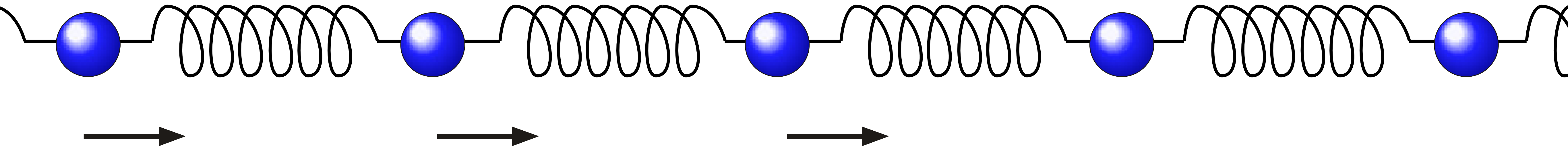
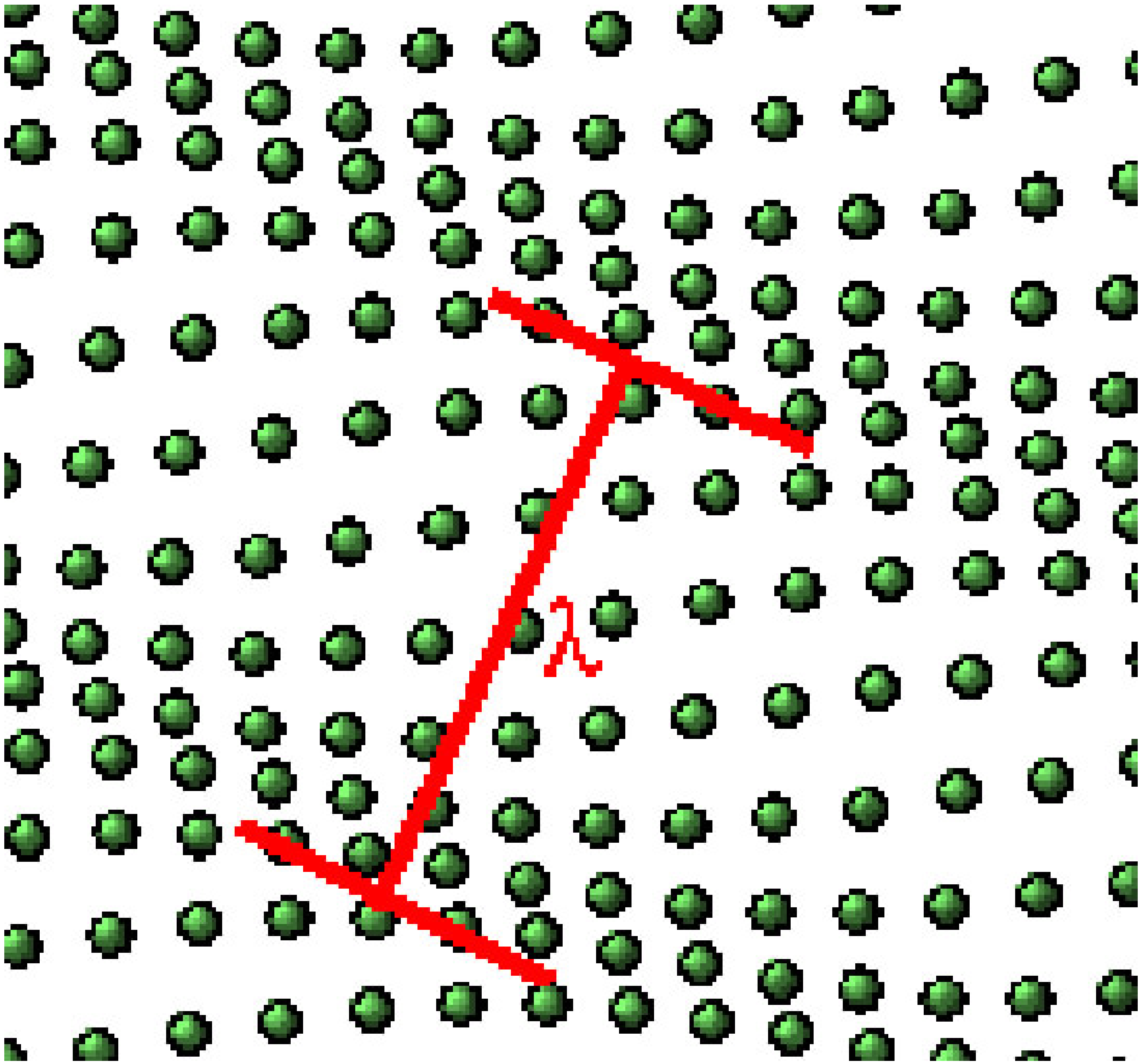
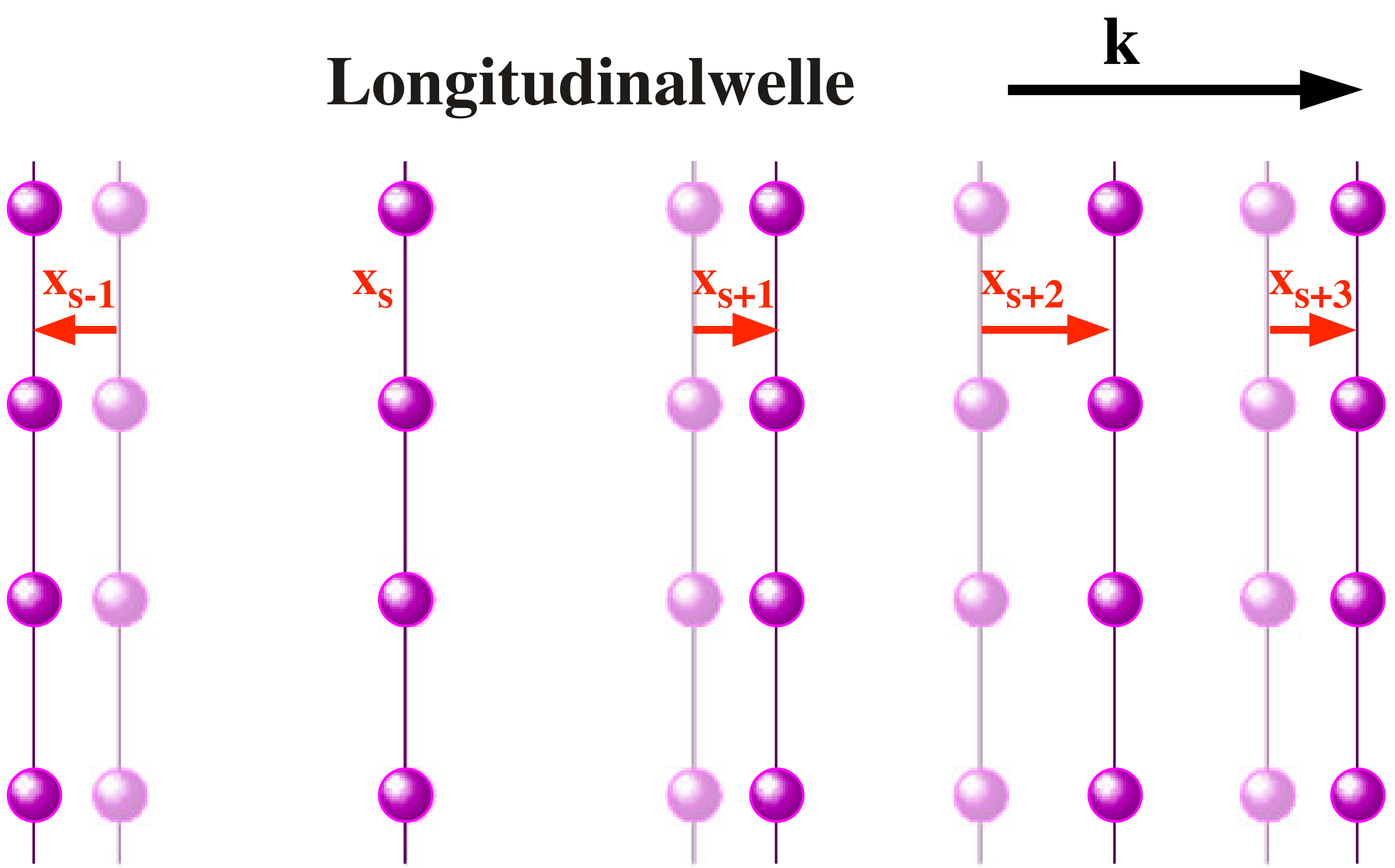
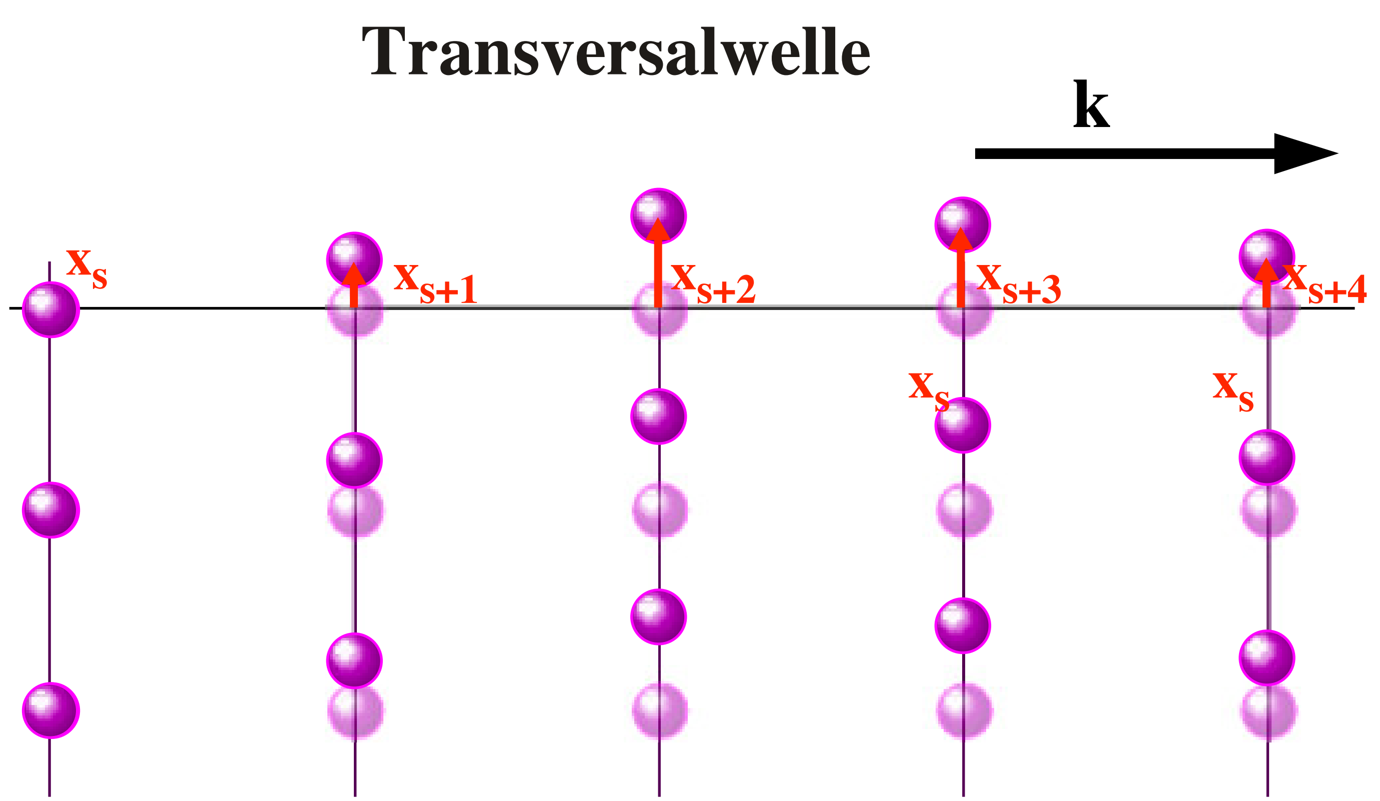
Schwingungen in diskreten Systemen

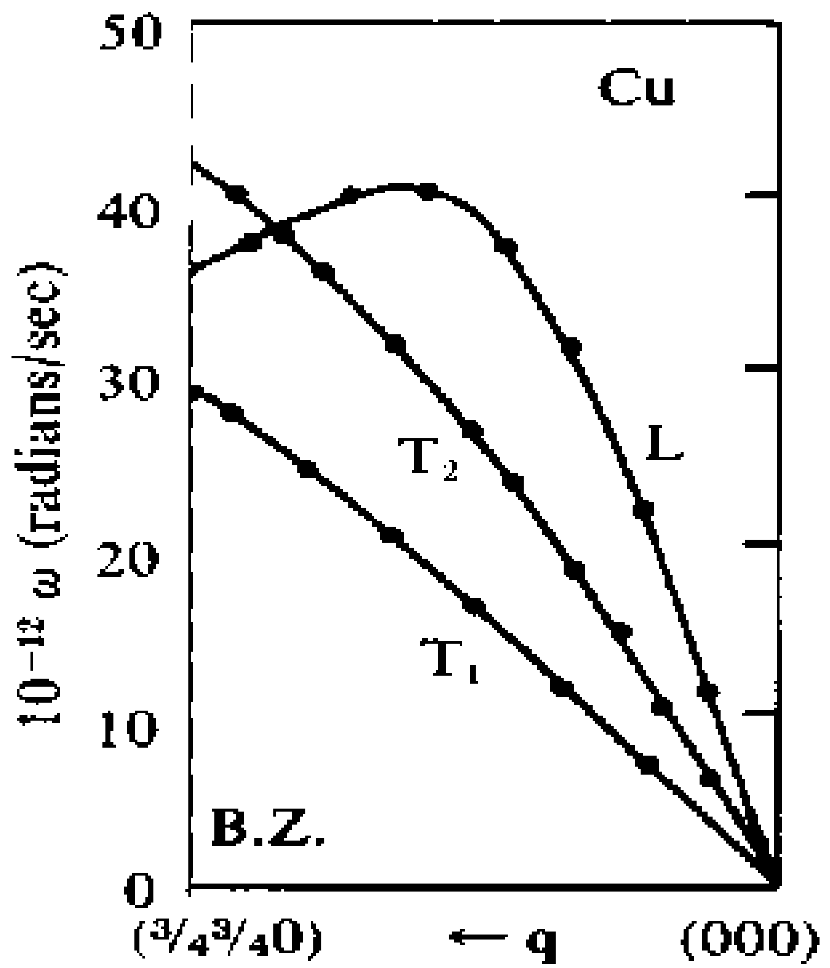




Longitudinal- und Transversalwellen

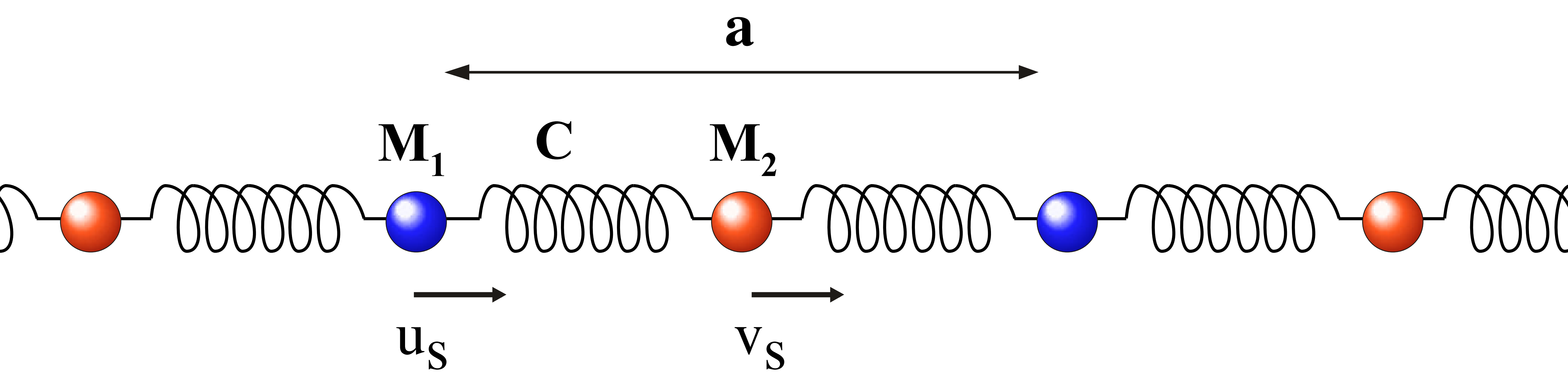






Werkstoff	Elastizitäts-Modul E in GN/m^2	Schub-Modul G in GN/m^2
Eis	9,9	3,7
Blei	17	5,5 bis 7,5
Al (rein)	72	27
Glas	76	33
Gold	81	28
Messing (kaltverf.)	100	36
Kupfer (kaltverf.)	126	47
V2A-Stahl	195	80

2 Atome pro Einheitszelle



Bewegungsgleichung

$$M_1 \ddot{u}_s = C(v_s + v_{s-1} - 2u_s)$$

$$M_2 \ddot{v}_s = C(u_{s+1} + u_s - 2v_s)$$

Lösungsansatz: ebene Welle

$$u_s = U_0\, e^{(iksa-\omega t)} \qquad v_s = V_0\, e^{(ik(s+1/2)a-\omega t)}$$

Lösung der Wellengleichung

Lösungsansatz: ebene Welle mit Wellenvektor k und Frequenz ω:

$$M_1 \ddot{u}_s = C(v_s + v_{s-1} - 2u_s)$$

$$M_2 \ddot{v}_s = C(u_{s+1} + u_s - 2v_s)$$

$$u_s = U_0\, e^{(iksa-\omega t)} \qquad v_s = V_0\, e^{(ik(s+1/2)a-\omega t)}$$

Einsetzen ergibt

$$-M_1 \omega^2 U_0 = 2CV_0 \cos(ka/2) - 2CU_0$$

$$-M_2 \omega^2 V_0 = 2CU_0 \cos(ka/2) - 2CV_0$$

Bedingung für Existenz einer Lösung:

$$\begin{vmatrix} 2C - M_1\omega^2 & -2C\cos(ka/2) \\ -2C\cos(ka/2) & 2C - M_2\omega^2 \end{vmatrix} = 0$$

oder

$$M_1 M_2 \omega^4 - 2C(M_1+M_2)\omega^2 + 4C^2(\sin^2(ka/2)) = 0$$

Somit:

$$\omega^2 = C\left(\frac{1}{M_1} + \frac{1}{M_2}\right) \pm C\sqrt{\left(\frac{1}{M_1} + \frac{1}{M_2}\right)^2 - \frac{4}{M_1 M_2}\sin^2(ka/2)}$$

Dispersionsrelation für 2-atomige Basis

$$\omega^2 = C\left(\frac{1}{M_1}+\frac{1}{M_2}\right) \pm C\sqrt{\left(\frac{1}{M_1}+\frac{1}{M_2}\right)^2 - \frac{4}{M_1 M_2}\sin^2(ka/2)}$$

Näherung für ka << 1

$$\omega^2 = C\left(\frac{1}{M_1}+\frac{1}{M_2}\right) \pm C\left[\left(\frac{1}{M_1}+\frac{1}{M_2}\right) - \frac{k^2a^2}{2(M_1+M_2)}\right]$$

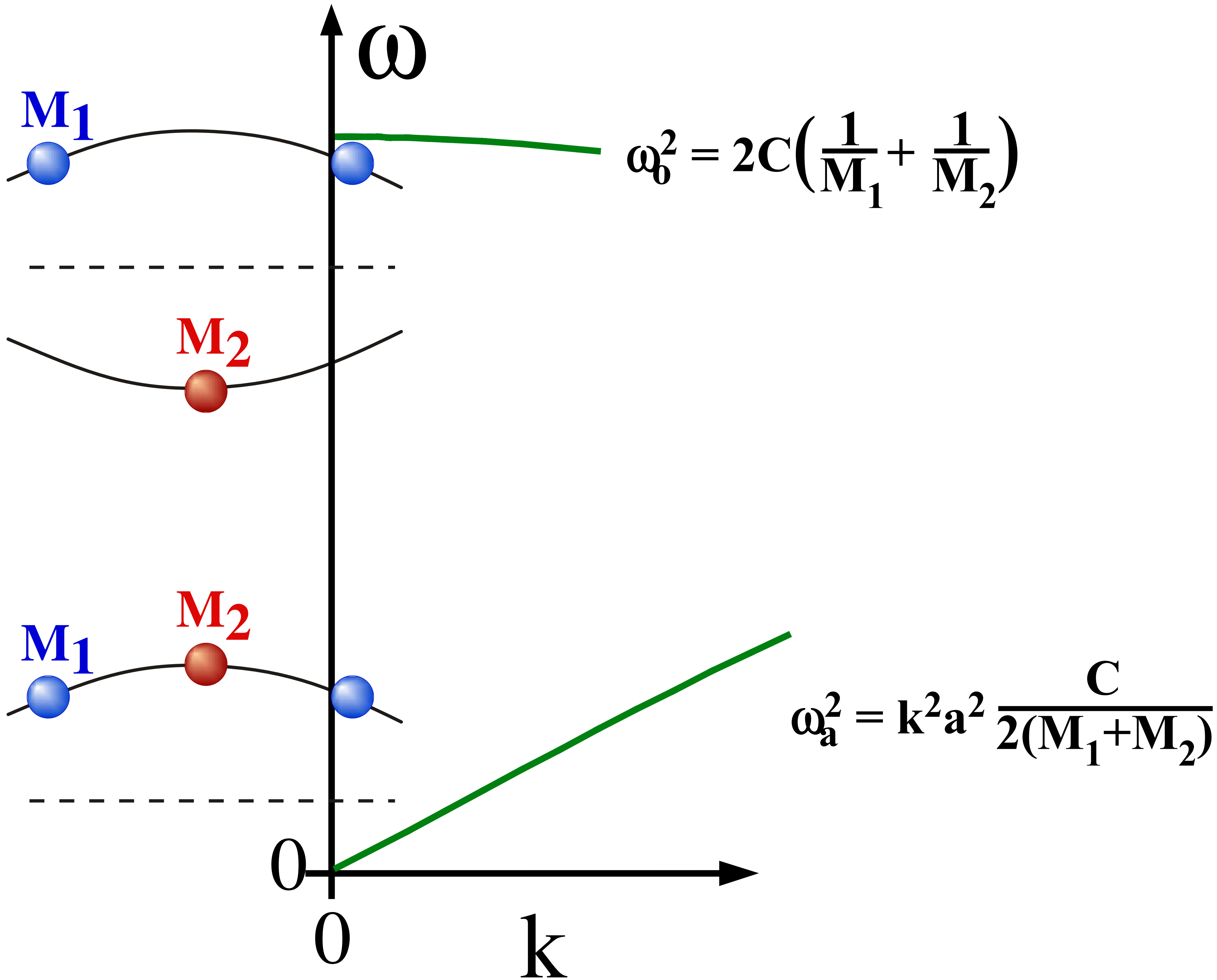
- Vorzeichen

$$\omega_a^2 = k^2a^2\,\frac{C}{2(M_1+M_2)}$$

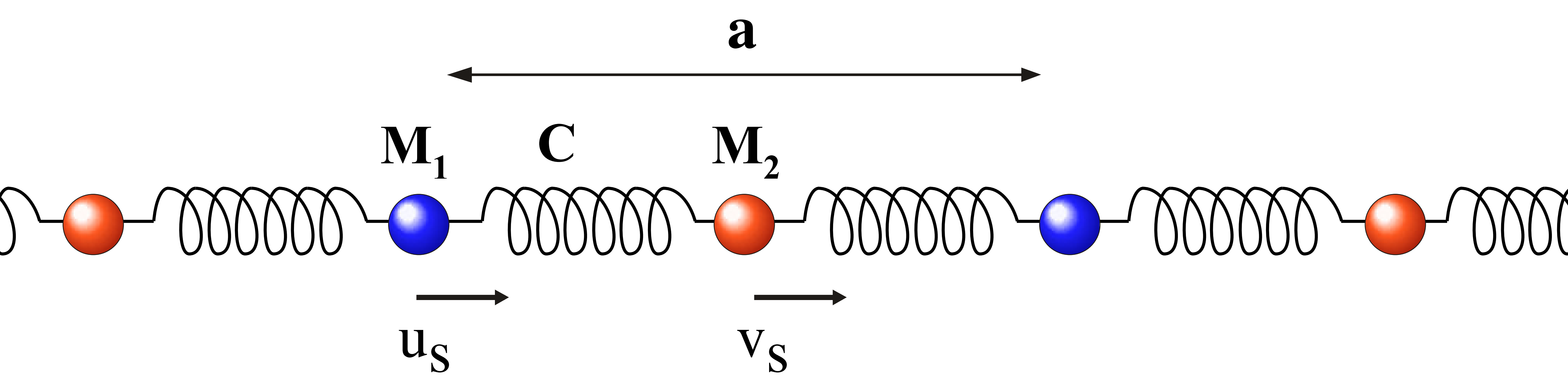
+ Vorzeichen

$$\omega_o^2 = 2C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)$$

Optische und akustische Schwingungen



2 Atome pro Einheitszelle



Bewegungsgleichung

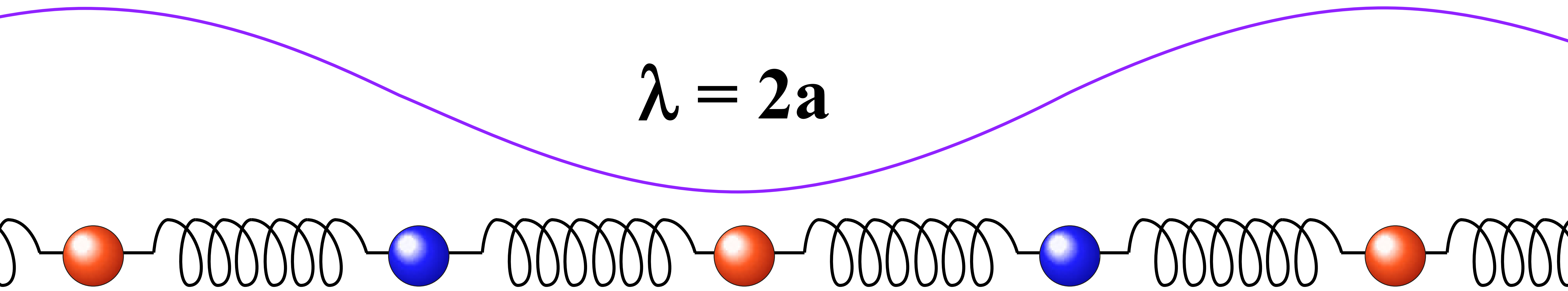
$$-M_1\,\omega^2\,U_0 = 2CV_0\cos(ka/2) - 2CU_0$$

$$-M_2\,\omega^2\,V_0 = 2CU_0\cos(ka/2) - 2CV_0$$

Dispersionsrelation

$$\omega^2 = C\left(\frac{1}{M_1}+\frac{1}{M_2}\right) \pm C\sqrt{\left(\frac{1}{M_1}+\frac{1}{M_2}\right)^2 - \frac{4}{M_1M_2}\sin^2(ka/2)}$$

2 Atome pro Einheitszelle



Bewegungsgleichung

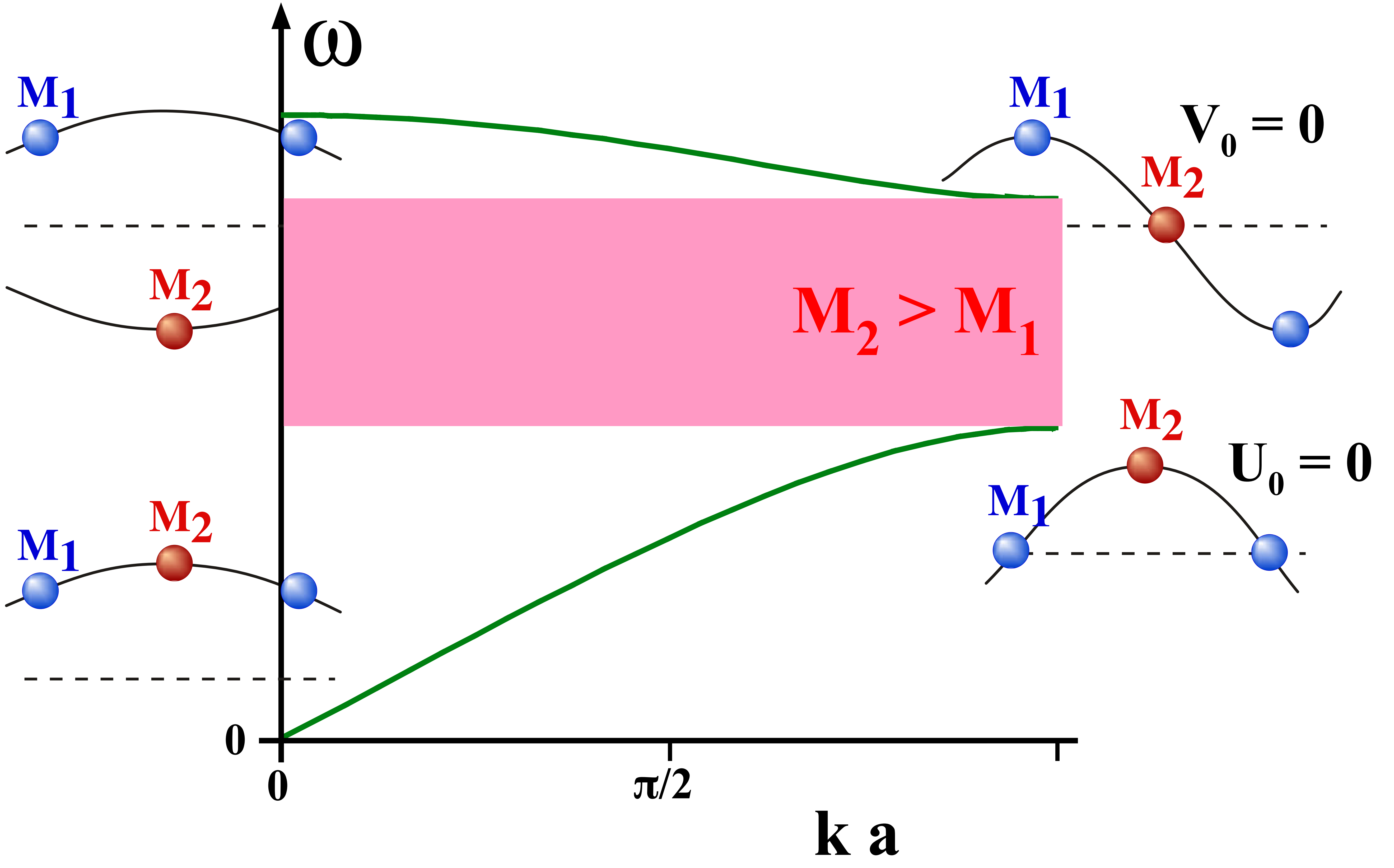
$$- M_1 \omega^2 U_0 = 2CV_0 \cos(ka/2) - 2CU_0$$

$$- M_2 \omega^2 V_0 = 2CU_0 \cos(ka/2) - 2CV_0$$

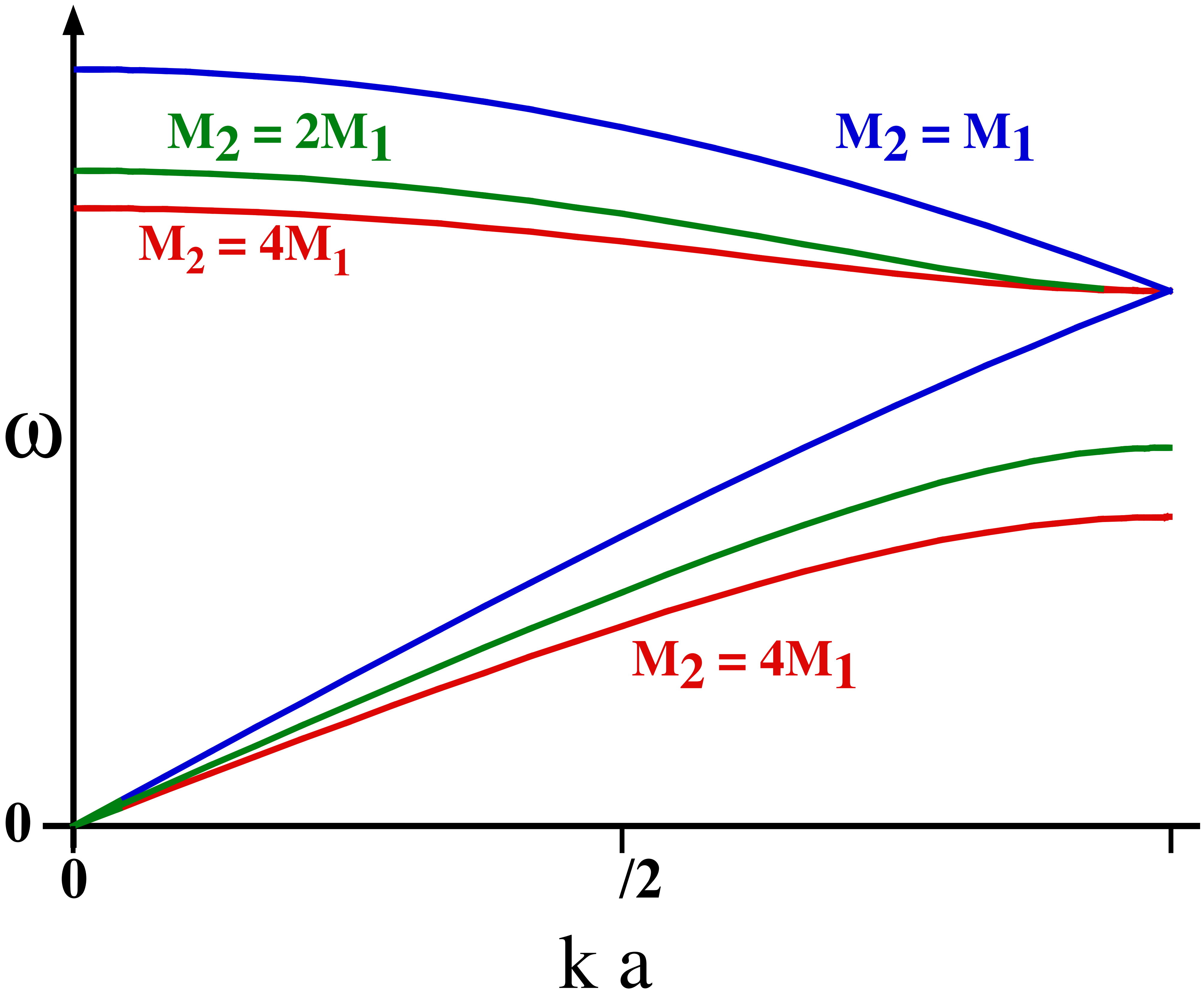
Dispersionsrelation

$$\omega^2 = C\left(\frac{1}{M_1} + \frac{1}{M_2}\right) \pm C\sqrt{\left(\frac{1}{M_1} + \frac{1}{M_2}\right)^2 - \frac{4}{M_1 M_2}\sin^2(ka/2)}$$

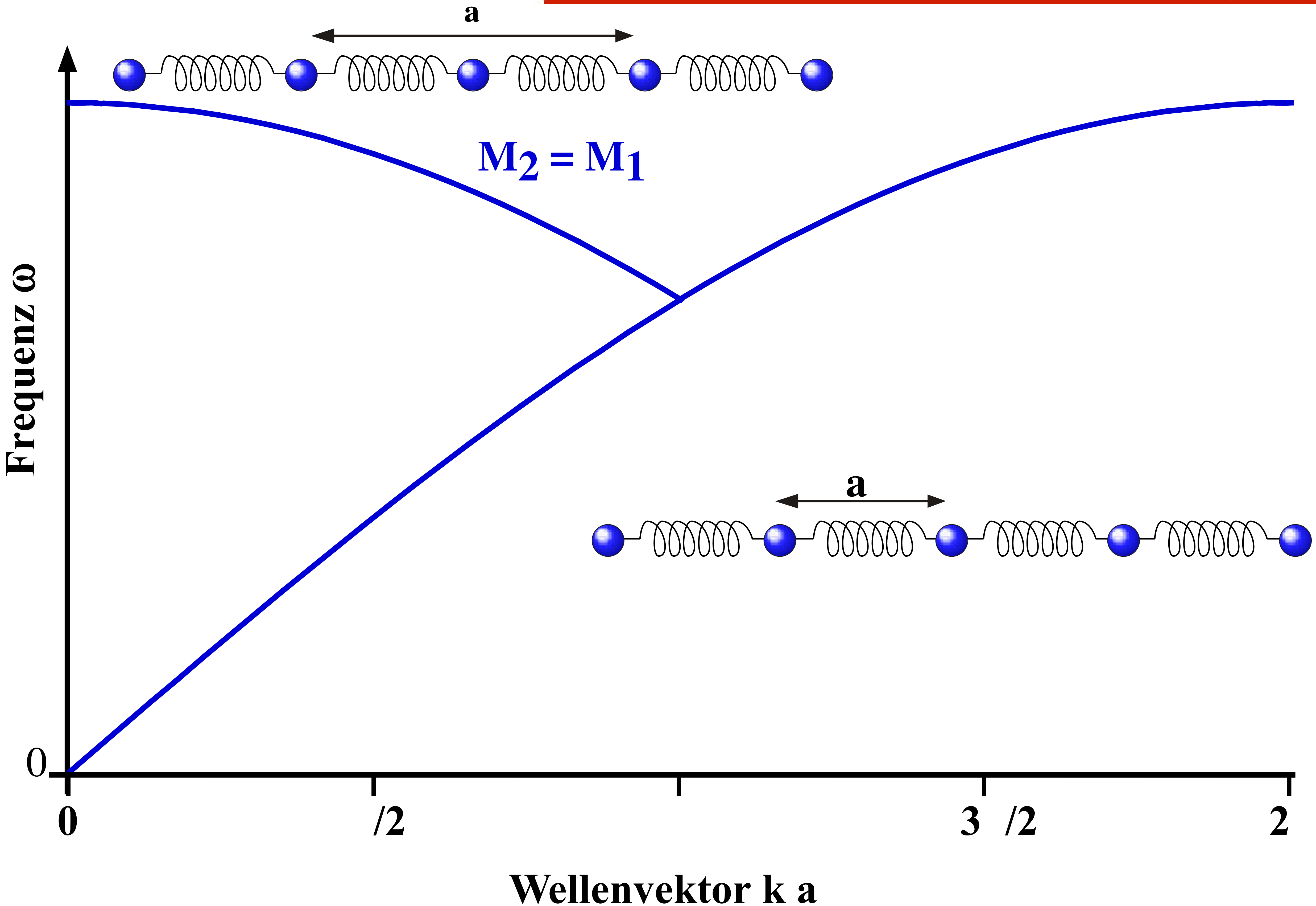
Optische und akustische Schwingungen



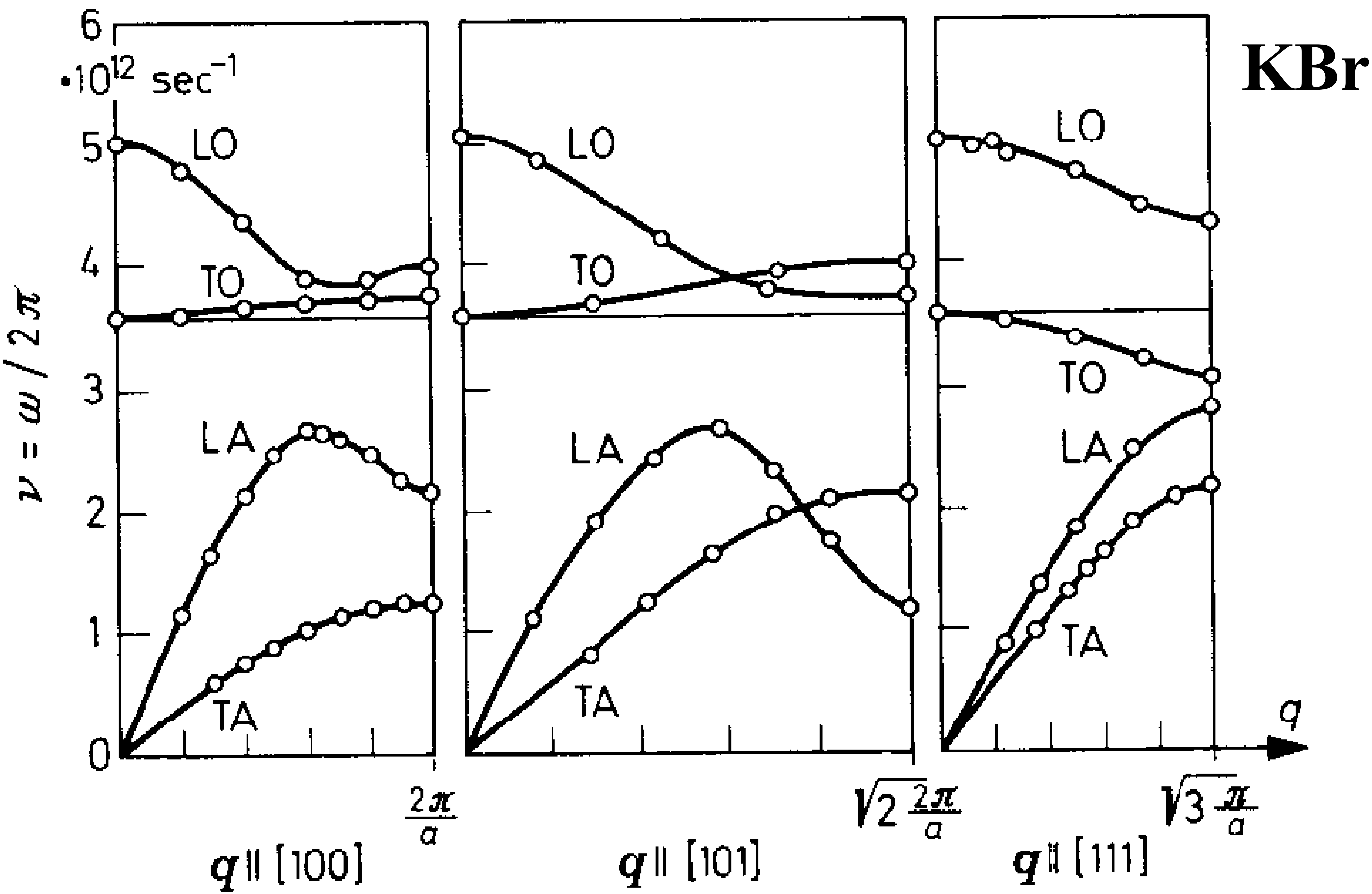
Massenverhältnis



Wahl der Einheitszelle

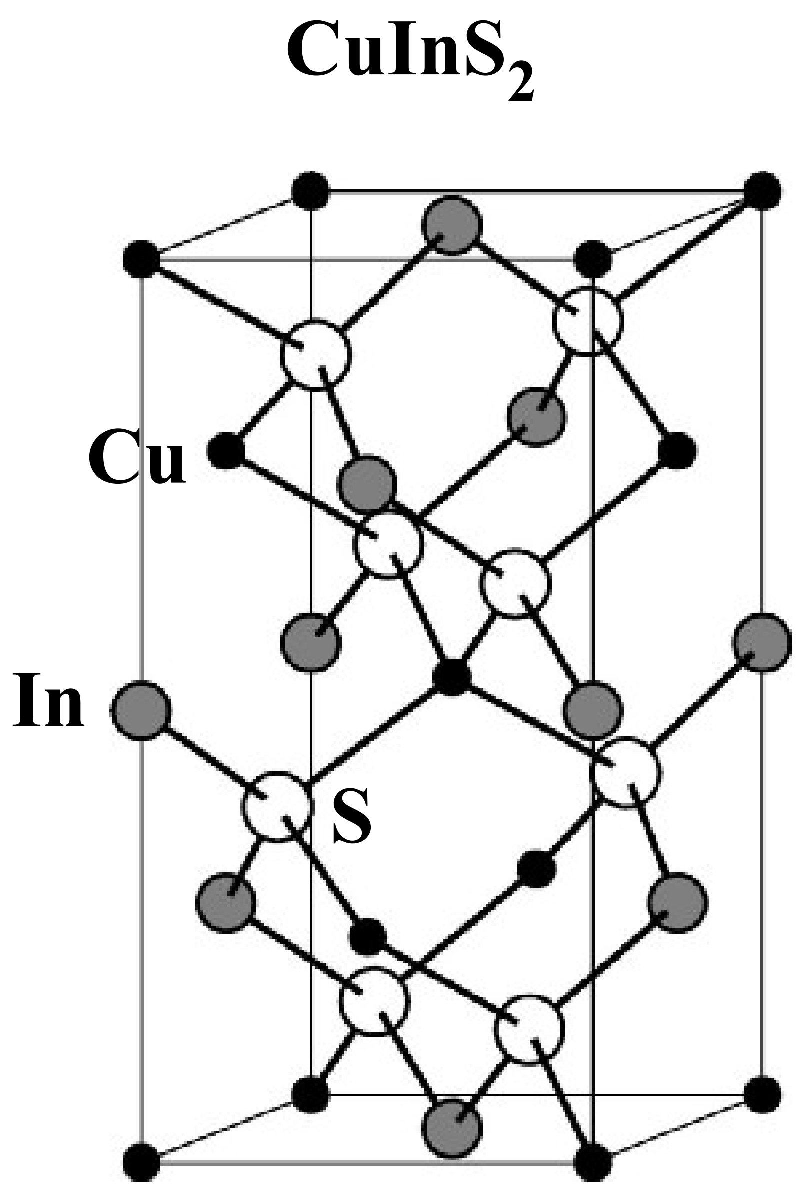


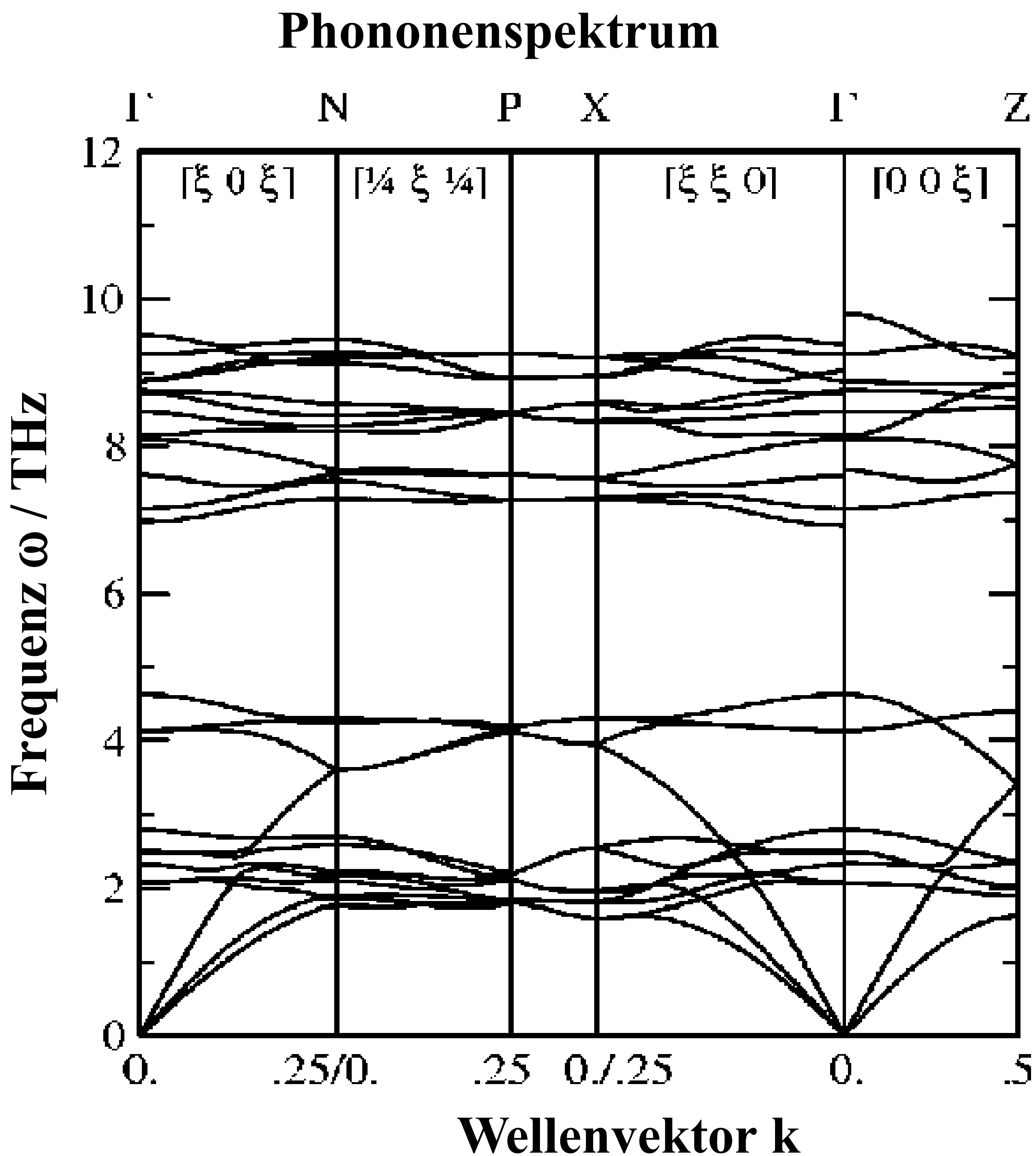
Akustische und optische Phonen



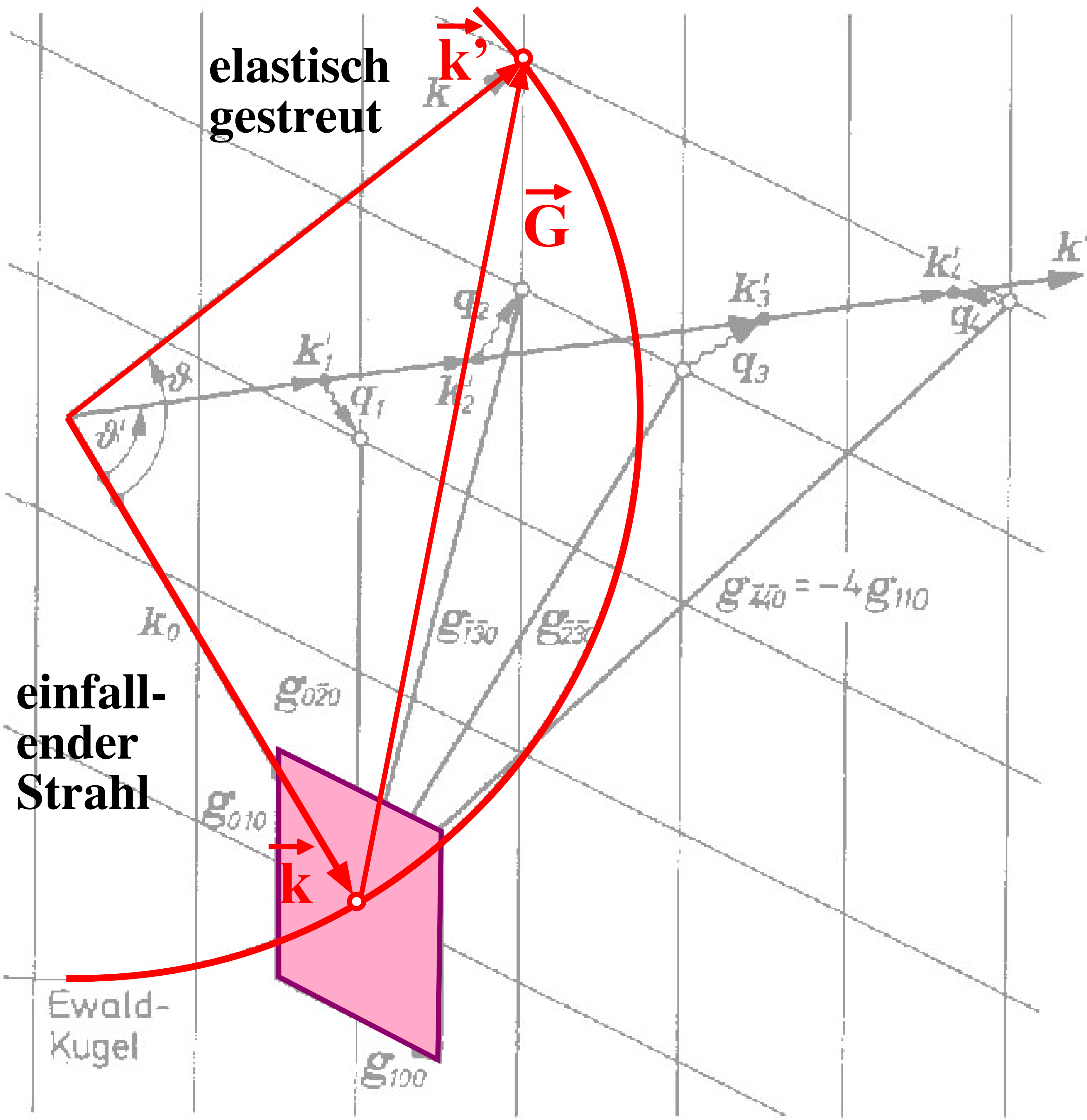
K.H. Hellwege, 'Einführung in die Festkörperphysik', Springer, Berlin (1976).

Mehrere Atome

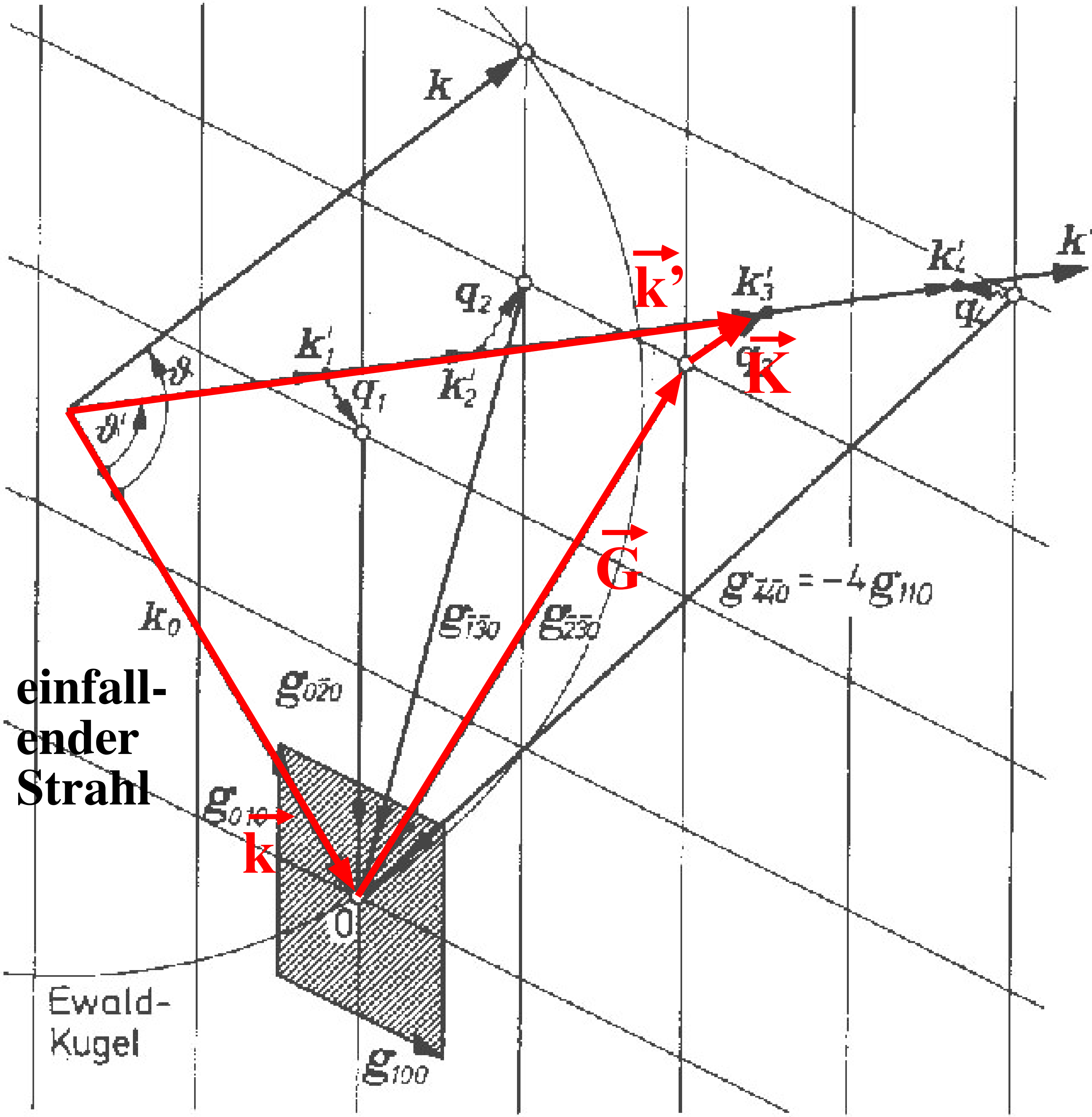




Elastische Streuung



Inelastische Streuung



zum
Detektor

$$\vec{k}' = \vec{k} + \vec{G} \pm \vec{K}$$

Neutronenstreuung

Neutronenspektrometer