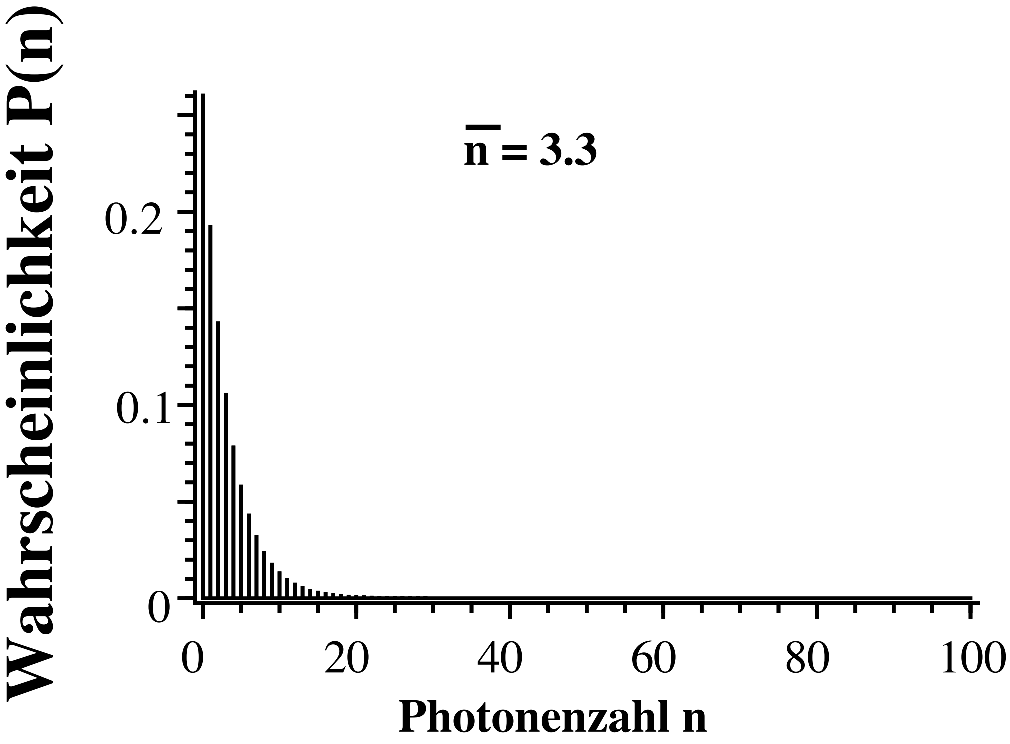


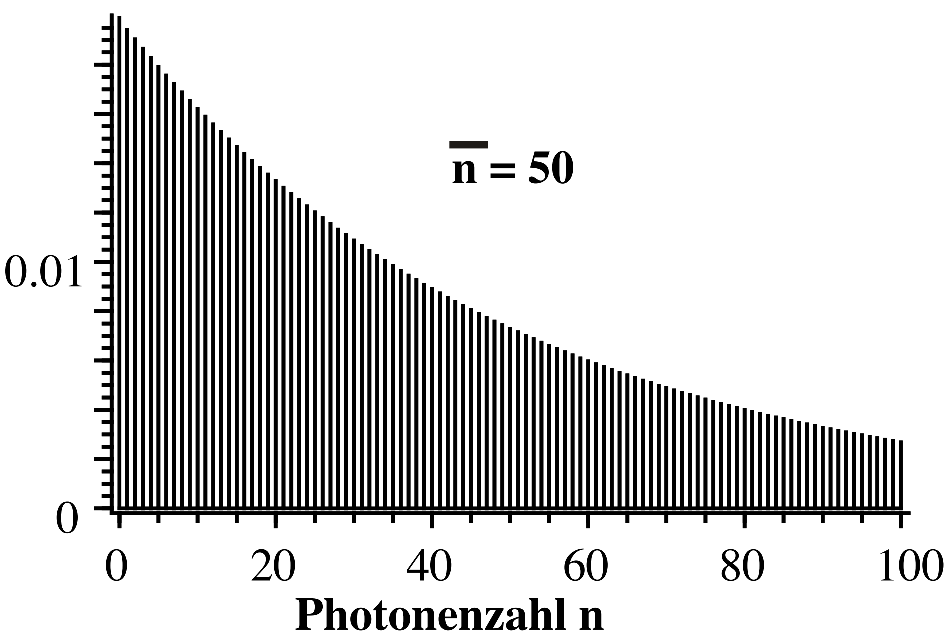

Besetzungswahrscheinlichkeit
thermisch
kohärent
$\sqrt{n}$
0
20
40
60
Photonenzah


Discoscan

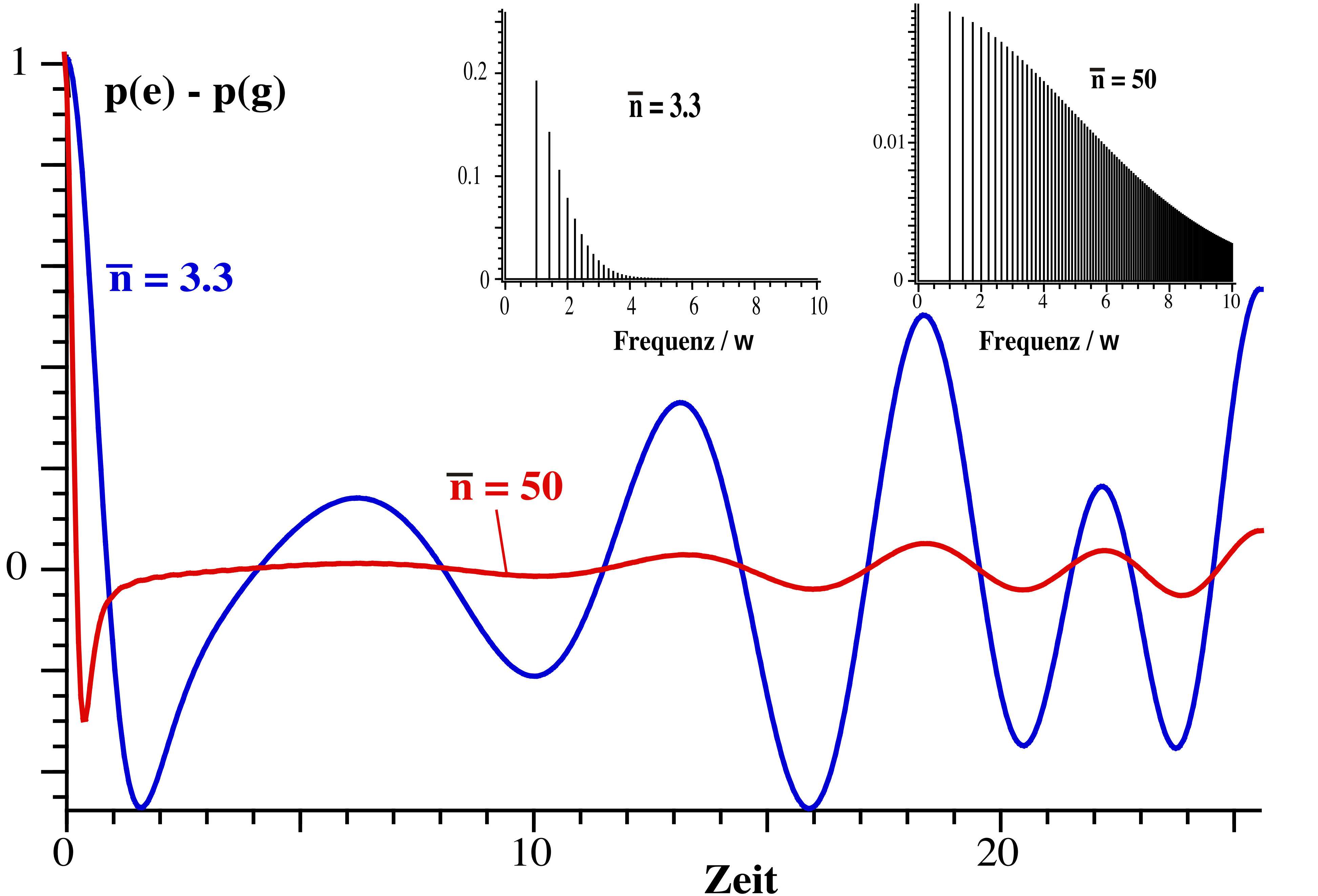
# Thermische Zustände

$$P(n) = \frac{\langle n \rangle^n}{(\langle n \rangle + 1)^{n+1}} \qquad \langle n \rangle = \frac{1}{e^{h\nu/kT} - 1}$$

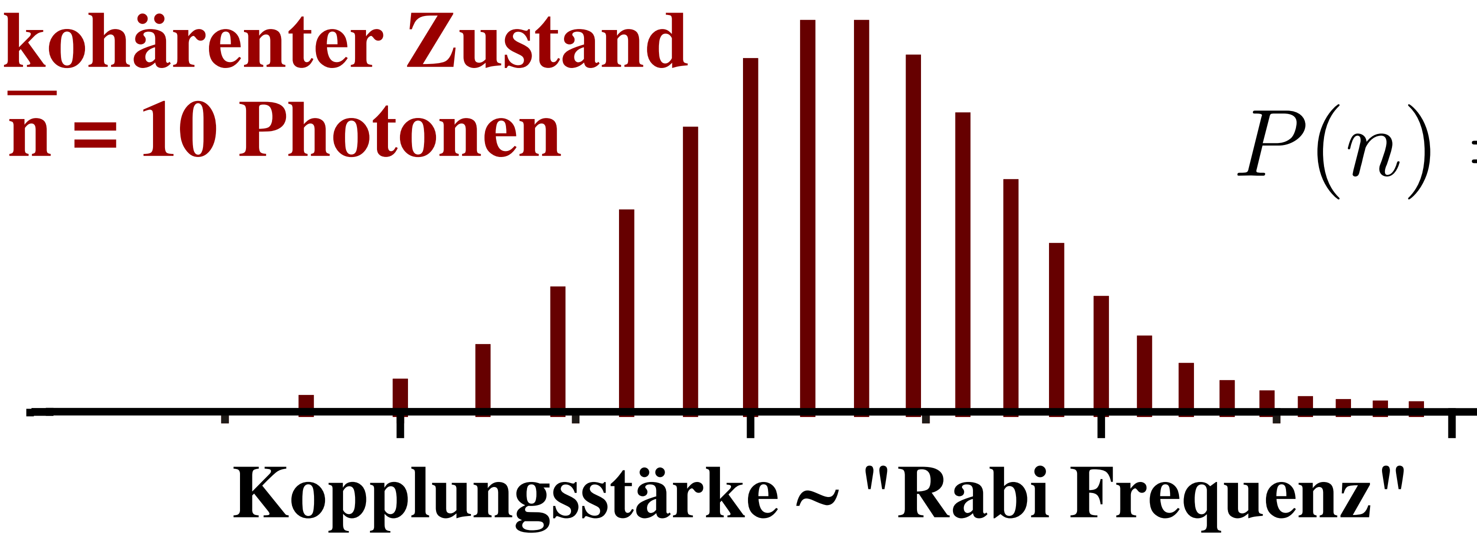


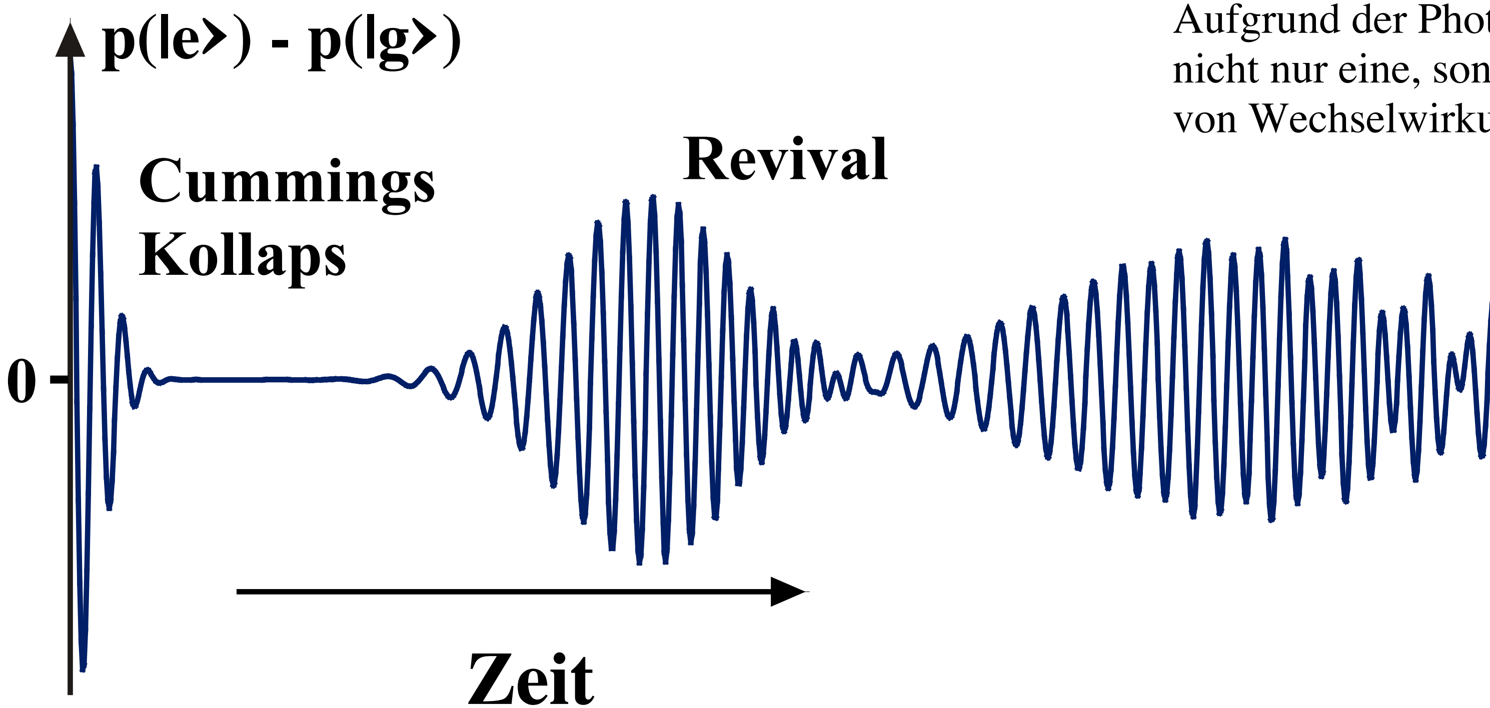
# Thermischer Kollaps



# Kohärenter Zustand



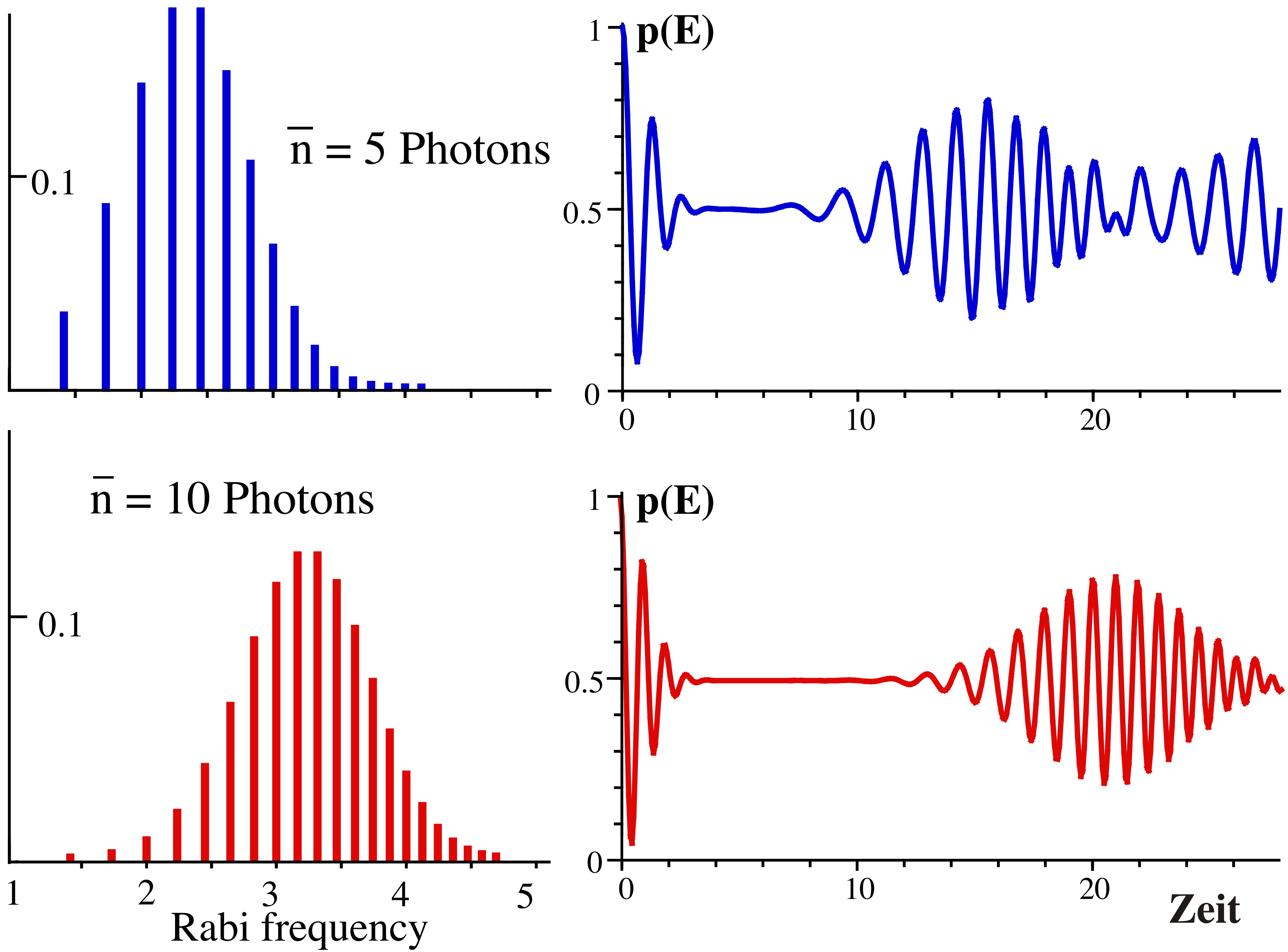

$$P(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$




Aufgrund der Photonenstatistik 'sieht' das Atom nicht nur eine, sondern eine ganze Verteilung von Wechselwirkungen unterschiedlicher Stärke

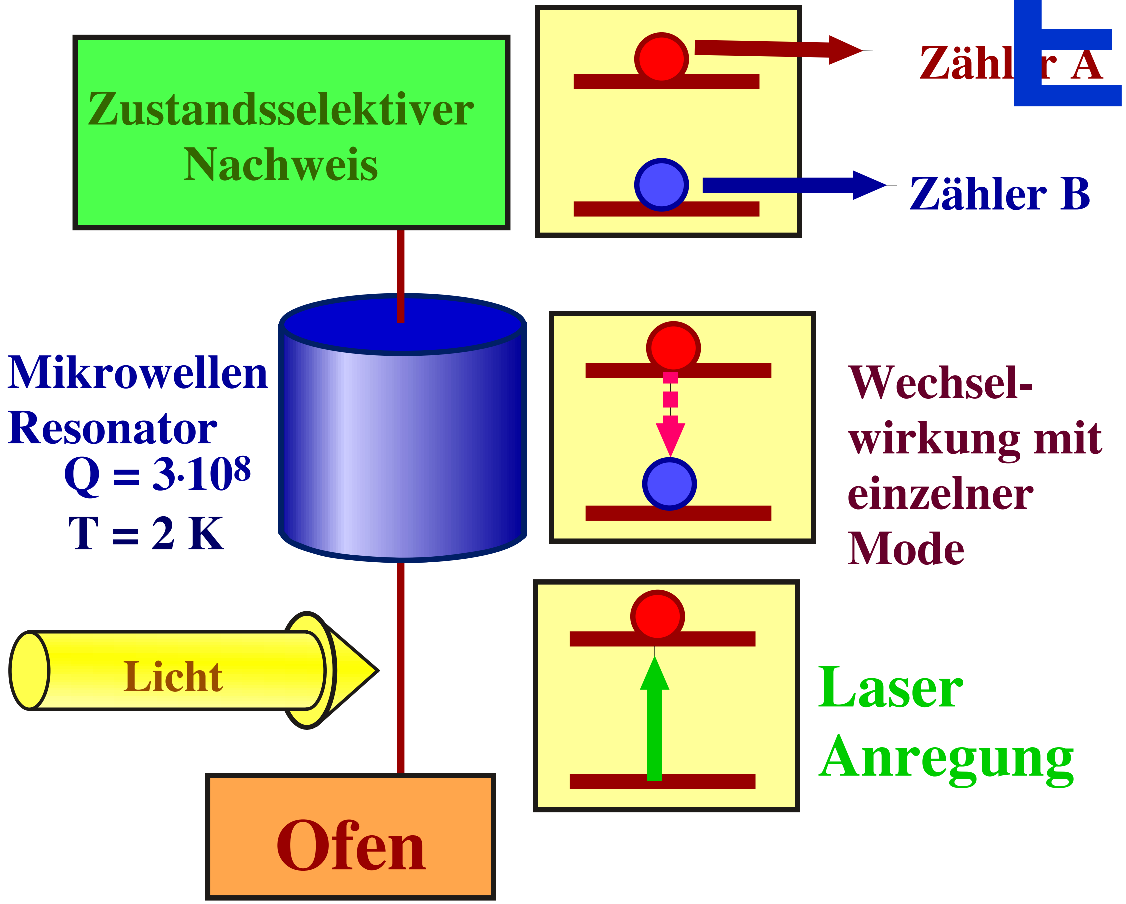
Dadurch geraten die Übergangswahrscheinlichkeiten außer Phase

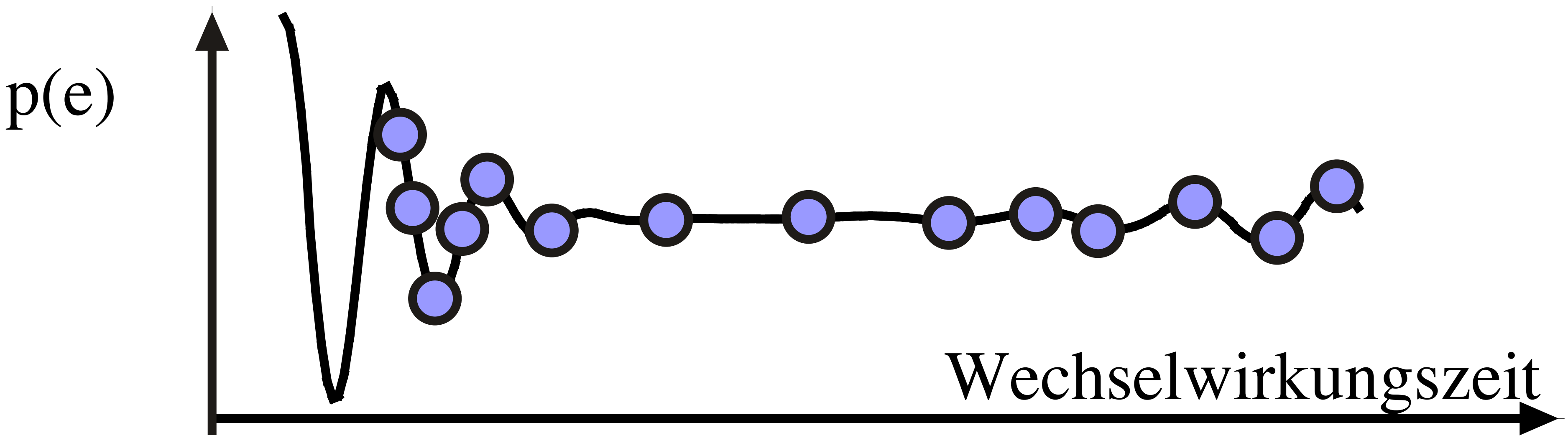
# Kohärente Zustände



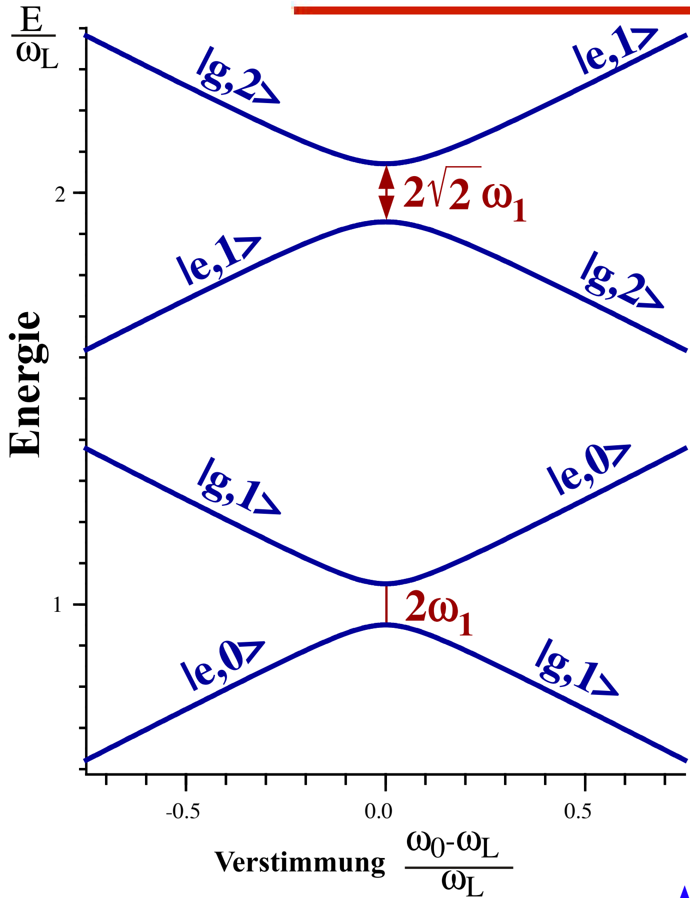
# Experiment

G. Rempe, H. Walther, und N. Klein, Phys. Rev. Lett. 58, 353 (1987).

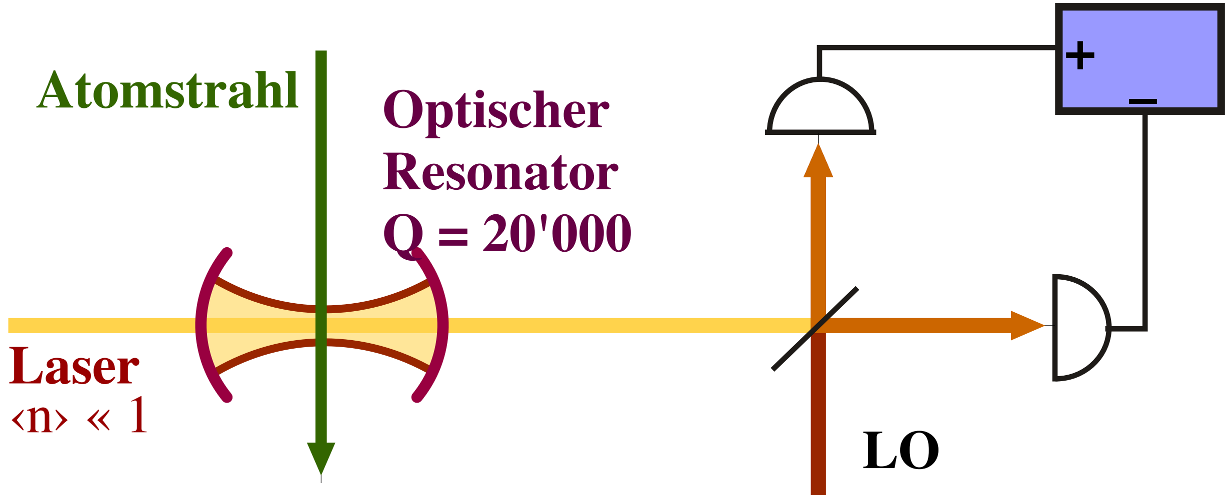


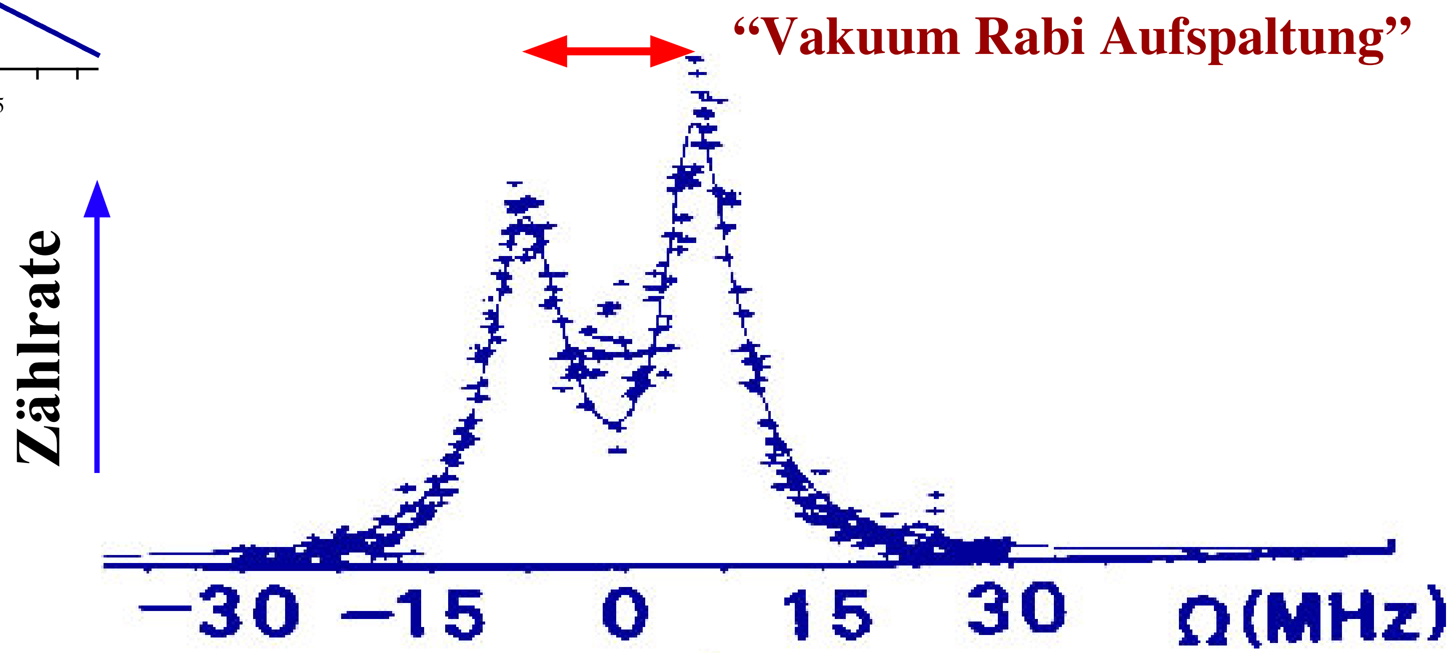
# Vakuum Rabi-Aufspaltung




## Experimenteller Aufbau



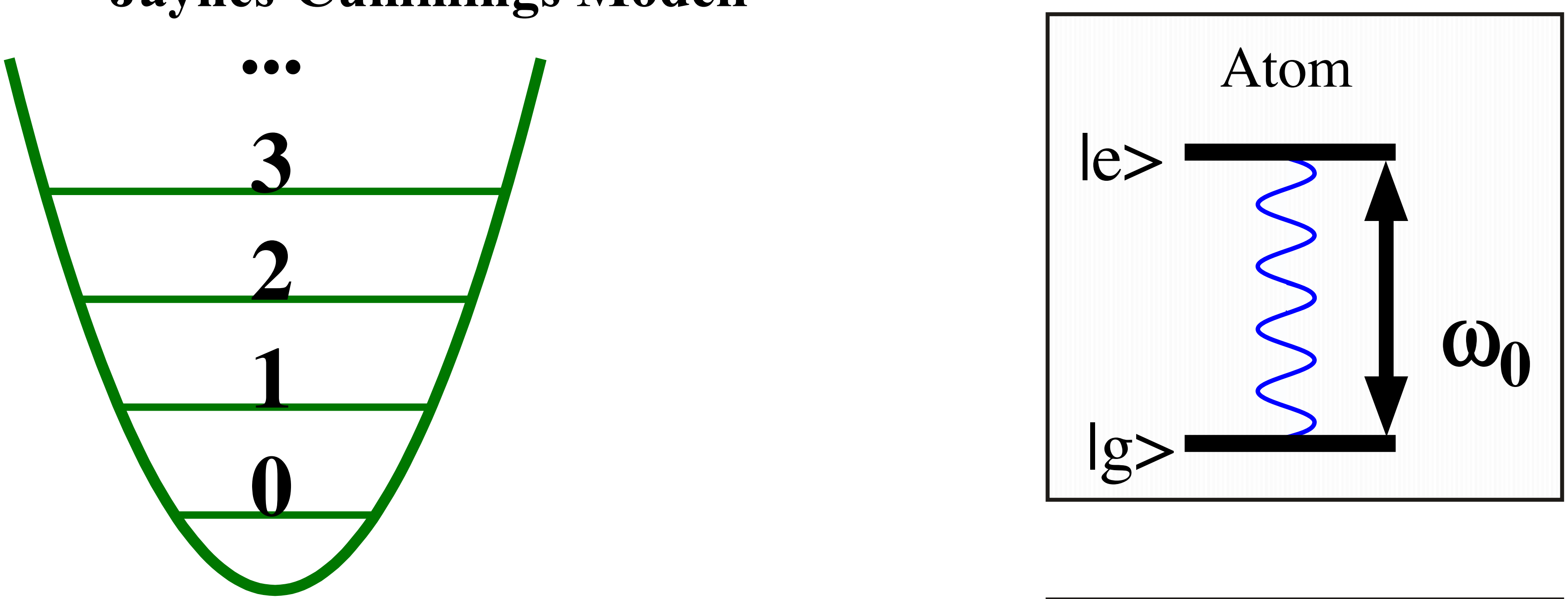

## Resultate



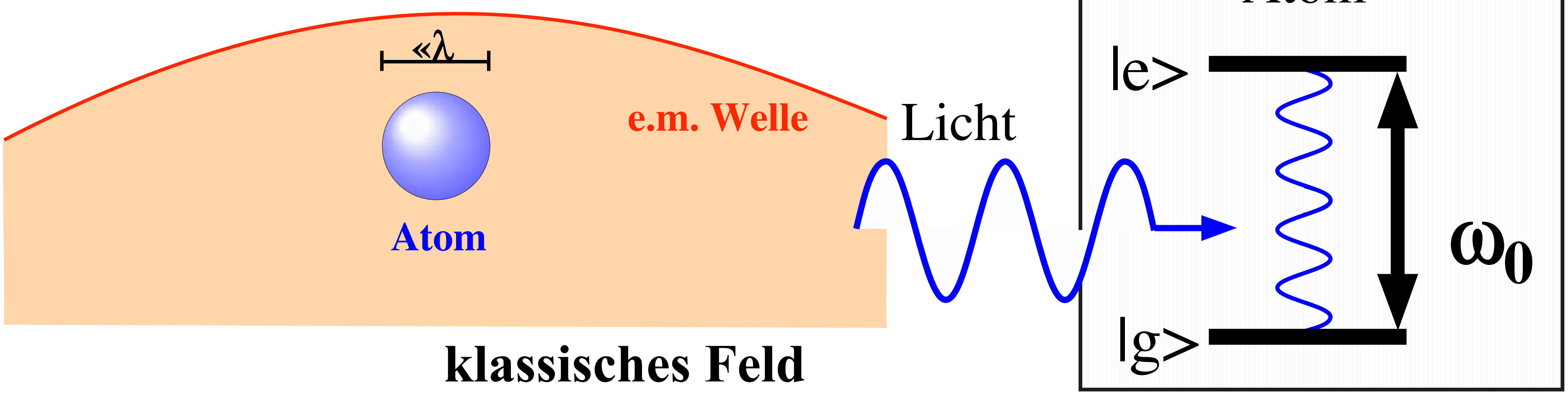

*M.G. Raizen, R.J. Thompson, R.J. Brecha, H.J. Kimble, and H.J. Carmichael Phys. Rev. Lett. 63, 240-243 (1989).*

# Halbklassisches Modell

**Jaynes-Cummings Modell**




**“Halbklassisch”**




**klassisches Feld**

**quantenmechanische Materie**

# 3.3 Halbklassisches Modell

**Literatur**

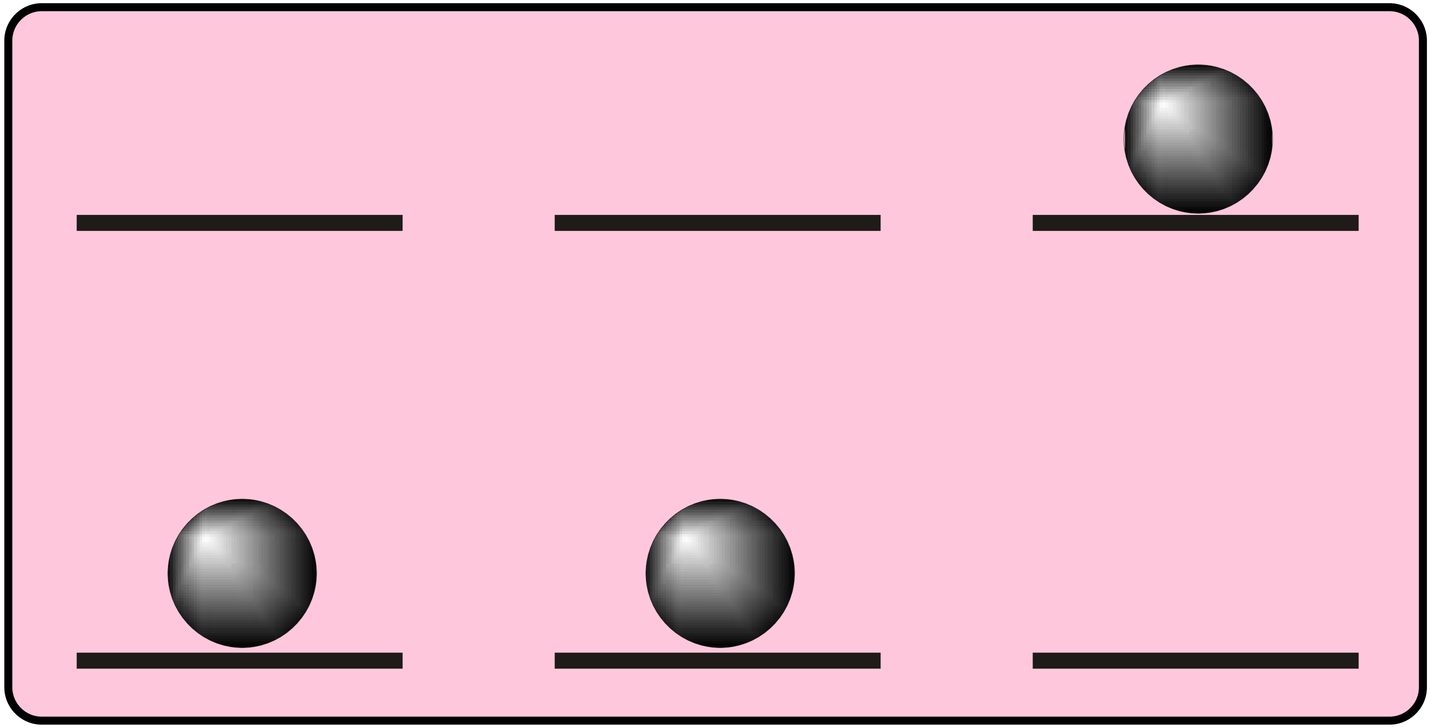

**R.P. Feynman, F.L. Vernon, and R.W. Hellwarth, ‘Geometrical representation of the Schrödinger equation for solving maser problems’, J. Appl. Phys. 28, 49-52 (1957).**

**R.G. Brewer, Coherent optical spectroscopy, in Frontiers of Laser Spectroscopy, Editor: R. Balian, S. Haroche, and S. Liberman, North Holland (1977).**

**L. Allen and J.H. Eberly, ‘Optical resonance and two-level atoms’, Dover Publications, Mineola, NY (1987).**

**Dieter Suter, ‘The Physics of Laser-Atom Interaction’, Cambridge University Press, Cambridge (1997). Chapter 2.**

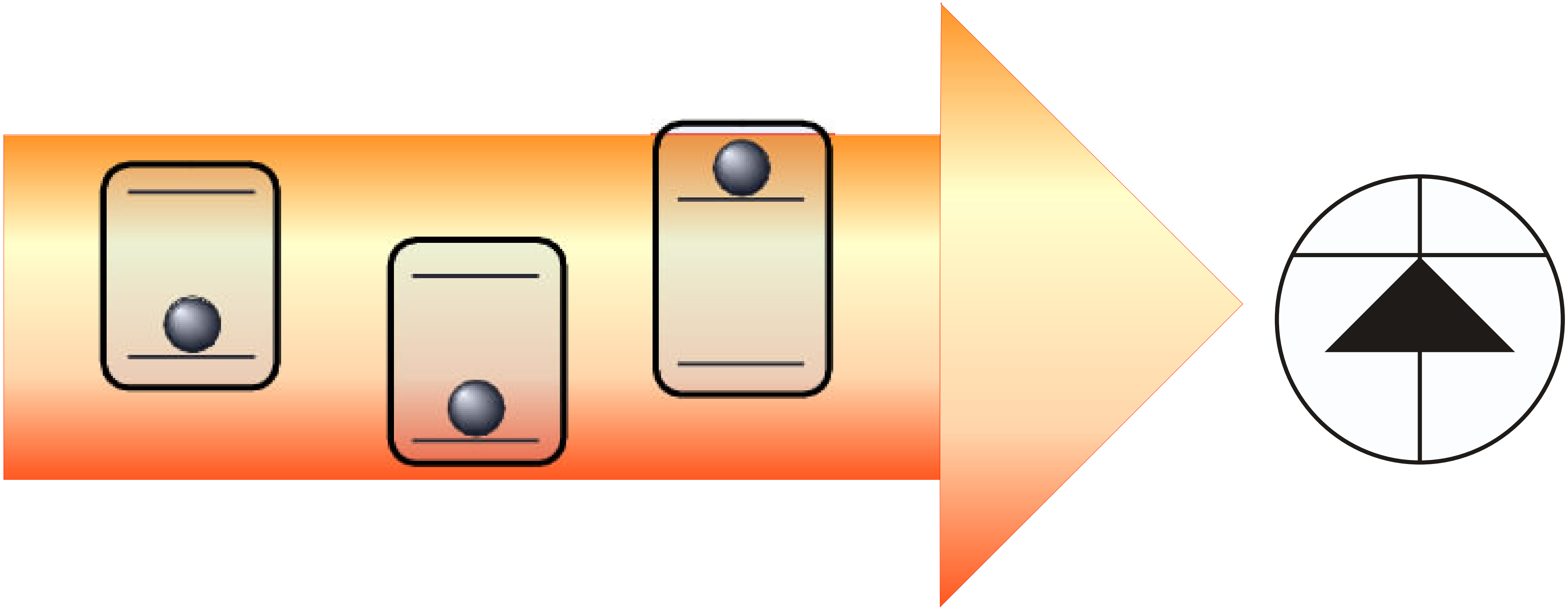
# Ensemble



$\Psi_1 = |g\rangle$ $\Psi_2 = |g\rangle$ $\Psi_3 = |e\rangle$

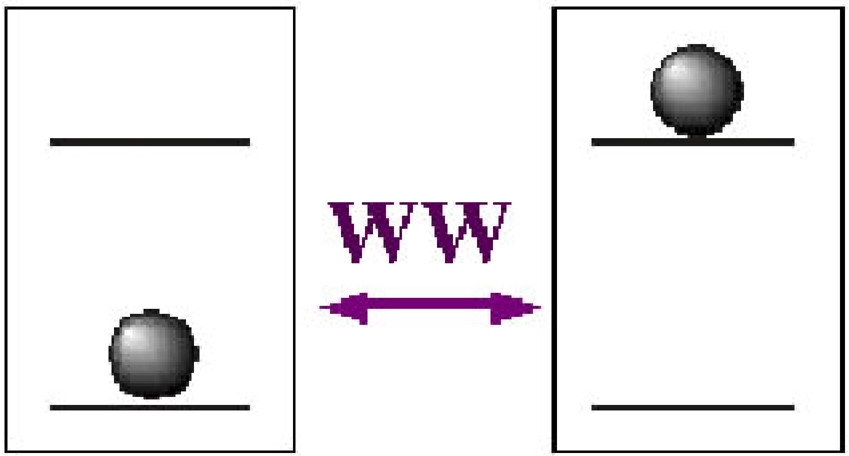
$\begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}0\\1\end{pmatrix}$

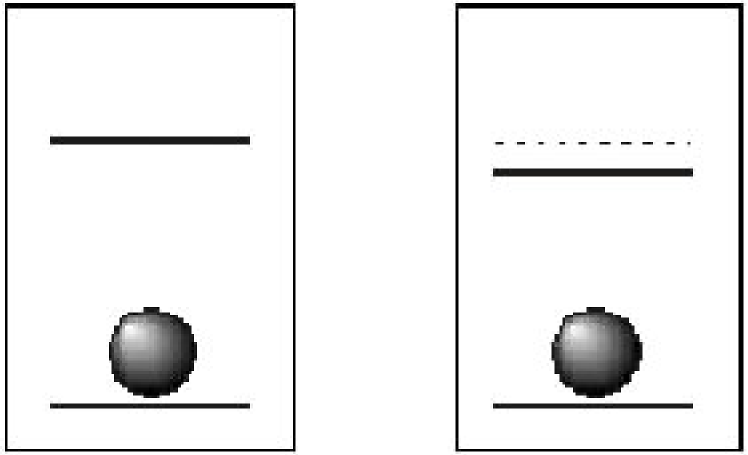
# Ensemble-Messung



# Ensemble-Messung

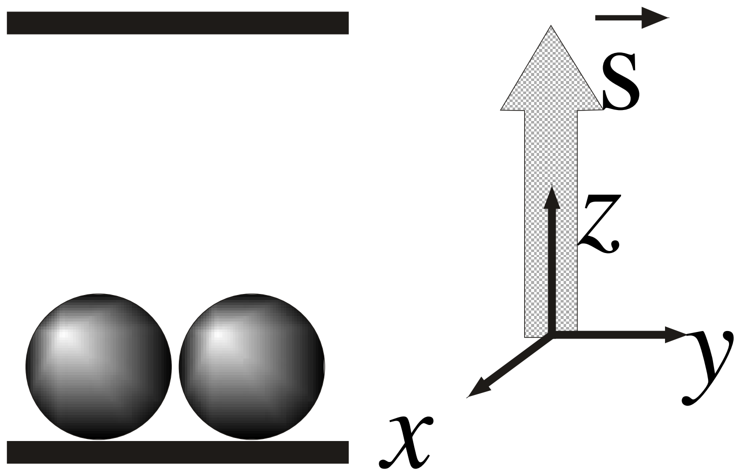
**Wechselwirkungen** **Inhomogenitäten**



# Dichteoperator

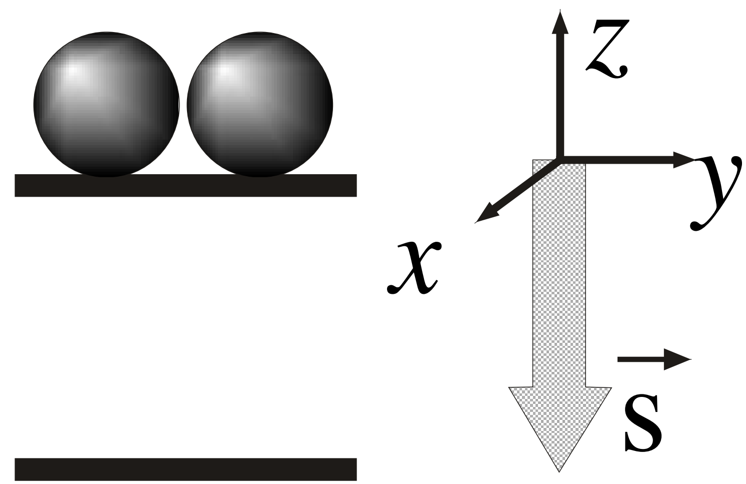
Atome im Grundzustand




$$\Psi_a = |g\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \rho'_a = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\rho_a = S_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Atome im angeregten Zustand

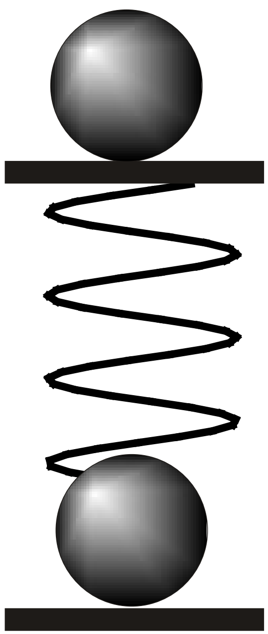


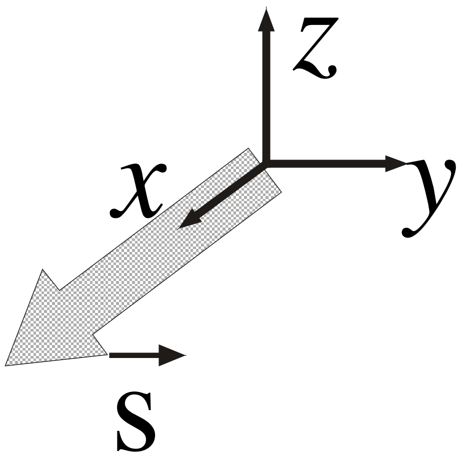

$$\Psi_b = |e\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \rho'_b = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rho_b = -S_z = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

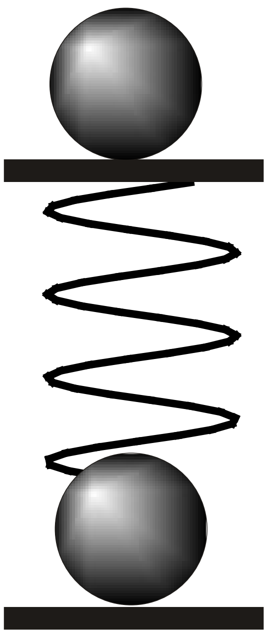
# Elektrisches Dipolmoment

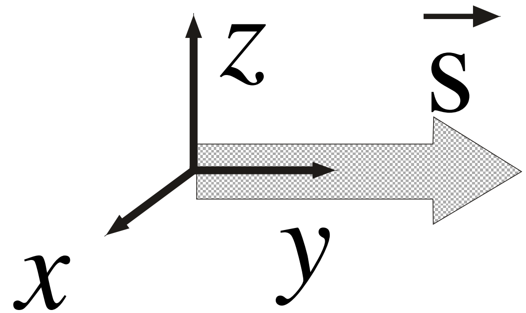
$$\Psi_a = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$






$$\Psi_b = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$$

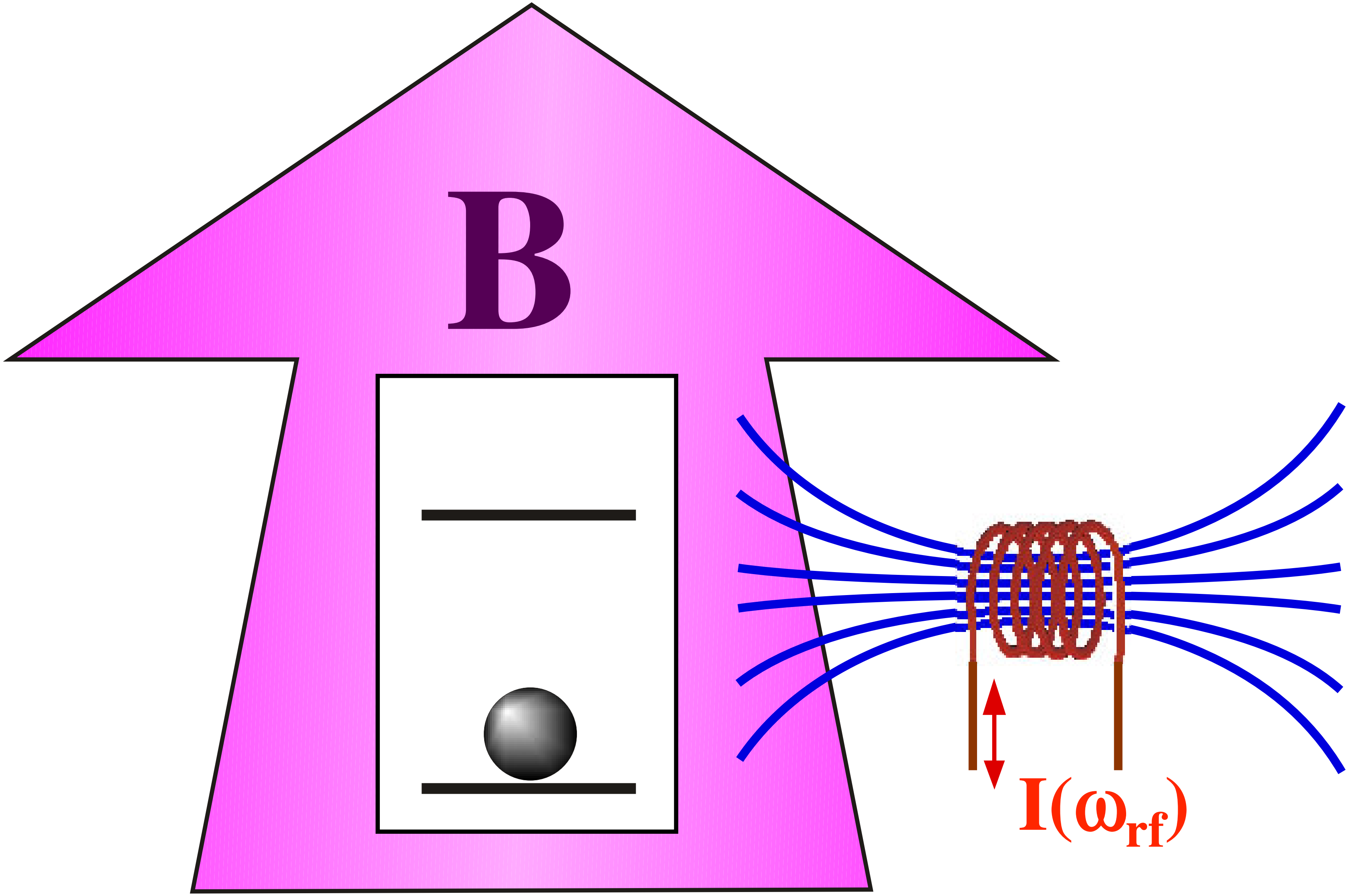





$$\rho_a = S_x = \frac{1}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$$

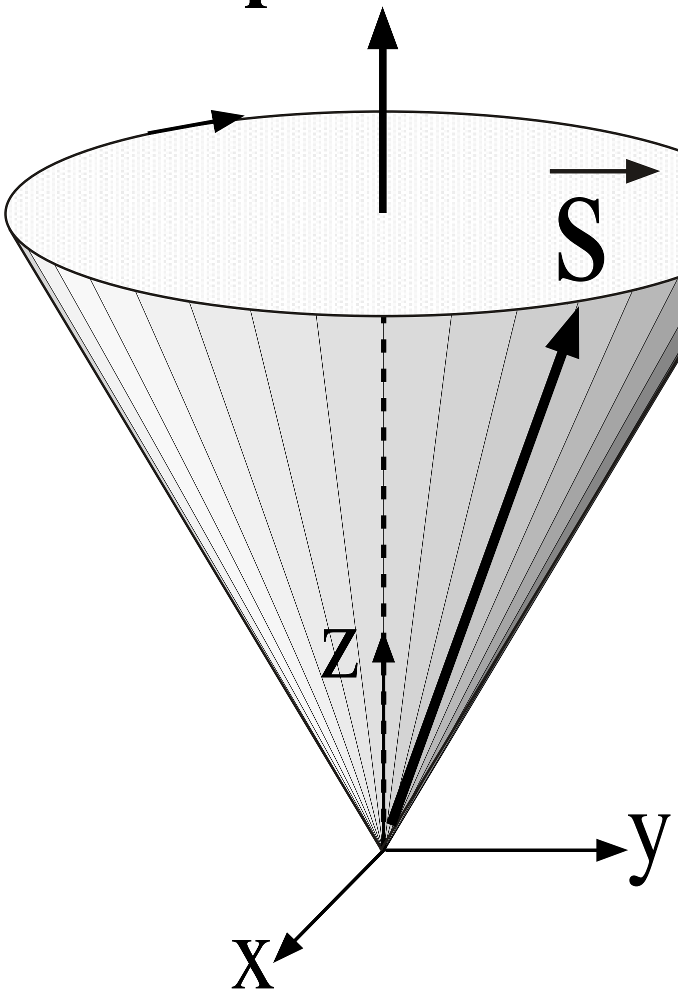
$$\rho_b = S_y = \frac{1}{2}\begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$$

# Pesudo-Spin-WW

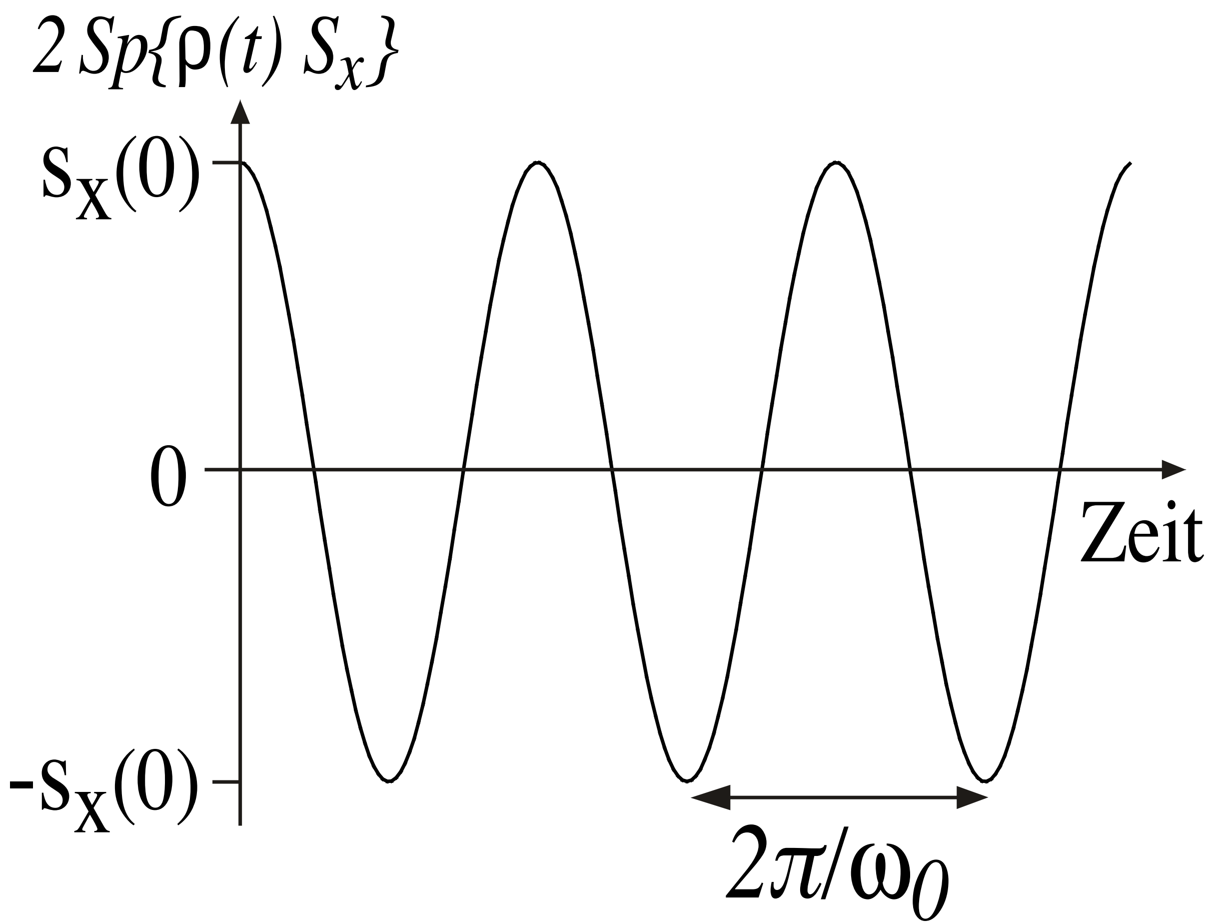


# Freie Präzession

**Pseudospin**



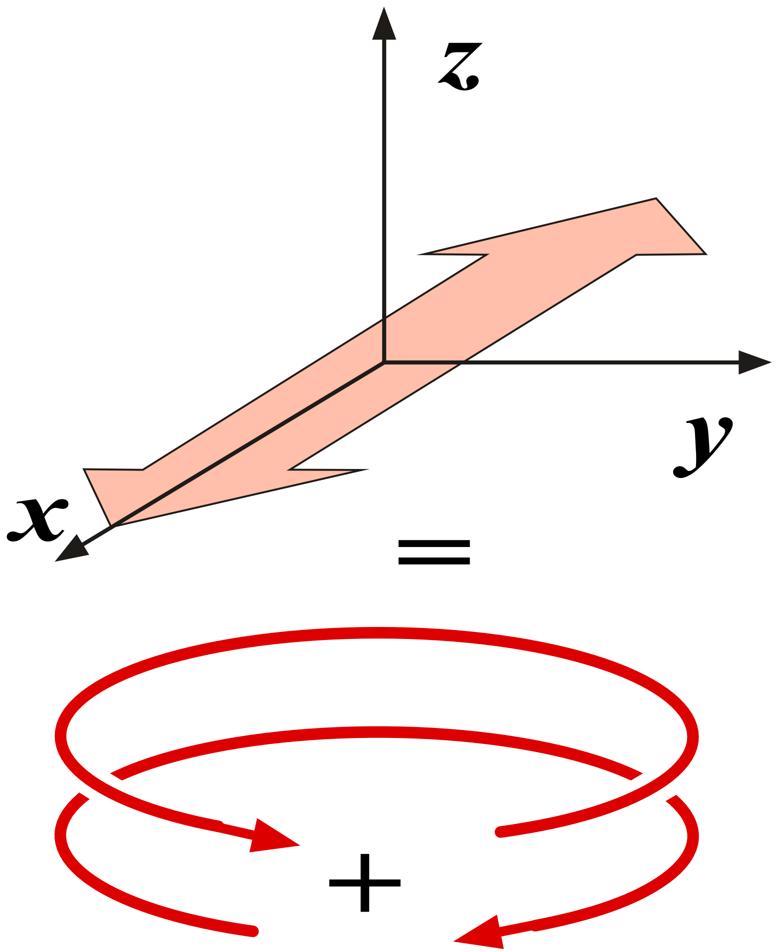

**Dipolmoment**



# Rotierendes Koordinatensystem

**Laborsystem**

$(\omega_x, 0, 0)\ 2\cos(\omega_L t)$

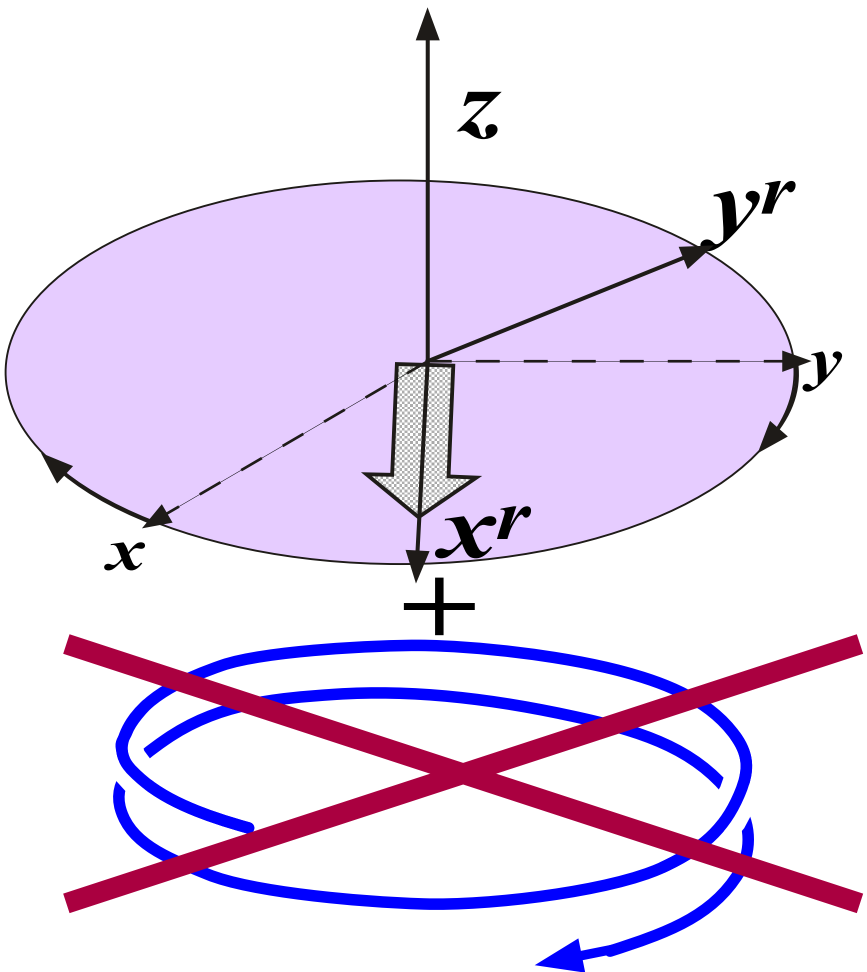



***Rotierendes Koordinatensystem***

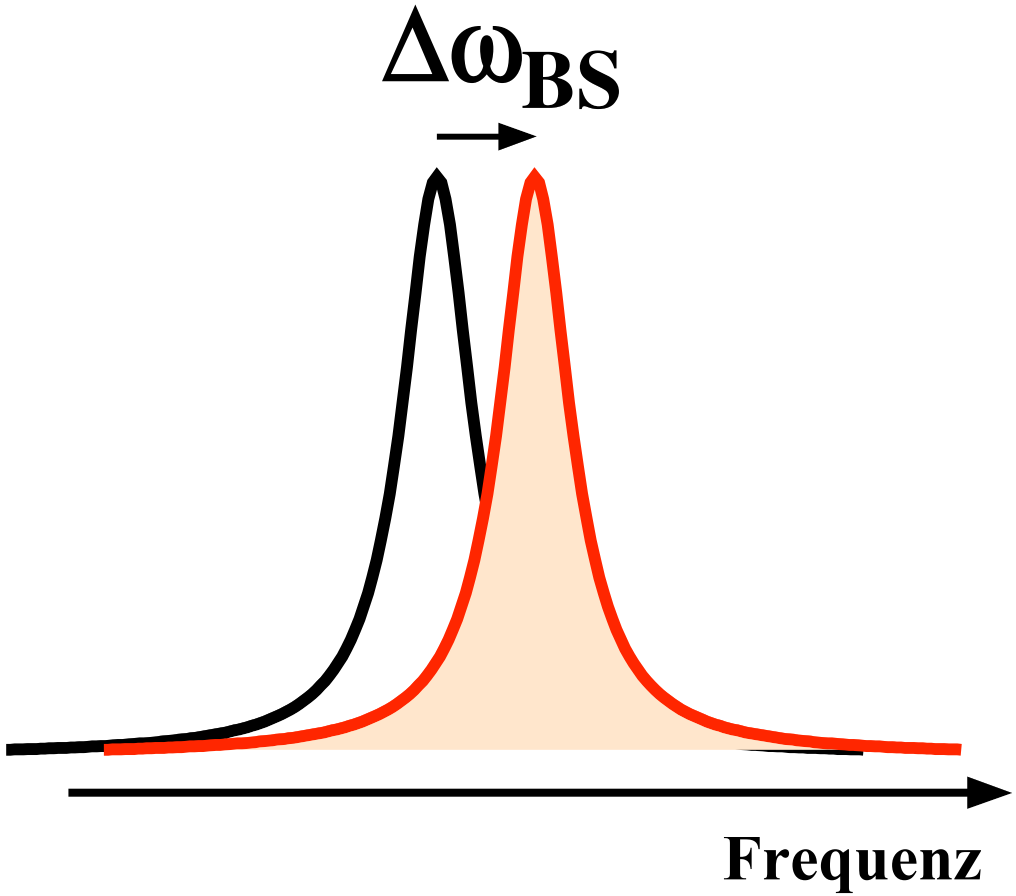
$(\omega_x, 0, 0)\ [1 + \cos(2\omega_L t)]$

$- (0, \omega_x, 0)\sin(2\omega_L t)$

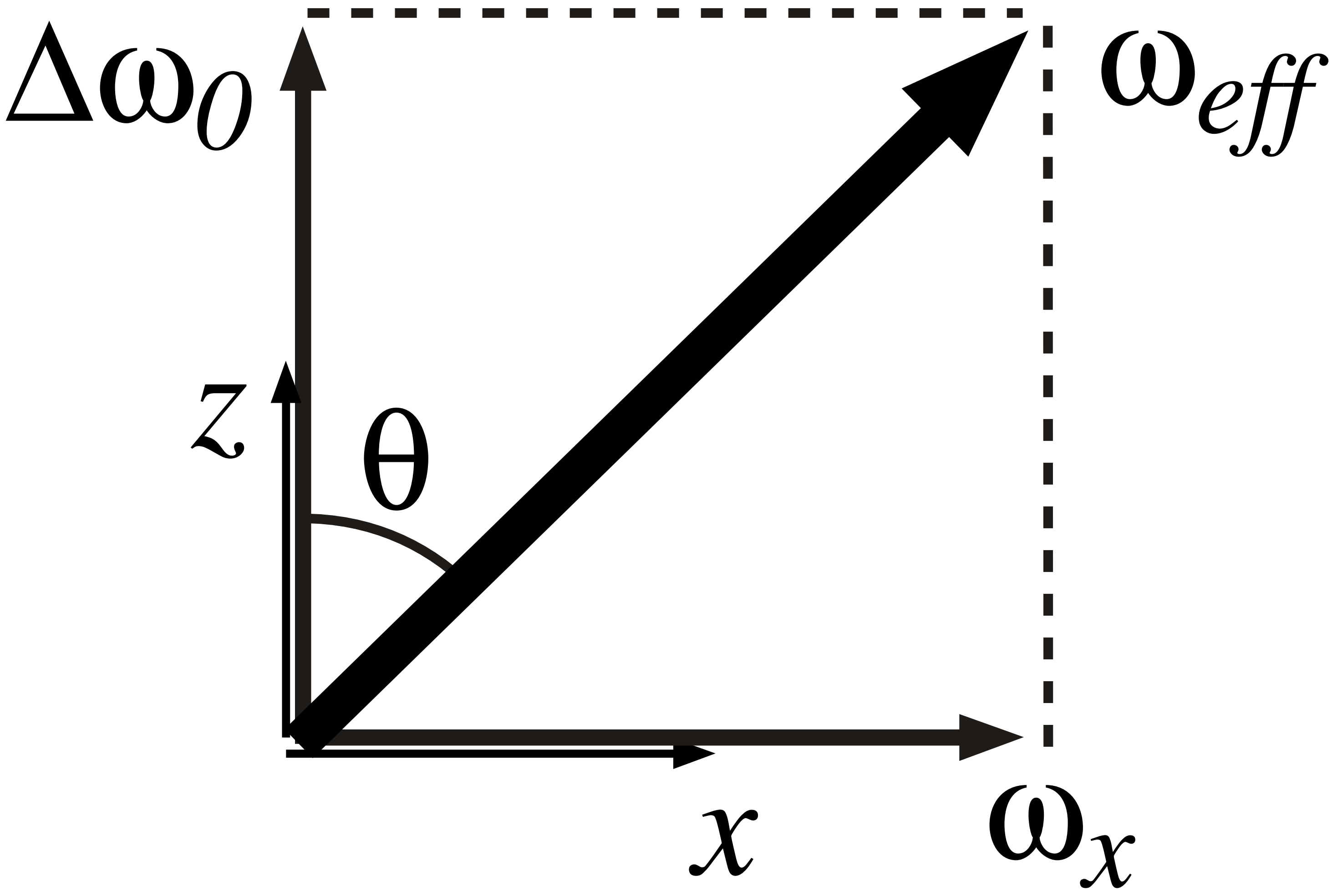
**nichtresonante Teile vernachläßigen**



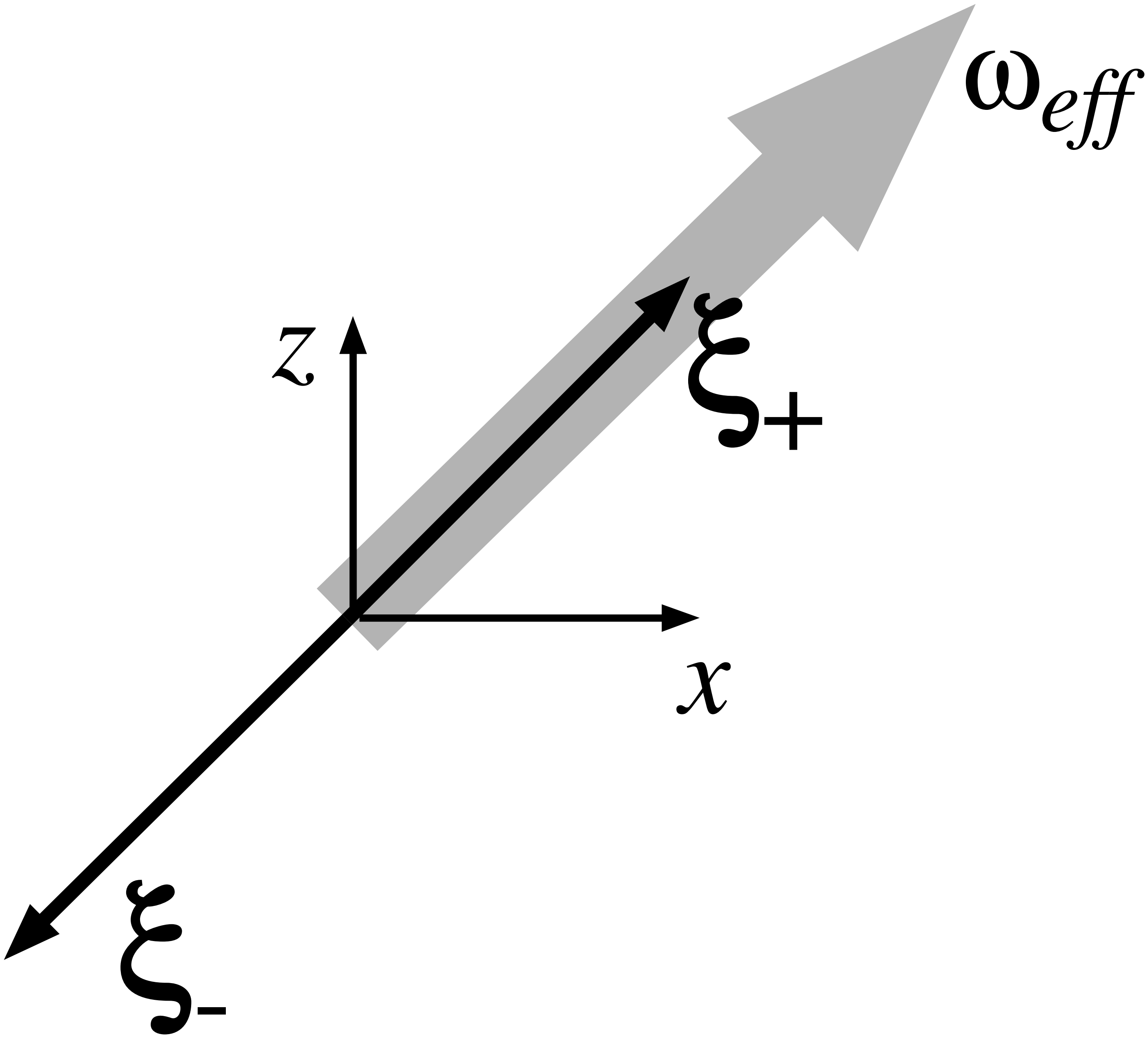
# Bloch-Siegert Shift



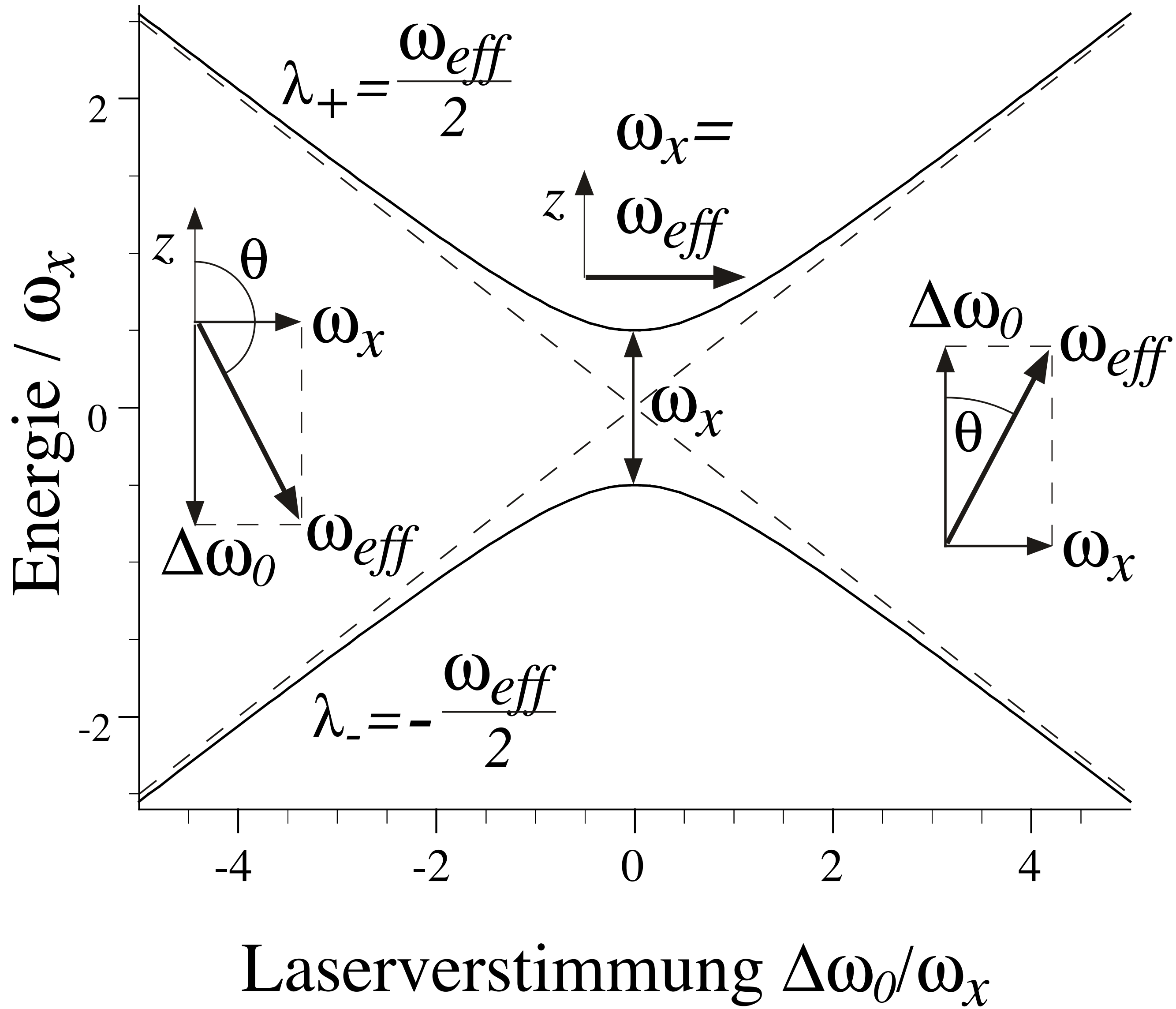
# Geometrische Lösung



# Eigenbasis



# Energien und Zustände

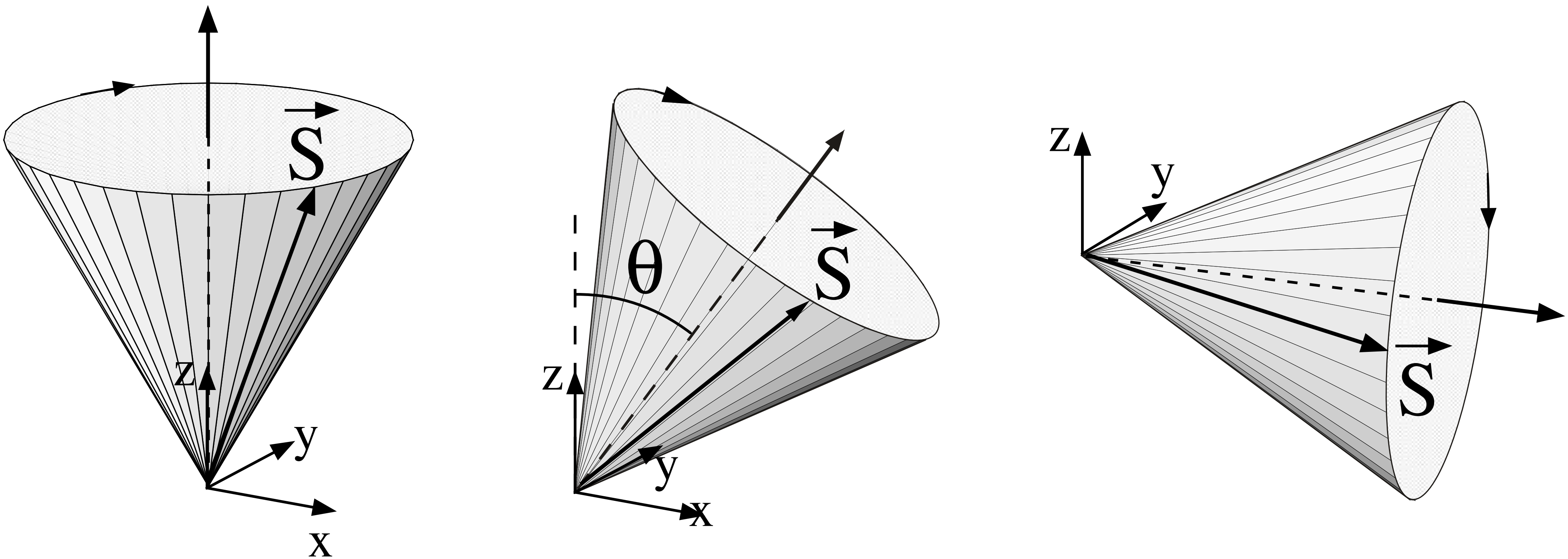


# Präzession

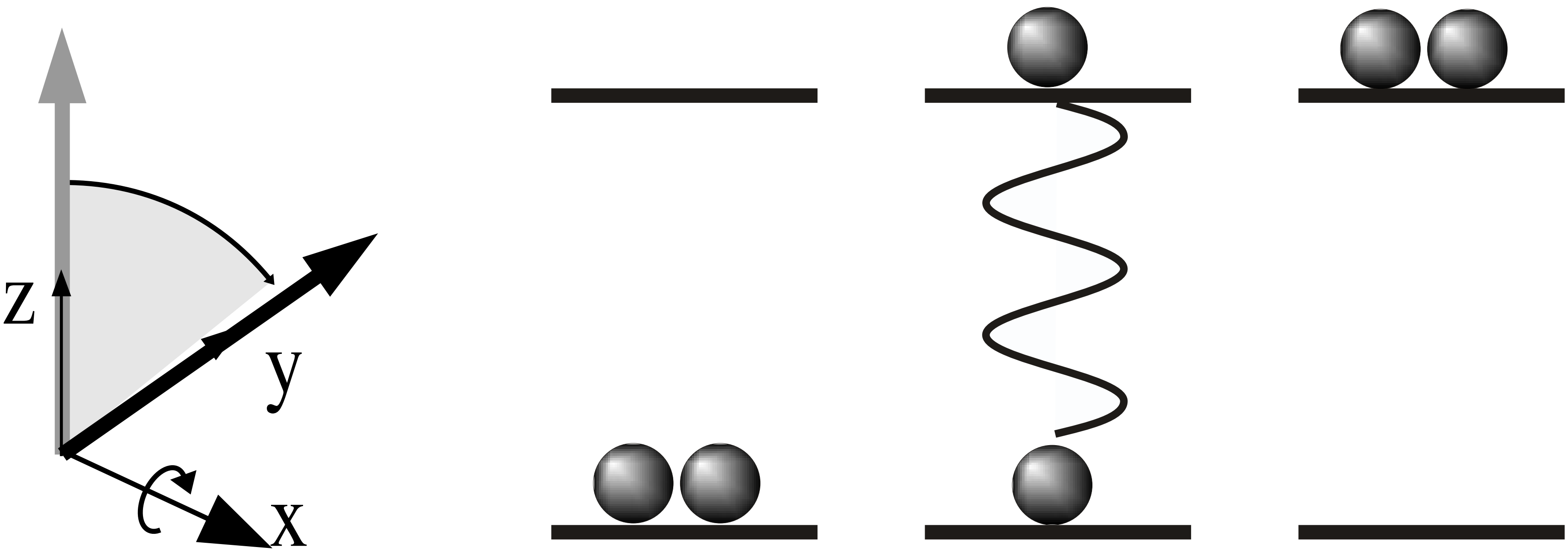
*a)* $\Delta\omega_0 \neq 0$, $\omega_x = 0$

*b)* $\Delta\omega_0 \neq 0$, $\omega_x \neq 0$

*c)* $\Delta\omega_0 = 0$, $\omega_x \neq 0$

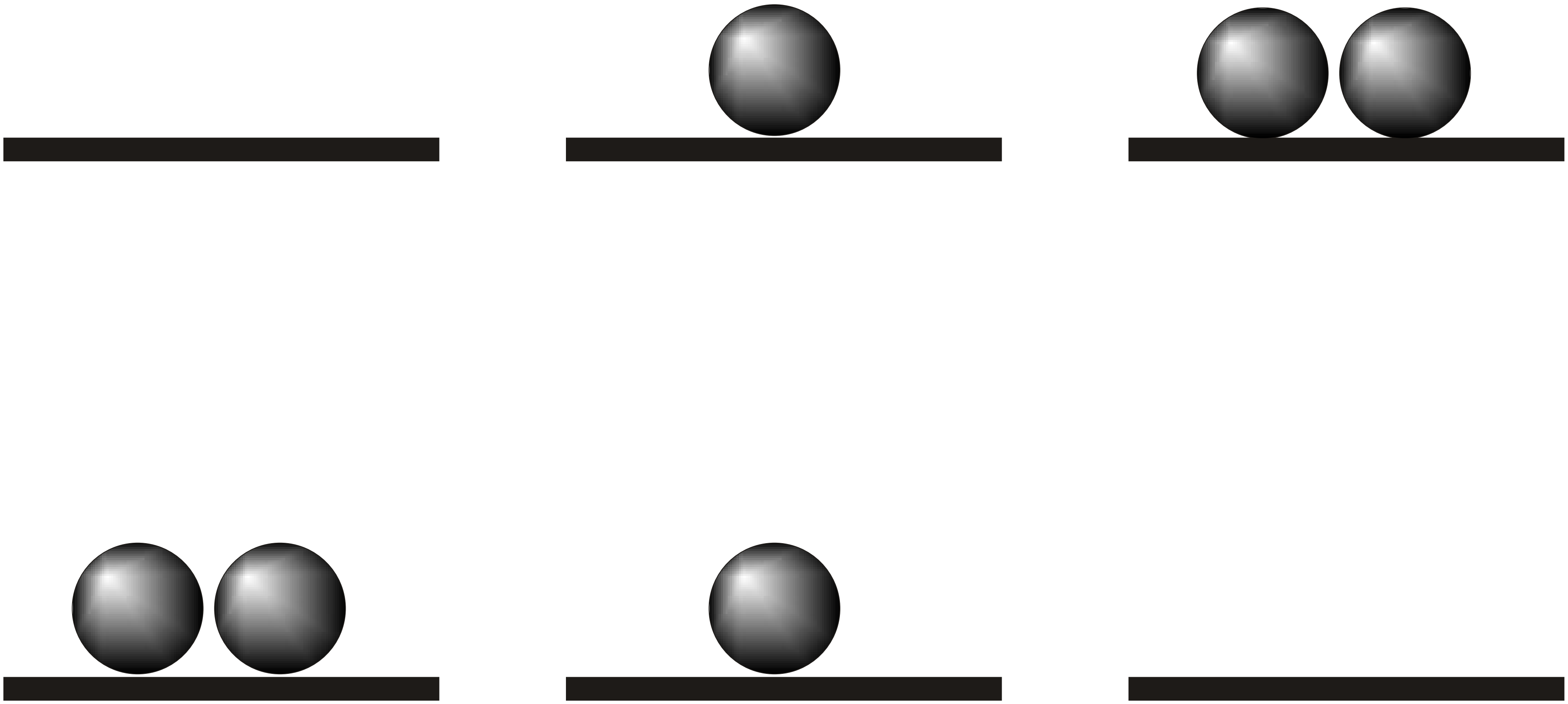


# Laserpuls

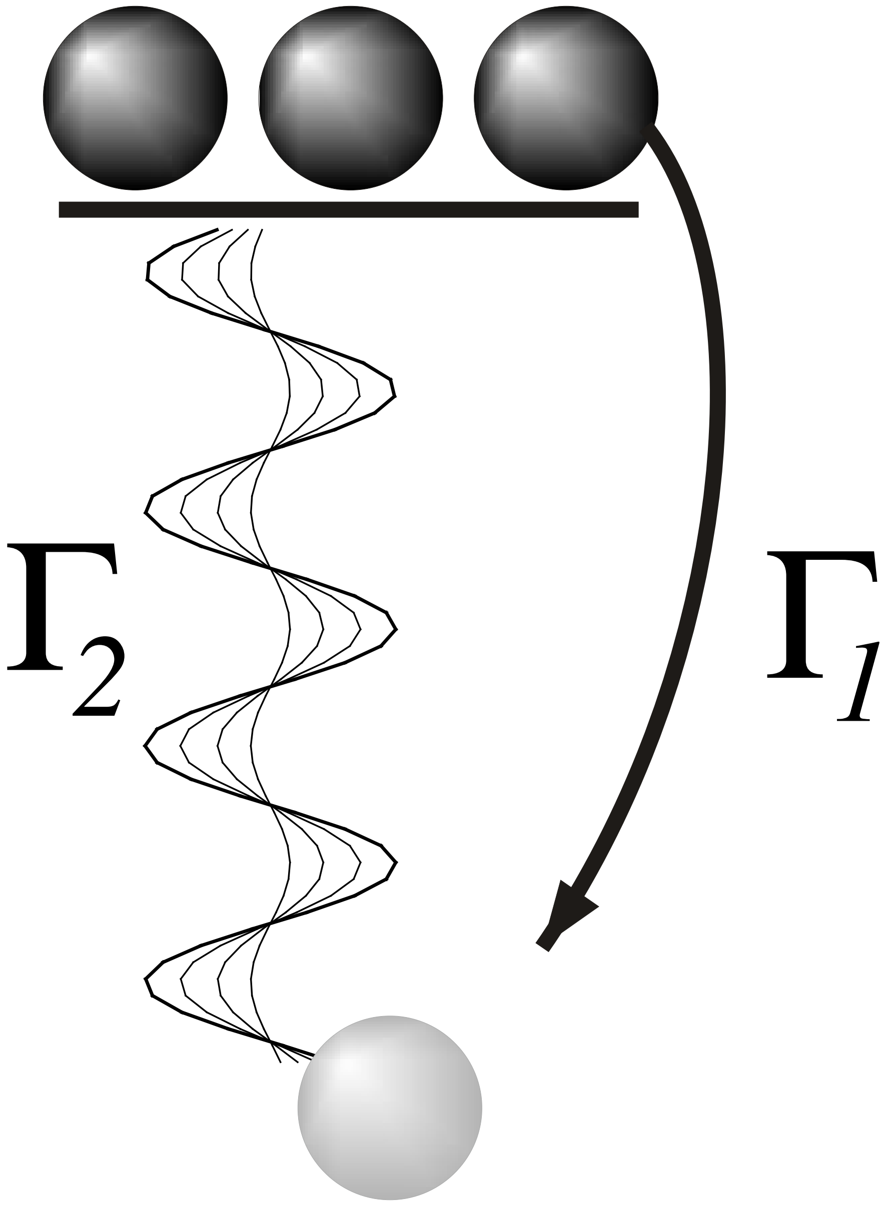


# Boltzmann-Temperatur

$T = 0$ $T = \infty$ $T < 0$



# Relaxation



# Optische Blochgleichung

$$\begin{aligned}
\dot{s}_x &= \Delta\omega_0 s_y - \Gamma_2 s_x \\
\dot{s}_y &= -\Delta\omega_0 s_x + \omega_x s_z - \Gamma_2 s_y \\
\dot{s}_z &= -\omega_x s_y + \Gamma_1 (1 - s_z)
\end{aligned}$$

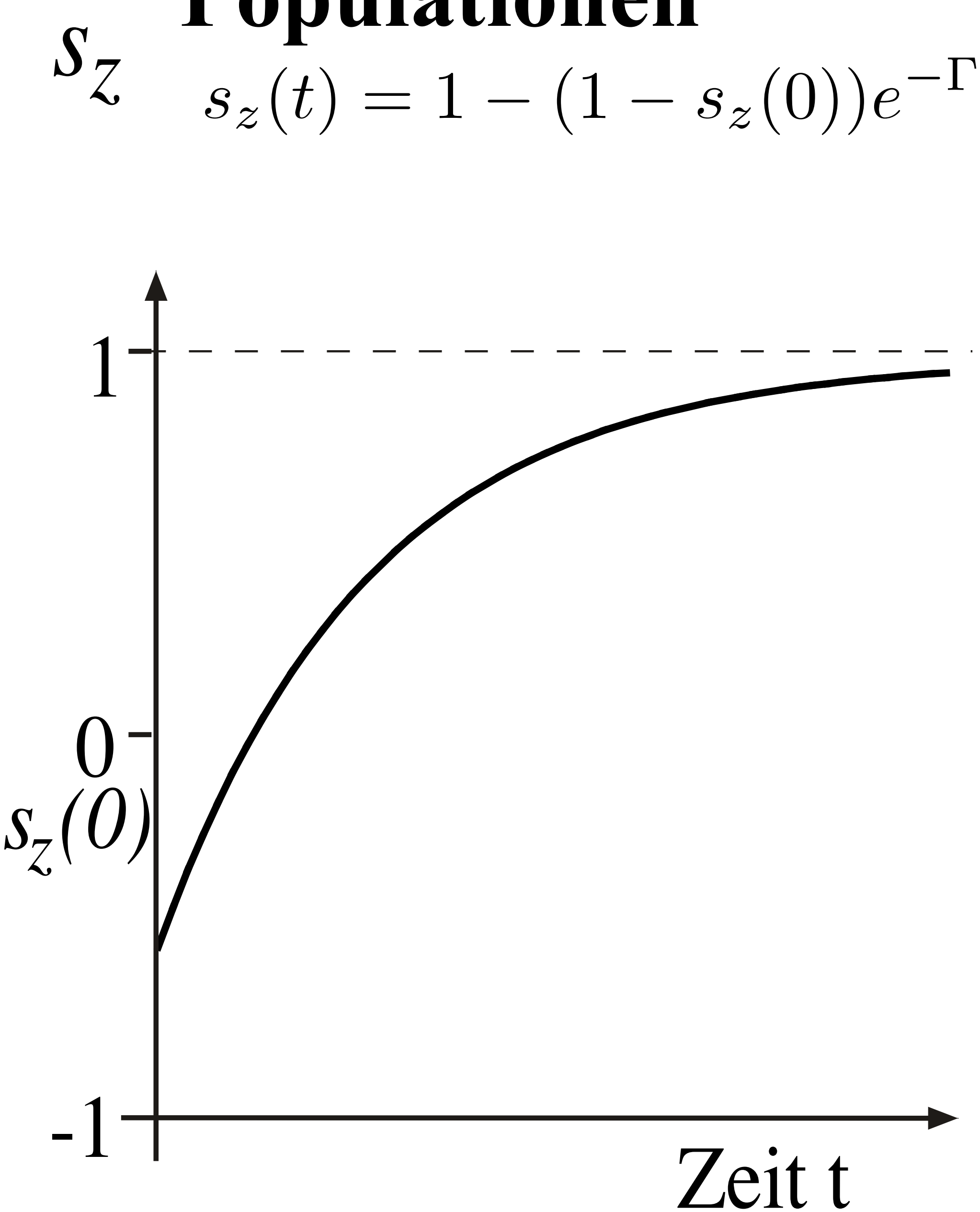
**F. Bloch, 'Nuclear induction', Phys. Rev. 70, 460-485 (1946).**

**R.P. Feynman, F.L. Vernon, and R.W. Hellwarth, 'Geometrical representation of the Schrödinger equation for solving maser problems', J. Appl. Phys. 28, 49-52 (1957).**

# Relaxation

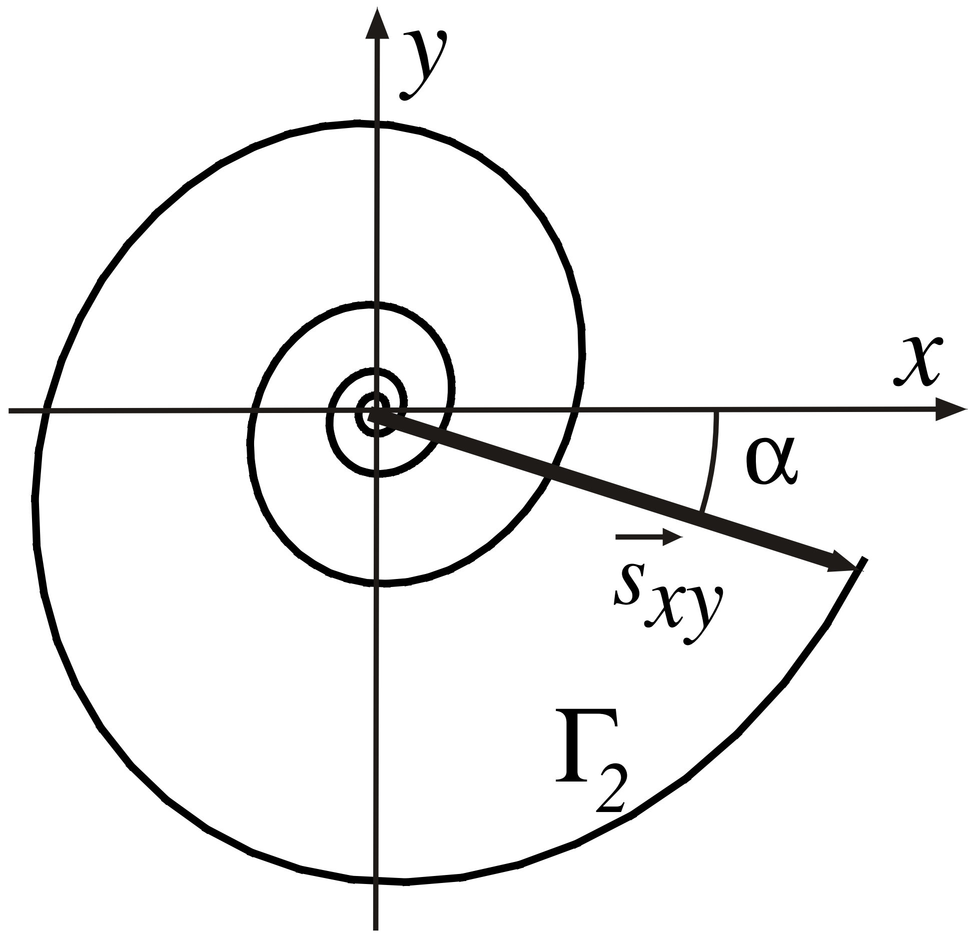
## Populationen

$$s_z(t) = 1 - (1 - s_z(0))e^{-\Gamma_1 t}$$



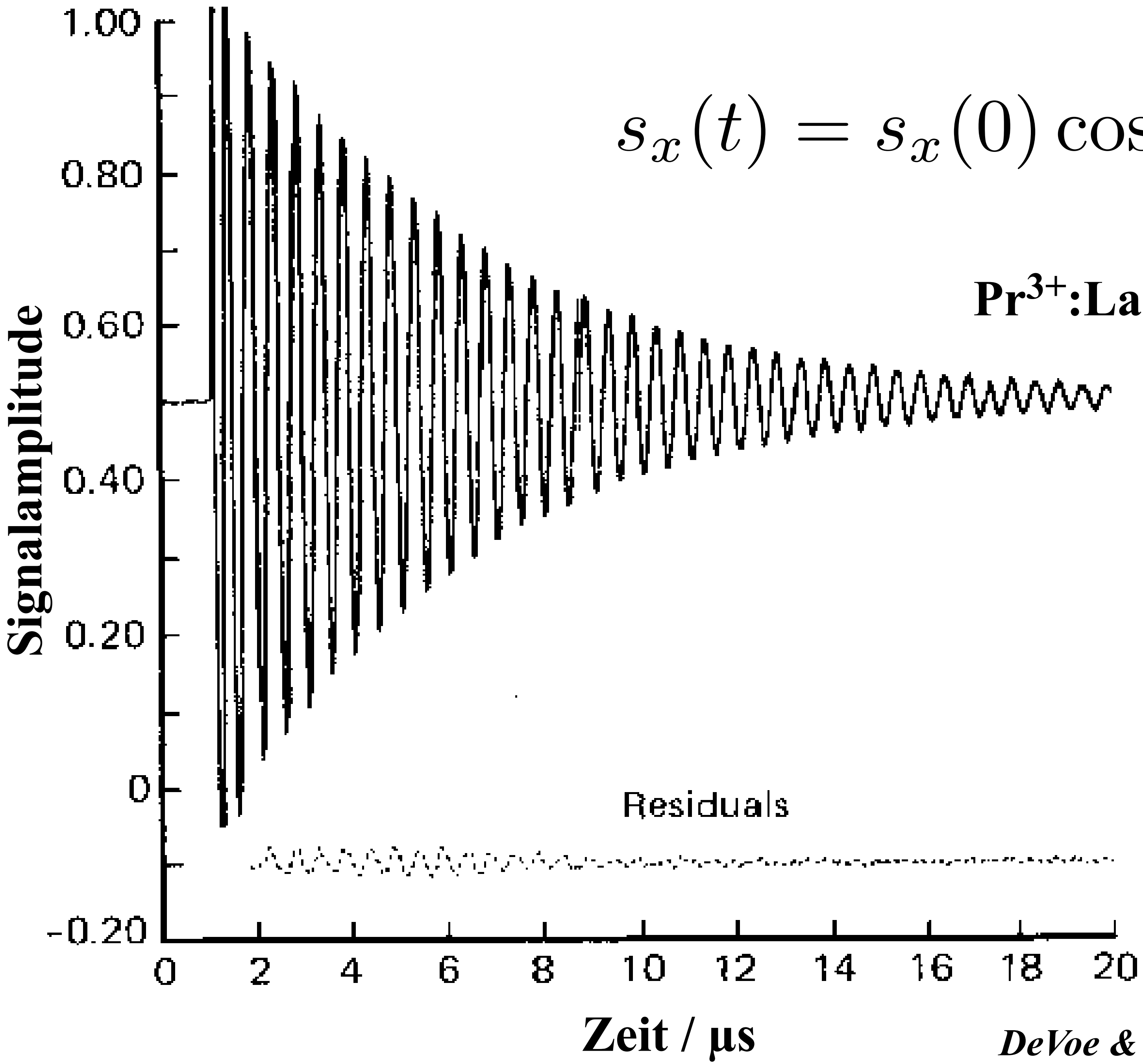

## Kohärenz

$$s_x(t) = (s_x(0)cos(\omega_0 t) + s_y(0)sin(\omega_0 t))e^{-\Gamma_2 t}$$
$$s_y(t) = (s_y(0)cos(\omega_0 t) - s_x(0)sin(\omega_0 t))e^{-\Gamma_2 t}$$



# Freie Präzession

$$s_x(t) = s_x(0)\cos(\omega_0 t)e^{-t/T_2}$$




*DeVoe & Brewer, PRL 50, 1269 (1984).*

# Stationäre Lösung

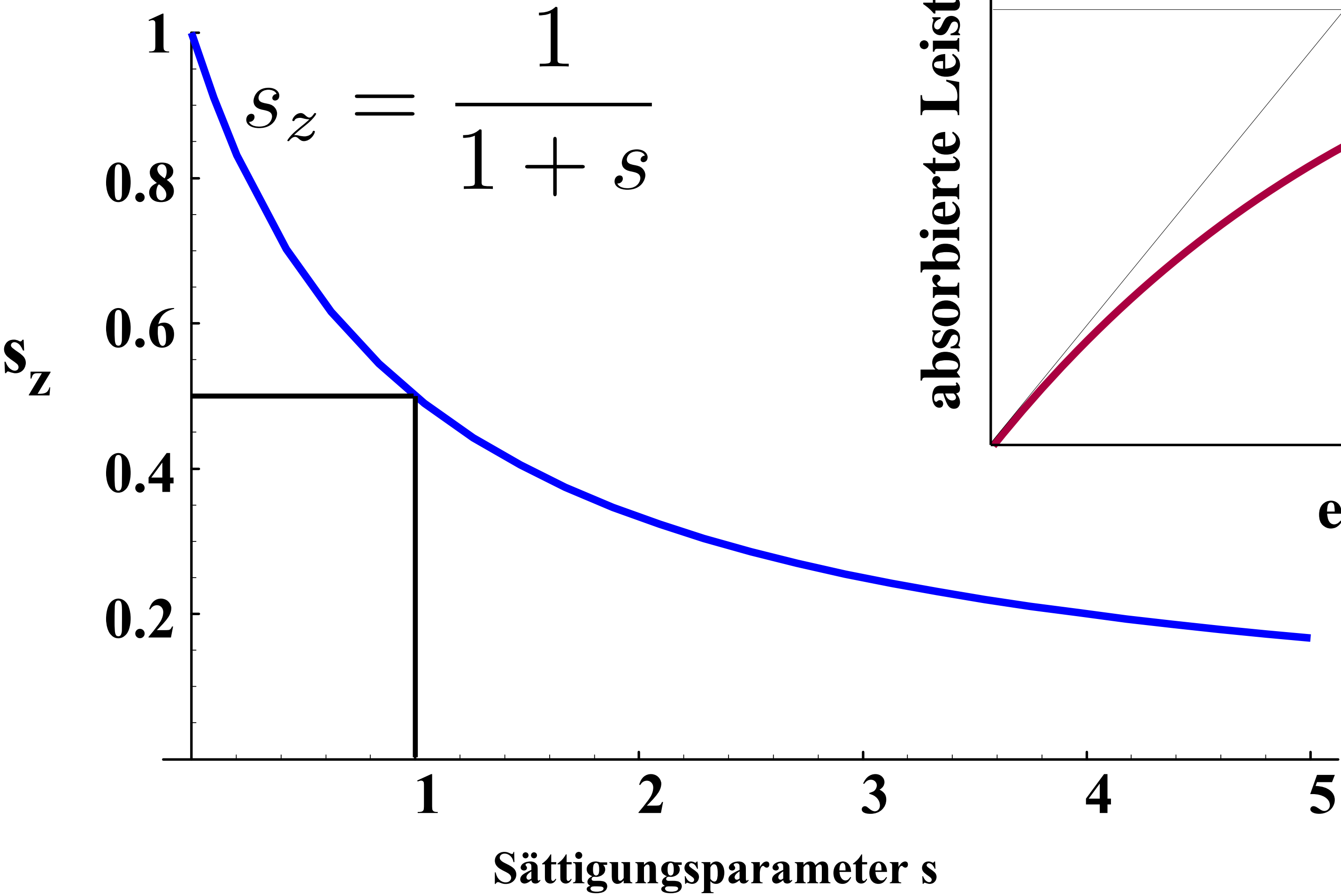
$$\begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix}_\infty = \frac{1}{\Gamma_2^2 + \Delta\omega_0^2 + \omega_x^2 \frac{\Gamma_2}{\Gamma_1}} \begin{pmatrix} \Delta\omega_0 \omega_x \\ \omega_x \Gamma_2 \\ \Gamma_2^2 + \Delta\omega_0^2 \end{pmatrix}$$

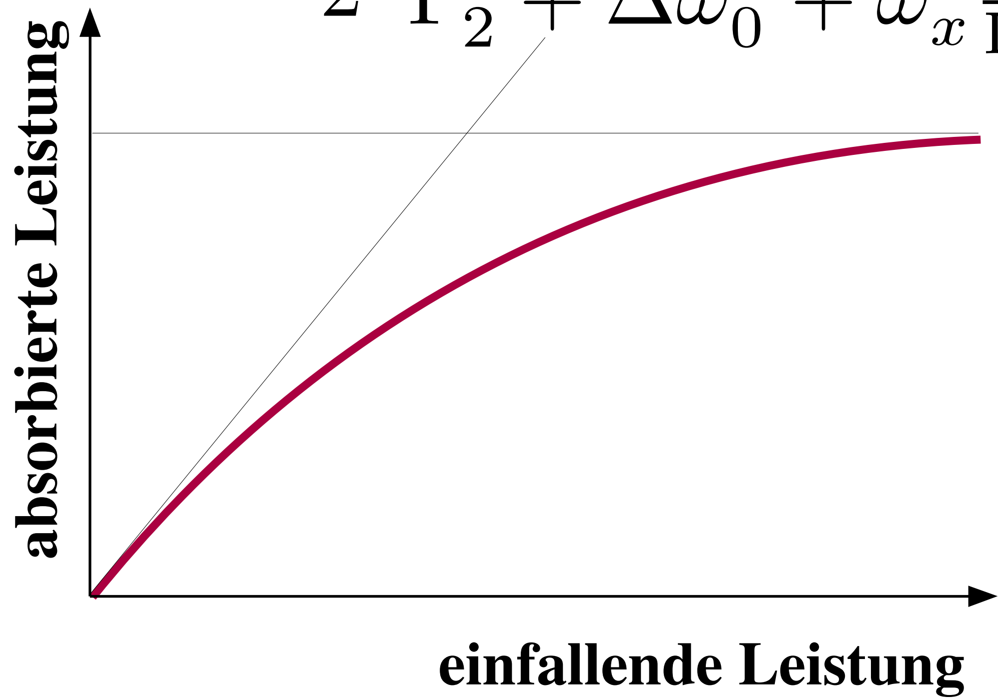
# Sättigung

**Sättigungsparameter**

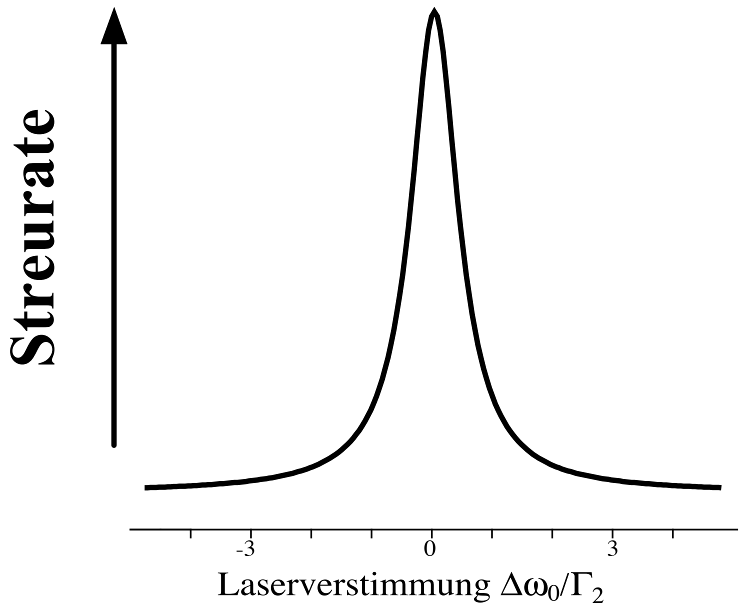
$$s = \frac{2\omega_x^2}{\Gamma_1^2 + 4\Delta\omega_0^2}$$

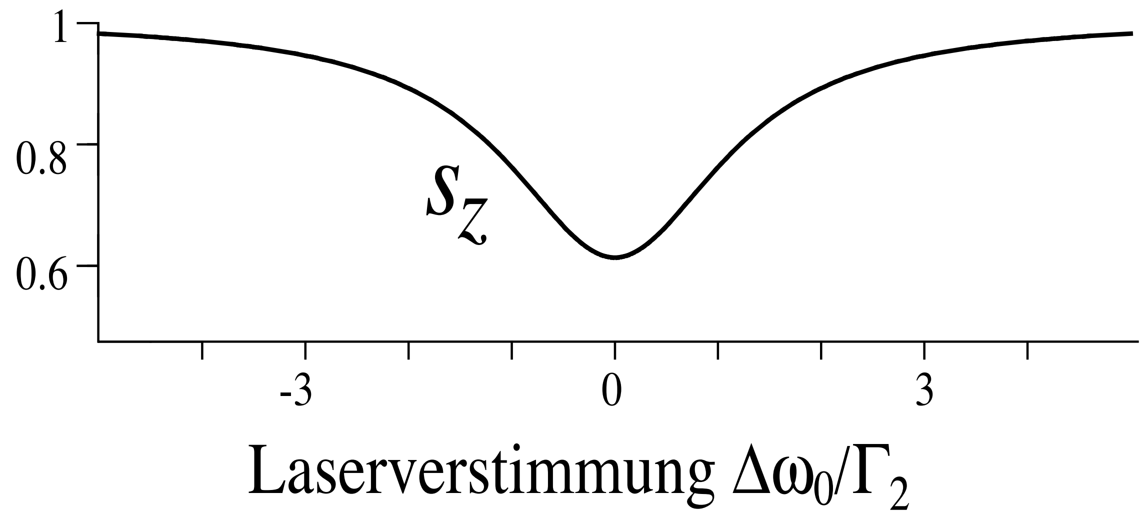
$$r = \frac{\Gamma_2}{2}\frac{\omega_x^2}{\Gamma_2^2 + \Delta\omega_0^2 + \omega_x^2 \frac{\Gamma_2}{\Gamma_1}}$$

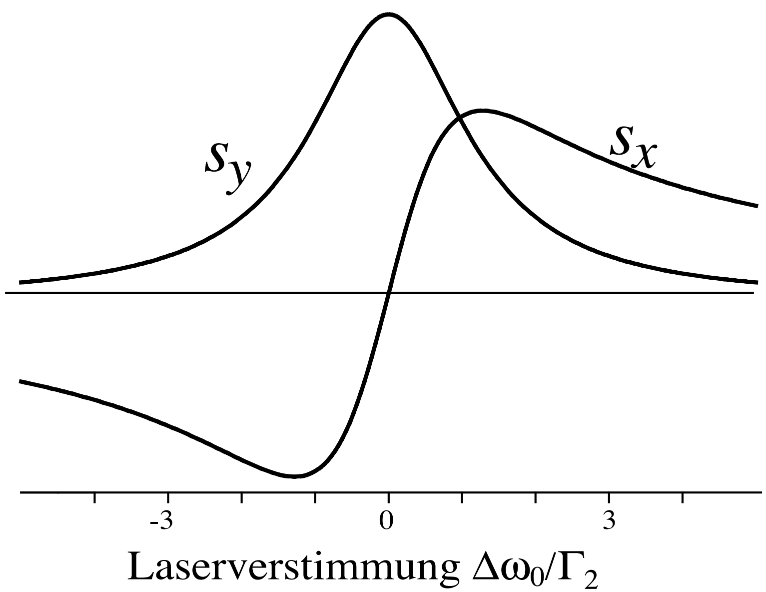


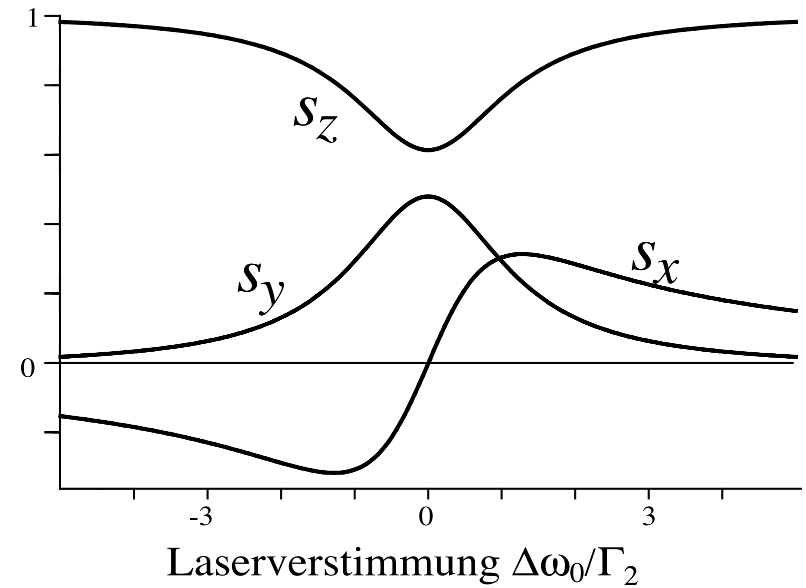



# Verstimmungsabhängigkeit

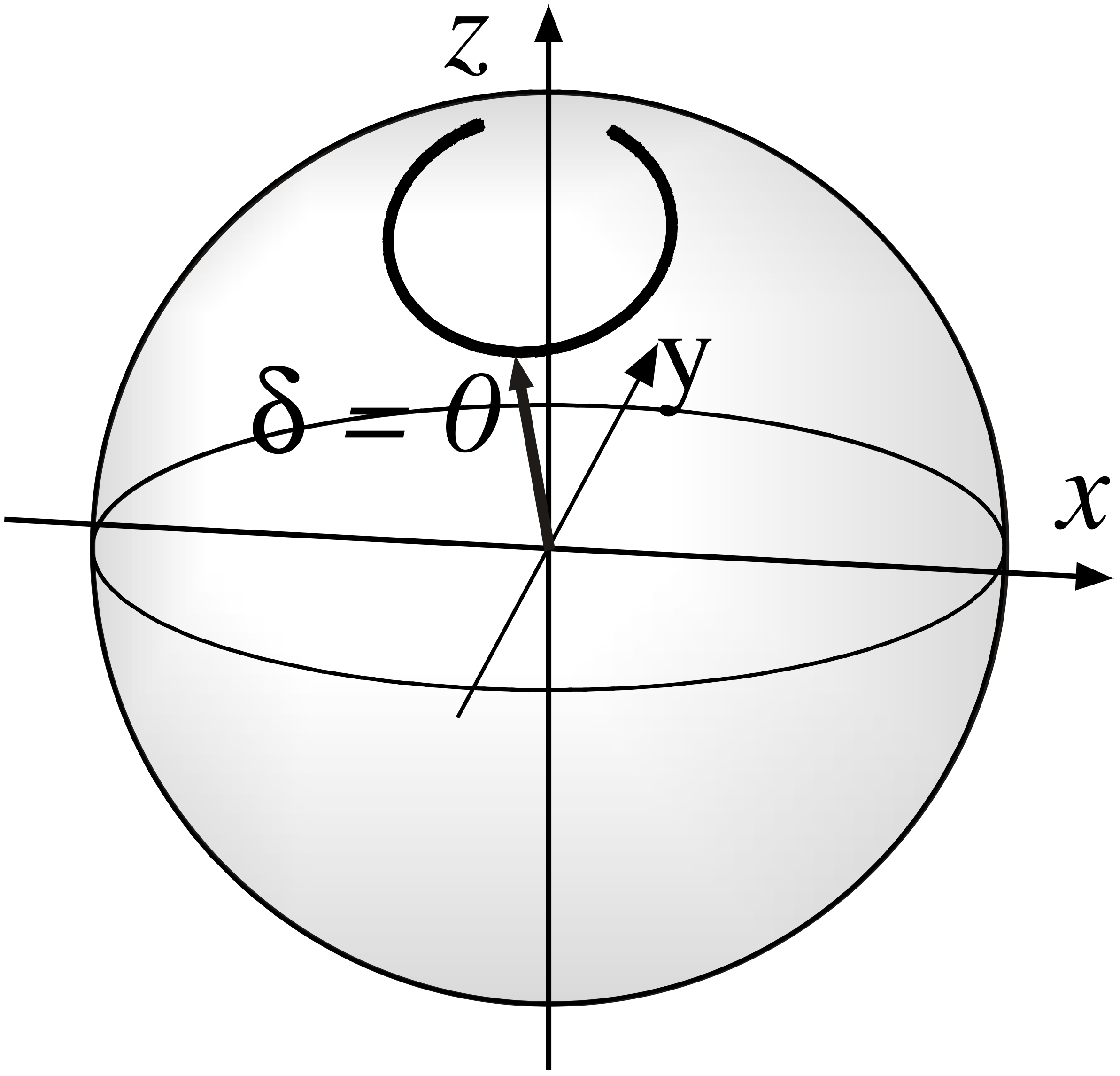


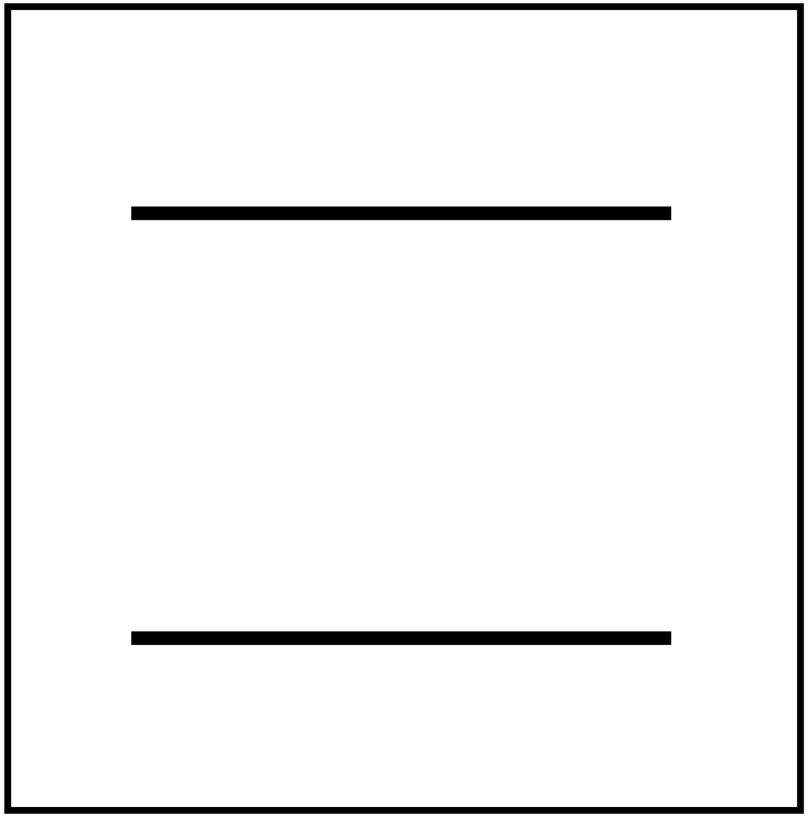



# Verstimmungsabhängigkeit



# Niveaustruktur

**2-Niveausystem**



**Harmonischer Oszillator**